FUZZY OPTIMIZATION FOR SUPPLY CHAIN PLANNING UNDER UNCERTAIN ENVIRONMENTS

BY

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ABSTRACT

This thesis develops advanced fuzzy optimization models to strengthen resilience in Supply Chain Aggregate Production Planning (SCAPP) by addressing uncertainties inherent in modern supply chains. Utilizing fuzzy logic, the model integrates uncertain parameters such as fluctuating demand, variable supplier reliability, and operational disruptions, providing approaches to managing unpredictability. This innovative framework is designed to tackle multiple conflicting objectives simultaneously, including cost minimization, resource allocation optimization, and risk mitigation, thereby enabling decision-makers to achieve balanced and efficient SCAPP. This advancement marks a departure from conventional approaches, which frequently focus on static assumptions and single-objective optimization.

By systematically quantifying uncertainties, the model ensures that supply chain strategies remain robust against external shocks and internal variabilities. Its ability to provide adaptive solutions to unexpected scenarios demonstrates its relevance in industries where supply chains face frequent disruptions due to market volatility, global uncertainties, and rapid technological changes. The empirical results confirm that the proposed models enhance operational efficiency, reduce the risk of cost fluctuations,

and improve resource utilization, making it a valuable tool for businesses aiming to maintain stability in volatile environments. By bridging the gap between theoretical advancements and practical applications, this study contributes to both scholarly discourse and industry practice, emphasizing the importance of adaptable and scalable solutions in dynamic supply chain environments.

The findings of this thesis go beyond theoretical advancements, offering practical insights that empower supply chain managers to make more informed and effective decisions. By addressing real-world complexities, the model demonstrates its versatility and applicability across various industries, serving as a crucial tool for organizations aiming to achieve both operational efficiency and long-term sustainability. Additionally, this research lays a strong foundation for future studies, encouraging the exploration of more advanced fuzzy optimization models and further integration of risk mitigation strategies into SCAPP frameworks.

Keywords: Supply Chain Management, Supply Chain Aggregate Production Planning, Multi-Criteria Decision-Making, Conflicting Objectives, Uncertainty, Risk of Uncertainty, Risk Assessment, Asymmetrical Triangular Distribution, Fuzzy Logic, Fuzzy Optimization, Fuzzy Linear Programming, Proportional Fairness Model, Robustness Model, Pareto Optimal Solution

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LIST OF SYMBOLS/ABBREVIATIONS

Symbols/Abbreviations	**Terms**
AHP	Analytic Hierarchy Process
ANP	Analytic Network Process
APP	Aggregate Production Planning
AUGMECON	Augmented Epsilon Constrained
CCP	Chance-Constrained Programming
CV	Coefficient of Variation
CvaR	Conditional Value-at-Risk
DMs	Decision Makers
ERP	Enterprise Resource Planning
EV	Expected Value
FLP	Fuzzy Linear Programming
FR	Fuzzy Ranking
FS	Fuzzy Set
GA	Genetic Algorithms
HWRP	Hard Worst Robust Programming
IFLP	Intuitionistic Fuzzy Linear Programming
IoT	Internet of Things
LHS	Left-Hand Side
LP	Linear Programming
MCVaRG	Mean-Conditional Value at Risk Gap
MOFLP	Multiple Objective Fuzzy Linear Programming
MOLP	Multiple-Objective Linear Programming
MRP	Material Requirement Planning
NLP	Non-Linear Programming

NIS	Negative Ideal Solution
PET	Polyethylene Terephthalate
PF	Proportional Fairness
PIS	Positive Ideal Solution
RHS	Right-Hand Side
RP	Robust Programming
RRP	Realistic Robust Programming
SC	Supply Chain
SCAPP	Supply Chain Aggregate Production Planning
SCPP	Supply Chain Production Planning
SD	Standard Deviation
SOLP	Single-Objective Linear Programming
SWRP	Soft Worst Robust Programming
TFNs	Triangular Fuzzy Numbers
TIFNs	Triangular Intuitionistic Fuzzy Numbers
TOPSIS	Technique for Order of Preference by Similarity to Ideal Solution
TrFNs	Trapezoidal Fuzzy Numbers
VaR	Value-at-Risk
WA	Weighted Average
ZM	Zimmermann

CHAPTER 1
INTRODUCTION

This chapter provides a thorough exploration of the research context, objectives, and contributions. It begins with a detailed research background, outlining the significance of the research within the broader field of production planning and supply chain management. The problem statement identifies the key challenges addressed by the research, specifically focusing on the complexities of managing supply chains under uncertainty. This chapter further highlights the research contributions, emphasizing the novel methodologies and frameworks proposed to improve the resilience and efficiency of supply chain operations. Finally, the thesis overview presents a roadmap of the subsequent chapters, offering a clear structure for the reader to follow as the study progresses from foundational concepts to advanced optimization techniques and practical applications.

1.1 Research Background

This research centers on the growing importance of Supply Chain Aggregate Production Planning (SCAPP) within the context of an increasingly complex and competitive marketplace. As businesses face increasing pressure from global competition, rapidly changing market conditions, and unpredictable demand patterns, SCAPP has become an essential tool for optimizing supply chain operations (Reyes et al. (2021); Ravindran et al. (2023)). Consequently, effective supply chain management presents significant challenges, primarily resulting from inherent uncertainties originating from diverse sources such as supply disruptions, fluctuating demand, and changing economic conditions.

