CHAPTER I

INTRODUCTION

The fixed-point iteration process for nonlinear operators in Hilbert spaces and Banach spaces including Mann, Halpern and Isikawa iterations process have been studied extensively by many authors to approximate fixed point of various classes of operators in both Hilbert spaces and Banach spaces. In 1952, Mann [1] defined Mann iteration in a matrix formulation. In 1967, Halpern [2] introduced the new innovation iteration process which resemble in Mann's iteration. In 1974, Ishikawa [3] introduced Ishikawa iteration and studied its strong convergence theorem for Lipschitzian pseudo-contractive mappings in Hilbert spaces. On the other hand, construction of fixed points of nonexpansive mappings and finds application in a number of applied areas, in particular, in image recovery and signal processing; see [4, 5].

Variational inequalities introduced by Stampacchia [6] in the early sixties have witnessed explosive growth in theoretical advances and applications across all disciplines of pure and applied sciences (see [6, 7] and the references therein). Analysis of these problems requires a blend of techniques from functional analysis, convex analysis, and numerical analysis. In recent years, variational inequality theory has been extended and generalized in several directions, using new and powerful methods, to study a wide class of unrelated problems in a unified and general framework.

In 1994, equilibrium problems were introduced by Blum and Oettli [8] and by Noor and Oettli [9] as optimization problems and generalizations of variational inequalities. The equilibrium problem theory provides a novel and united treatment of a wide class of problems which arise in finance, economics, ecology, elasticity, transportation, network and optimization. This theory has had a great impact and

influence in the development of several branches of pure and applied sciences.

Motivated and inspired by the above works, we introduce new iterative schemes for finding a common solution of fixed point problems, variational inequality problems and equilibrium problems of some nonlinear mappings and we prove existence theorems of generalized equilibrium problems.

This thesis is divided into 7 chapters. Chapter 1 is an introduction to the research problem. Chapter 2 is dealing with some preliminaries and give some useful results that will be deplicated in later Chapter.

Chapter 3, 4, 5 and 6 are the main results of this research. Precisely, in section 3.1 we introduce two iterative schemes (one implicit and one explicit) for finding a common element of the set of solutions of the generalized equilibrium problems and the set of all common fixed points of a nonexpansive semigroup in a real Hilbert space. In section 3.2, we introduce a general composite algorithm for finding a common element of the set of solutions of an generalized equilibrium problem and the common fixed point set of a finite family of asymptotically nonexpansive mappings in Hilbert spaces.

In section 4.1 we consider a hybrid projection algorithm basing on the shrinking projection method for two families of relatively weak quasi-nonexpansive mappings. In section 4.2 we introduce a hybrid projection algorithm based on the shrinking projection method for two relatively weak nonexpansive mappings.

In section 5.1 we introduce a general iterative method for finding solutions of a general system of variational inclusions with Lipchitzian relaxed cocoercive mappings. In section 5.2 we consider a countable family $\{T_n\}_{n=1}^{\infty}$ of strictly pseudocontractions, a strong convergence of viscosity iteration is shown in order to find a common fixed point of $\{T_n\}_{n=1}^{\infty}$ in either p-uniformly convex Banach space which admits a weakly continuous duality mapping or p-uniformly convex Banach space with uniformly Gâteaux differentiable norm.

In section 6.1 we use the KKM technique, we obtain the existence of so-

lutions for the generalized mixed equilibrium problem in a Banach space. Furthermore, we also introduce a hybrid projection algorithm for finding a common element in the solution set of a generalized mixed equilibrium problem and the common fixed point set of a countable family of nonexpansive mappings. In section 6.2 we first prove the existence of a solution of the generalized equilibrium problem (GEP) by using the KKM mapping in a Banach space setting. We construct a hybrid algorithm for finding a common element in the solution set of a generalized equilibrium problem and the fixed point set of a countable family of nonexpansive mappings in Banach spaces.

The conclusion of research is in Chapter 7.