The complexities of modern supply chains are further compounded by the need to handle imprecise data that conventional deterministic models often fail to capture. This influences supply chain decisions, limiting their practical applicability. As such, there has been a growing interest in incorporating uncertainty into SCAPP through advanced methodologies that provide more flexible and adaptive solutions. This research focuses on utilizing fuzzy set theory and optimization techniques to address these challenges, offering a robust framework for decision-making in uncertain environments. Fuzzy set theory, which addresses the representation of uncertain and vagueness information, has demonstrated its effectiveness as a robust tool for managing uncertainty in supply chain planning. This has been demonstrated by numerous researchers over the years, including pioneers such as Lotfi A. Zadeh, who originally introduced the concept of fuzzy sets in 1965, and later scholars like Tanaka et al. (1974), who pioneered the application of fuzzy set theory in linear programming, and more recently such as Tuan et al. (2021) and Mohamed et al. (2023), who have shown its relevance in modeling and optimizing uncertain parameters in supply chain and production planning environments. Their work has contributed significantly to establishing fuzzy set theory as a robust framework for supporting decision-making under uncertainty. Specifically, fuzzy numbers, such as triangular and intuitionistic triangular fuzzy numbers, allow for the representation of uncertain parameters, enabling decision-makers to model supply chain variables more accurately. By integrating fuzzy logic with optimization models, this research aims to enhance the reliability and operational efficiency of SCAPP under conditions of uncertainty and conflicting objectives by developing advanced fuzzy optimization models.

Moreover, this research significantly contributes to the academic understanding of how advanced mathematical tools can be utilized to address the practical difficulties faced by modern supply chains. This research also emphasizes the importance of bridging the gap between theoretical advancements in optimization and their practical applications. By doing so, it equips supply chain managers with the necessary insights and tools to sustain operations even under adverse conditions. In essence, this research offers new pathways for enhancing resilience, adaptability, and sustainability in supply chain management, providing organizations with the strategies they need to not only survive but thrive in an increasingly unpredictable global marketplace.

1.2 Problem Statement

This research also emphasizes the importance of bridging the gap between theoretical advancements in optimization and their practical applications. By doing so, it equips supply chain managers with the necessary insights and tools to sustain operations even under adverse conditions. In essence, this research offers new pathways for enhancing resilience and adaptability in supply chain management, providing organizations with the strategies they need to not only survive but thrive in an increasingly unpredictable global marketplace.

Uncertainty in SCAPP arises from numerous unpredictable factors, including supplier delays, economic fluctuations, geopolitical tensions, and global crises. These factors further complicate the already difficult task of aligning production capacities with demand forecasts, leading to suboptimal resource utilization and increased risk exposure. Moreover, the presence of conflicting objectives adds a layer of complexity to the decision-making process. conventional planning methods often lack the flexibility and adaptability required to balance these competing demands and uncertainties effectively. These shortcomings significantly obstruct the ability of supply chain managers to make informed decisions and maintain operational continuity in the face of unforeseen events. Without the adoption of advanced tools and models that can better address the complexities of modern supply chains, organizations risk facing inefficiencies and disruptions that could threaten their long-term sustainability.

To overcome these limitations, innovative approaches are urgently needed that integrate uncertainty and conflicting objectives into the planning process in a more comprehensive and effective manner. There is a clear need for methodologies that not only account for the inherent uncertainties in supply chains but also provide flexible, adaptive solutions that allow businesses to respond swiftly to dynamic market conditions. Therefore, this research seeks to fill these critical gaps by developing advanced fuzzy optimization models tailored for SCAPP. These models aim to enhance decision-making in environments characterized by uncertainty and conflicting objectives. By utilizing fuzzy logic, these models will enable supply chain managers to better quantify and incorporate uncertainties into the planning process, leading to more reliable, flexible, and adaptive production plans. Through this research, it is anticipated that organizations will be better equipped to foster resilience and improve responsiveness in their supply chain operations.

1.3 Research Contributions

The potential research contributions are identified as follows:

1. **Development of Advanced Fuzzy Optimization Models for SCAPP**

   This research contributes to supply chain management field by developing advanced fuzzy optimization models tailored specifically for SCAPP. These models integrate uncertainty and conflicting objectives, providing supply chain managers with more reliable and adaptive decision-making tools under dynamic market conditions.

2. **Improvement of Supply Chain Resilience and Flexibility**

   This research enhances resilience of supply chain by providing models that can quickly adapt to sudden market changes, external crises, and unforeseen events. By focusing on flexibility, this research empowers organizations to respond to supply chain volatility and shifting market conditions more effectively, improving overall operational continuity.

3. **Introducing Downside Risk Management to SCAPP**

   This study introduces Mean-Conditional Value at Risk Gap (MCVaRG) as a novel downside risk measure to capture and minimize the risk of uncertainty in decision-making under ambiguity. Unlike existing risk measures, MCVaRG focuses on the gap between expected outcomes and extreme losses in the lower tail of the distribution, offering a more sensitive and targeted assessment of downside risk. This is the first study to apply MCVaRG within an optimization framework for SCAPP, providing a unique approach that enhances both the reliability and robustness of decisions under uncertainty.

4. **Bridging the Gap Between Theory and Practice in SCAPP**

   This research addresses the gap between theoretical advancements in fuzzy optimization and their practical applications. By developing models that are both theoretically sound and practically applicable, this research offers actionable insights and tools that supply chain managers can directly implement in real-world settings.

1.4 Thesis Overview

Following this introductory chapter, the remaining chapters of the thesis are organized as outlined below:

- **Chapter 2: Review of Literature**

  This chapter reviews key literature on supply chain management, uncertainty, fuzzy set theory, and optimization techniques. It begins with an overview of supply chain fundamentals and Supply Chain Aggregate Production Planning (SCAPP) as a strategy for balancing supply and demand. The discussion then examines uncertainty, its associated risks, and the challenges of conflicting objectives in decision-making. To address these issues, the chapter introduces Fuzzy Set Theory, including Fuzzy Numbers and Skewness Degree, as tools for modeling imprecise data. It also explores Fuzzy Mathematical Models, Defuzzification Approaches, and Pareto Optimal Solutions for optimizing trade-offs in multi-objective decision-making. Finally, Risk Measurement techniques are reviewed for assessing and mitigating potential negative outcomes. This literature review establishes the theoretical foundation for managing uncertainty in supply chain optimization.

- **Chapter 3: Research Methodologies and Case Studies**

  This chapter presents the various research methodologies employed to address optimization problems under uncertainty, particularly focusing on fuzzy optimization approaches. It begins with an exploration of the conventional specific fuzzy optimization approach, laying the groundwork for understanding the foundational techniques used in modeling uncertainty and imprecision. This chapter then progresses to introduce a five-phase hybrid fuzzy optimization approach, combining elements of multiple methodologies to enhance decision-making processes. Following this, a unified fairness and robustness fuzzy optimization approach is discussed, providing a structured framework for tackling unfair and sensitive optimization challenges. Then, this chapter presents a downside risk mitigation approach, providing a way to handle the risk of uncertainty. Each of these methodologies contributes to the advancement of optimization practices, particularly in environments characterized by uncertainty and vagueness. Finally, this chapter presents an introduction and outlines the contributions of three case studies, each examining a distinct methodological approach to Supply Chain Aggregate Production Planning (SCAPP) under conditions of uncertainty.

- **Chapter 4: Results**

  This chapter presents a comprehensive analysis of three case studies that explore distinct methodological approaches to Supply Chain Aggregate Production Planning (SCAPP) under uncertainty. Each case study introduces a unique framework designed to optimize production planning while addressing critical challenges such as cost efficiency, fairness, robustness, and risk mitigation. Case 1 introduces a five-phase hybrid fuzzy optimization approach that integrates multiple optimization techniques to enhance decision-making in SCAPP. Case 2 proposes a unified fairness and robustness fuzzy optimization approach, ensuring equitable resource distribution while maintaining resilience against uncertainties. Case 3 focuses on mitigating downside risk by incorporating advanced risk measurement techniques to minimize potential financial losses arising from fluctuations in supply chain operations. The structure of each case study includes a detailed formulation of the mathematical model, a description of the problem, an analysis of the results, and a discussion of key findings. The comparative insights drawn from these cases provide a holistic understanding of how various optimization strategies can be employed to improve SCAPP under uncertain circumstances.

- **Chapter 5: Discussion and Conclusions**

  This chapter provides a comprehensive analysis of the study's findings, drawing meaningful conclusions and highlighting their broader implications. The discussion and conclusion section synthesizes key insights, interpreting the results in relation to existing literature and the research objectives. The managerial implications section explores the practical significance of the findings, offering strategic recommendations for decision-makers in supply chain management. Finally, the limitations and further study section acknowledges the study's constraints and proposes directions for future research to enhance the robustness and applicability of the proposed methodologies.

CHAPTER 2
REVIEW OF LITERATURE

This chapter provides a comprehensive review of the key concepts, methodologies, and recent advancements that form the foundation of this research. It begins with an in-depth exploration of SCAPP, emphasizing the importance of aligning production capacity with resource allocation to optimize operational efficiency.

2.1 Supply Chain

A Supply Chain (SC) is a system of interconnected organizations, resources, processes, and technologies that collaboratively manage the flow of goods and services from initial suppliers to end consumers (Stevens (1989)). The primary objective of a SC is to efficiently satisfy customer demand while minimizing costs, optimizing resource utilization, and maintaining the flexibility to respond to market changes. An effectively managed SC can confer a competitive advantage to organizations by enhancing product availability, shortening lead times, and improving customer satisfaction.

In addition to the physical flow of goods, modern supply chains also involve significant information flow (Kumar (2001)). Efficient information sharing and real-time data access across all stakeholders enable better forecasting, decision-making, and performance monitoring. Technologies such as Enterprise Resource Planning (ERP) systems, Internet of Things (IoT) sensors (Yesodha et al. (2023)), and blockchain have become integral to ensuring smooth information flow throughout the SC.

The structure of a typical supply chain can be visualized in Figure 2.1 (New & Payne, (1995); Shukla et al. (2011)).

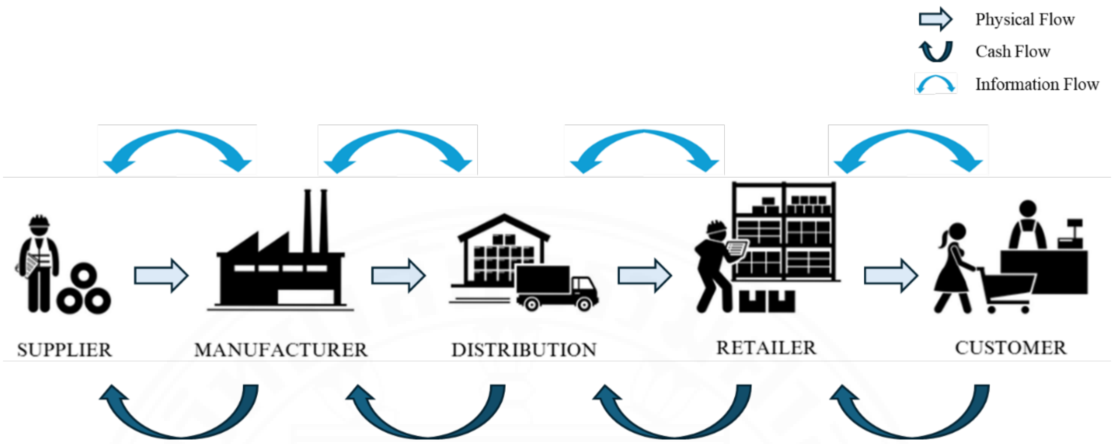

Figure 2.1 The structure of a typical supply chain.

The fundamental components of a supply chain are demonstrated as follows:

- **Suppliers:** Supply raw materials, components, and services essential for production.
- **Manufacturers:** Convert raw materials into finished or semi-finished products.
- **Distributors and Wholesalers:** Oversee the storage and transportation of goods to retailers or directly to consumers.
- **Retailers:** Offer products for sale to the final customers.
- **Consumers:** Represent the ultimate end-users of the products.

The processes in a SC can be represented as a series of interconnected stages, starting from raw material sourcing and ending with product delivery to the consumer (Christopher et al., (1998); Tan (2001)). Simple supply chain processes are typically explained as follows:

- **Procurement:** Sourcing and acquisition of raw materials or components.
- **Production:** Converting raw materials or components into finished products.
- **Logistics:** Coordinating transportation, storage, and distribution.

- **Demand Forecasting:** Predicting customer needs to optimize planning.
- **Inventory Management:** Balancing stock levels to meet demand without excess.

Modern supply chains face numerous challenges as follows:

- **Demand Uncertainty:** Fluctuating customer demand can complicate production and inventory management.
- **Supply Chain Disruptions:** Incidents such as geopolitical conflicts, natural catastrophe, and supplier failures can lead to delays or shortages.
- **Globalization:** Managing complex, multi-tier supply chains that span multiple countries, with different regulations and cultural expectations.
- **Sustainability:** Integrating eco-friendly practices to reduce social and environmental impacts.

2.1.1 Supply Chain Aggregate Production Planning

Supply Chain Aggregate Production Planning (SCAPP) is a comprehensive framework that integrates production planning with supply chain management to optimize resources and meet customer demand efficiently (Mendoza et al. (2014)). The core concept centers on the coordination of supply chain activities such as procurement, manufacturing, inventory management, and distribution to fulfill organizational objectives. (Heizer & Render (2004)).

The structure of SCAPP is built around a hierarchical planning process that encompasses strategic, tactical, and operational levels (Muriel & Simchi-Levi (2003); Bashiri et al. (2012)):

- **Strategic Level:** At this level, long-term decisions are made concerning the overall configuration of the SC, including facility locations, production capacities, and supplier selection. These decisions set the foundation for tactical and operational planning and typically span several years.
- **Tactical Level:** The tactical level focuses on medium-term planning, translating strategic decisions into actionable plans. This process includes aggregate production planning, which sets production volumes, inventory targets, and

labor needs over a planning period typically spanning several months to a year. Tactical SCAPP also addresses Material Requirement Planning (MRP) and capacity planning to ensure that resources are optimally allocated.

- **Operational Level:** At the operational level, short-term plans are developed to execute tactical plans efficiently. This includes detailed scheduling of production activities, inventory replenishment, and order fulfillment. Real-time monitoring and adjustments are often necessary to address unforeseen disruptions or changes in demand.

2.2 Uncertainty

Uncertainty is a concept with diverse interpretations and applications across a wide range of disciplines, each offering unique insights and contextual emphases (Klir (1995)). In the physical sciences and engineering, uncertainty often relates to measurement inaccuracies and variability in experimental outcomes, reflecting the inherent limitations of instruments and natural processes. In statistics, it captures the probabilistic nature of data and the challenges of drawing inferences from incomplete or imperfect information. Economics and finance approach uncertainty in terms of market volatility, forecasting challenges, and risk management, emphasizing its implications for strategic planning and investment. The insurance industry focuses on uncertainty through risk assessment and actuarial models to quantify and manage potential losses. Philosophy focuses on the epistemological dimensions of uncertainty, exploring questions about the limits of human knowledge, the nature of truth, and how certainty is constructed or perceived. Meanwhile, psychology examines uncertainty from a behavioral perspective, investigating how individuals perceive, interpret, and respond to ambiguous or unpredictable situations in their decision-making processes.

To synthesize these perspectives, uncertainty can be broadly defined and understood through the following dimensions:

- **The state of not being known or clearly determined:** This reflects the absence of definitive knowledge or clarity about a situation or outcome.
- **A condition of instability or change:** Uncertainty arises in dynamic contexts where variables are subject to fluctuation and unpredictability.
- **A situation where the probability of outcomes is unknown:** This denotes circumstances where it is difficult or impossible to assign precise probabilities to potential events.
- **Vagueness or ambiguity:** Uncertainty often derives from imprecise, incomplete, or conflicting information, leading to multiple interpretations.
- **A lack of confidence or sureness:** This form of uncertainty is experienced as doubt or hesitation, whether about a person, process, or forecast.

It is important to distinguish that uncertainty arises from both objective and subjective sources. In some cases, uncertainty is inherent to physical systems, as in the randomness of a dice toss or quantum phenomenon, where outcomes are governed by probabilistic laws independent of human perception. In such instances, uncertainty exists regardless of whether a human observer is present. However, in many real-world applications, especially those involving decision-making, planning, or forecasting, uncertainty is closely tied to human perception, understanding, and limitations of knowledge. For example, uncertainty may emerge due to incomplete data, cognitive biases, or linguistic vagueness, which shape how information is interpreted and acted upon. Thus, while not all uncertainties originate from human cognition, many practical expressions of uncertainty in fields such as economics, supply chain management, and risk analysis are influenced by the way humans perceive, process, and evaluate information. Recognizing this distinction helps avoid overgeneralization and ensures a more precise interpretation of uncertainty across different contexts.

In this research, uncertainty is defined as the inherent unpredictability and variability present in real-world situations. It reflects the limitations of available information and acknowledges the potential for unforeseen events or outcomes. This definition emphasizes that uncertainty is not only an abstract concept, but also a practical challenge influencing decision-making, risk assessment, and strategic planning.

The phenomenon of uncertainty arises from a confluence of factors, primarily deriving from a lack of understanding, incomplete knowledge, insufficient data, and the inherent variability present within natural processes. This state of uncertainty is further exacerbated by several key sources, as identified by Lawrence & Lorsch (1967) and Duncan (1972). Firstly, uncertainty arises when crucial information regarding environmental factors remains inaccessible or unattainable, preventing thorough evaluations of potential outcomes. Secondly, it surfaces when the anticipated results of a decision remain ambiguous, limiting the formulation of clear and informed strategies. Finally, uncertainty becomes more pronounced when assigning a degree of confidence to a given scenario proves ineffective, causing decision-making processes to be inherently volatile and precarious.

In the context of business and supply chain operations, uncertainty typically originates from two primary sources: environmental uncertainty and system uncertainty (Cha-ume & Chiadamrong, 2012).

- **Environmental Uncertainty:** This form of uncertainty derives from external factors that influence the SC and are often beyond the control of the business. One of the main contributors is the performance of suppliers, which can fluctuate due to various reasons such as production delays, financial instability, or logistical challenges. Another influential factor is customer behavior, particularly in terms of supply and demand dynamics. Shifts in consumer preferences, market trends, or changes in the economic landscape can lead to unpredictable demand patterns, making it challenging for businesses to accurately forecast needs and plan accordingly. Moreover, environmental uncertainty also encompasses broader geopolitical events, regulatory changes, and natural disasters, all of which can create sudden and significant disruptions in supply chain operations.

- **System Uncertainty:** System uncertainty arises from internal organizational factors that contribute to unpredictability. This type of uncertainty often derives from the unreliability and uncontrollability of internal processes. Issues such as machinery breakdowns, software malfunctions, or human errors can disrupt the flow of operations, leading to unexpected delays or failures. Furthermore, inefficiencies within workflows, a lack of coordination between departments, or insufficient resource management practices can amplify system uncertainty, making it difficult for an organization to achieve consistent performance. This type of uncertainty also includes the challenges of aligning organizational strategies with rapidly changing internal and external conditions, such as fluctuating workforce availability or changes in production capabilities.

Uncertainty within organizational decision-making and operations can be categorized into four distinct types: data uncertainty, model uncertainty, parameter uncertainty, and scenario uncertainty.

- **Data Uncertainty:** This type of uncertainty arises from limitations or inaccuracies in the available data, which affects the reliability or validity of the information on which decisions are based. Whether the data is outdated, incomplete, or corrupted, data uncertainty limits organizations from making confident and accurate assessments, ultimately impacting strategic planning and operational efficiency (Jones et al., 2020).
- **Model Uncertainty:** Model uncertainty refers to the uncertainty associated with the mathematical or computational models used to represent real-world systems. These models, whether mathematical, computational, or statistical, are simplified representations of complex environments, and their structure may not fully capture the complexity of the real world. As a result, model uncertainty reflects the limitations of these representations and their inability to account for all possible variables or scenarios, which can lead to inaccurate predictions or conclusions (Brown & Johnson, 2018).

- **Parameter Uncertainty:** Parameter uncertainty arises from the uncertainty surrounding the values or estimates of parameters within a model. These parameters often involve estimates based on historical data, expert judgment, or assumptions that may carry inherent variability or imprecision. In many cases, slight changes in the values of key parameters can lead to significant variations in the model's output, highlighting the challenge of ensuring precision in parameter estimation (Chen et al., 2021; Li and Wu (2006)).
- **Scenario Uncertainty:** Scenario uncertainty pertains to the uncertainty related to future conditions or scenarios that could affect outcomes. This includes a broad range of factors such as changes in market conditions, technological advancements, regulatory shifts, or unforeseen geopolitical events. Since these factors are often unpredictable and can evolve rapidly, scenario uncertainty plays a critical role in strategic decision-making, requiring businesses to plan for a variety of potential futures rather than a single anticipated outcome (Li & Wang, 2017).

Recognizing and understanding these diverse types of uncertainty is crucial for organizations operating in complex and dynamic environments. By acknowledging the different sources and categories of uncertainty, businesses can develop more robust strategies for managing risks, enhancing decision-making, and improving their ability to adapt to unforeseen changes. This understanding allows organizations to implement comprehensive risk management frameworks, which not only account for known variables but also prepare for the unpredictable factors that can impact performance, market stability, and long-term growth. In turn, businesses can adopt a proactive approach to navigating uncertainty, positioning themselves to thrive through volatility and change.

2.2.1 Risk of Uncertainty

The concept of risk, as discussed by Rachev et al. (2011), derives from inherent uncertainty and the likelihood of adverse exposure. The presence of uncertainty often leads to the perception of risk, which reflects the potential for adverse effects on individuals or organizations. According to Rachev et al., (2011) risk encompasses both the element of uncertainty where future events or conditions are unpredictable and the exposure to potential negative consequences that may result from these uncertain factors. This perspective highlights that risk cannot be fully understood or managed without considering the inherent uncertainty in any given situation.

Uncertainty refers to the incomplete knowledge of future outcomes or conditions, which introduces variability and unpredictability into decision-making processes. When uncertainty exists, it implies that there are multiple possible outcomes, each with varying probabilities. Risk, therefore, emerges because of this uncertainty, as it represents the potential for adverse outcomes resulting from the unknown (Head (1967)). For instance, in financial investments, uncertainty about market movements creates the risk of financial loss. Similarly, in project management, uncertainty about resource availability or project timelines introduces the risk of delays and cost overruns.

The subjective nature of risk underscores that different individuals or organizations may perceive and respond to risk differently based on their own experiences, knowledge, and risk tolerance. This subjectivity means that the same level of uncertainty can be viewed as more or less risky depending on the context and perspective of the decision-makers (Toma et al. (2012)). For example, a high-risk investment may be perceived as attractive to a risk-tolerant investor but as too risky for a more conservative investor.

Effective risk management, therefore, involves not only understanding and quantifying uncertainty but also addressing how this uncertainty impacts exposure to potential negative outcomes. Accordingly, various risk measurement methods have been proposed to assist decision makers gain a comprehensive perception of the risks they encounter and to formulate effective mitigation or management strategies.

2.2.2 Risk Measurement

Risk measurement is a critical process in risk management that involves quantifying the potential impact of uncertainties on an organization or investment. This process aims to provide a clear and objective assessment of risk exposure by evaluating the likelihood and severity of adverse outcomes. Risk measurement helps decision-makers understand the extent of potential losses or damage and aids in developing strategies to mitigate these risks effectively (McGoun (1995)).

One fundamental approach to risk measurement involves calculating the probability and impact of different risk events. Probability quantifies the likelihood of a risk event occurring, whereas impact evaluates the potential severity or consequences should the risk materialize. Techniques such as risk assessments, simulations, and statistical analyses are commonly used to evaluate these factors (Fishburn (1984)). For instance, Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) are pivotal tools in risk management and decision analysis, widely used to quantify uncertainty in environments where financial or operational risks prevail (Jorion, 2007). Value-at-Risk (VaR) is a statistical metric that estimates the maximum potential loss within a specified time frame at a predetermined confidence level. (Marshall & Siegel, 1997). It establishes a threshold beyond which the probability of experiencing larger losses is relatively low. Although VaR effectively captures the probability of losses, it does not convey information regarding the magnitude of losses that exceed the VaR threshold. This limitation is a significant drawback, as it fails to describe the magnitude of extreme events, a critical consideration, particularly in high-risk situations where large losses can have catastrophic consequences (Artzner et al., 1999). To better handle this limitation, CVaR (Expected Shortfall) evaluates the anticipated loss assuming losses exceed the VaR threshold, thereby offering a more informative risk assessment (Rockafellar & Uryasev, 2000). CVaR addresses the limitations of VaR by concentrating on the tail of the loss distribution, thereby offering a more comprehensive assessment of risks linked to extreme events. By capturing the average of the worst losses beyond the VaR point, CVaR not only estimates the likelihood of severe losses but also quantifies their potential magnitude, making it a more robust risk measure, especially for institutions or systems exposed to extreme risk (Acerbi & Scandolo, 2008). The strength of CVaR lies in its mathematical properties, which ensures that diversification of risk leads to a reduction in total risk, a feature that is not guaranteed by VaR.

An additional critical element of risk measurement involves employing metrics and indicators to continuously monitor and manage risk over time. These metrics can include measures such as standard deviation, which gauges the volatility of returns, or the beta coefficient, which assesses the sensitivity of an asset's returns to market movements (Szegö (2005)). By tracking these indicators, organizations can identify changes in risk levels and make adjustments to their risk management strategies accordingly. Additionally, scenario analysis and stress testing are valuable tools for examining how different scenarios or extreme conditions might impact risk, allowing organizations to prepare for and mitigate potential adverse outcomes.

In summary, risk measurement is a crucial component of risk management that involves quantifying the probability and impact of potential adverse events. By employing various techniques and metrics, organizations can assess their risk exposure, monitor changes over time, and develop strategies to mitigate potential losses. Integrating quantitative data with qualitative insights provides a more holistic view of risk, enhancing decision-making and helping to safeguard against uncertainties.

2.3 Fuzzy Set Theory

Fuzzy logic, introduced by Lotfi Zadeh in the 1960s, revolutionized computational science by providing a mathematical framework for handling uncertainty. In contrast to conventional set theory, which employs binary logic to categorize information as either true or false, fuzzy logic accommodates varying degrees of truth, thereby capturing the ambiguity inherent in real-world situations. This approach has been extensively utilized across various fields such as data mining, artificial intelligence, control systems, and decision-making. By enabling machines to process imprecise or incomplete data, fuzzy logic improves the adaptability and robustness of intelligent systems, making them more capable of reasoning in uncertain environments.

Essentially, fuzzy logic enhances classical binary logic by allowing truth values to vary continuously between 0 and 1, which supports more subtle and complex reasoning. Instead of categorizing statements as entirely true or false, fuzzy logic introduces linguistic variables and fuzzy rules that better capture human reasoning. This flexibility makes it particularly useful in modeling complex systems where precise categorization is impractical. Applications of fuzzy logic span diverse domains, including climate modeling, risk analysis, and customer behavior prediction, where uncertainty is a fundamental challenge. By simulating human-like decision-making processes, fuzzy logic provides a more realistic and effective approach to handling vague or imprecise data.

The introduction of fuzzy logic into optimization methods further expanded its applicability. In 1974, Tanaka et al. incorporated fuzzy set theory into Linear Programming (LP), allowing for fuzzy goals and constraints to model uncertainty in optimization problems. This advancement enabled decision-makers to incorporate imprecise parameters into optimization models, making them more reflective of real-world conditions. Building on this, Bellman and Zadeh (1970) developed fuzzy decision models that linked fuzzy set theory with optimization techniques, demonstrating its effectiveness in addressing vagueness. The mathematical foundations of fuzzy logic, including constructs like Triangular Fuzzy Numbers (TFNs) and Trapezoidal Fuzzy Numbers (TrFNs), provide structured methods for representing uncertain data. These developments have solidified fuzzy logic as a critical tool in computational science, influencing fields ranging from industrial control systems to decision support tools.

2.3.1 Fuzzy Number

Fuzzy number possesses a set of well-defined mathematical properties that make them suitable for representing imprecise numerical values, where each element is assigned a membership degree ranging from 0 to 1. Unlike crisp numbers with precise values, fuzzy numbers accommodate uncertainty and imprecision, making them more suitable for modeling real-world scenarios where exact values are difficult to determine (Dubois & Prade, 1993). A fuzzy number is defined by a membership function that assigns varying degrees of certainty to values within its range. A membership degree of 1 indicates full membership, while 0 represents complete non-membership, with intermediate values capturing partial truth. This flexibility allows fuzzy numbers to extend beyond conventional binary logic, incorporating a spectrum of possibilities that better reflect uncertain circumstances (Heilpern, 1997).

In addition, its special properties include normality, convexity, boundedness, and upper semi-continuity. Normality ensures that the fuzzy number has at least one value with full membership (i.e., a membership degree of 1), representing the most plausible or core value. Convexity implies that all α-cuts of the fuzzy number form convex sets, which guarantees that the degree of membership does not increase once it begins to decrease, preserving the intuitive idea of gradual uncertainty around the core. Boundedness refers to the requirement that the support of the fuzzy number; the set of all values with non-zero membership, is finite, ensuring that the fuzzy number remains computationally manageable. Additionally, the membership function of a fuzzy number must be upper semi-continuous, which prevents abrupt increases in membership grades and ensures mathematical stability. These foundational properties are essential for enabling consistent fuzzy arithmetic operations and integration into fuzzy optimization models (Dubois & Prade, 1978).

Fuzzy numbers play an essential role in fuzzy logic and fuzzy set theory, providing a structured way to handle vague or incomplete data (Zadeh, 1988). They are widely applied in supply chain management field, where they model fluctuating demand and unpredictable lead times, enhancing decision-making in uncertain environments. Their ability to integrate ambiguity into mathematical models makes them valuable for optimizing processes that require adaptability and robustness. By allowing for a more realistic representation of dynamic systems, fuzzy numbers contribute to improved planning and operational efficiency, particularly in complex and data-limited scenarios.

A fuzzy number is a special type of fuzzy set defined over the real numbers. Formally, a fuzzy number $\tilde{A}$ is a convex and normalized fuzzy set that satisfies specific conditions. First, there exists at least one real number $x_0 \in R$ such that the membership function $\mu_{\tilde{A}}(x_0) = 1$, meaning the degree of membership of x_0 in the fuzzy set is maximal. Second, the membership function $\mu_{\tilde{A}}(x_0)$ must be piecewise continuous over the real numbers, ensuring the fuzzy number is well-defined and mathematically tractable. These properties ensure that the fuzzy number represents imprecise but bounded and meaningful quantities in decision-making and optimization contexts.

- **Triangular Fuzzy Number**

Triangular Fuzzy Numbers (TFNs) are a widely used type of fuzzy number in fuzzy set theory, representing uncertainty in a structured way (Shyamal & Pal (2007)). In the context of minimization, TFNs are defined by three key parameters: a^o (the optimistic value), a^m (the most likely value), and a^p (the pessimistic value), where $a^o \leq a^m \leq a^p$ Conversely, for maximization problems, the order of these parameters is reversed. The triangular shape of the membership function for a TFN reflects the assumption that the most likely value (a^m) has the highest degree of membership, while values closer to the lower and upper bounds (a^o and a^p) gradually have decreasing degrees of membership (Anand & Bharatraj (2017); Hierro et al. (2023)) as presented in Figure 2.2.

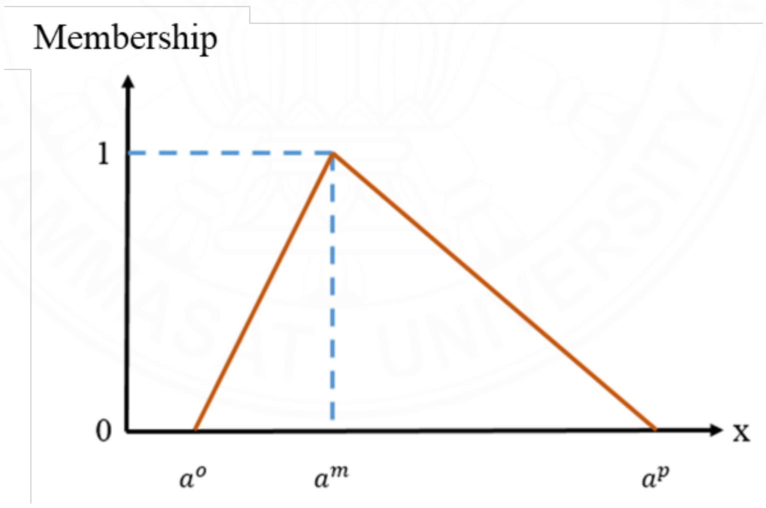

Figure 2.2 Triangular Distribution.

The three parameters can be described as follows:

1. a^o is an optimistic value that represents the best case (for minimization context) and the worst case (for maximization context). It has a very low likelihood or possibility degree equal to 0.
2. a^m is the most likely value that represents the normal case. It has a very high likelihood or possibility degree equal to 1.
3. a^p is a pessimistic value that represents the worst case (for minimization context) and the best case (for maximization context). It has a very low likelihood or possibility degree equal to 0.

A commonly used representation of fuzzy numbers is the Triangular Fuzzy Numbers (TFNs). The membership function of a triangular fuzzy number $\tilde{A}$ is defined as:

$$\mu_{\tilde{A}}(x_0) = \begin{Bmatrix} 1 - \frac{a^m - x}{a^o}, & a^m - a^o \le x \le a^m \\ 1 - \frac{x - a^m}{a^p}, & a^m \le x \le a^m + a^p \\ 0, & otherwise \end{Bmatrix} \tag{2.1}$$

Here, $a^m$is the most likely value, while a^o and a^p represent the left and right spreads, respectively. These parameters describe the uncertainty range around the central value. A triangular fuzzy number can be concisely denoted as $\tilde{A} = (a^o, a^m, a^p)$. The set of all such triangular fuzzy numbers defined on real number *R* is represented as $F(R)$.

A fuzzy number $\tilde{A}$ is said to be nonnegative if all values with a degree of membership greater than zero are nonnegative. Formally, this means that $\mu_{\tilde{A}}(x_0) = 0$ for all$x < 0$. For a triangular fuzzy number $\tilde{A} = (a^o, a^m, a^p)$, this condition translates to $a^m - a^o \ge 0$, ensuring the entire support of the fuzzy number lies in the nonnegative real domain.

Two triangular fuzzy numbers $\tilde{A} = \left(a_A^o, a_A^m, a_A^p\right)$ and $\tilde{B} = \left(a_B^o, a_B^m, a_B^p\right)$ are equal if and only if their respective modal values and spreads are identical. That is, $\tilde{A} = \tilde{B}$ if and only if $a_A^o = a_B^o, a_A^m = a_B^m$, and $a_A^p = a_B^p$.

A triangular fuzzy number is considered symmetric when the left and right spreads from the modal value are equal. Formally, a TFN $\tilde{A} = (a^o, a^m, a^p)$ is symmetric if $a^m - a^o = a^p - a^m$. This symmetry implies that the uncertainty surrounding the modal value is balanced on both sides.

For computational purposes and model formulation, basic arithmetic operations on triangular fuzzy numbers are defined through their parameters. Let $\tilde{A} = \left(a_A^o, a_A^m, a_A^p\right)$ and $\tilde{B} = \left(a_B^o, a_B^m, a_B^p\right)$ be two triangular fuzzy numbers. Then the operations are defined as follows:

- **Addition:**
  $\tilde{A} + \tilde{B} = \left(a_A^o + a_B^o, a_A^m + a_B^m, a_A^p + a_B^p\right)$
- **Subtraction:**
  $\tilde{A} - \tilde{B} = \left(a_A^o + a_B^p, a_A^m + a_B^m, a_A^p + a_B^o\right)$
- **Multiplication (assuming all values are non-negative):**
  $\tilde{A} \times \tilde{B} \approx \left(a_A^o \times a_B^o, a_A^m \times a_B^m, a_A^p \times a_B^p\right)$
- **Scalar Multiplication (for a positive scalar $\boldsymbol{\lambda} > 0$):**
  $\lambda \times \tilde{A} = \left(\lambda a_A^o, \lambda a_A^m, \lambda a_A^p\right)$
- **Division:**
  $\tilde{A} \div \tilde{B} = (a_A^m \div a_B^m, a_A^o \div a_B^o, a_A^p \div a_B^p,)$ where $a_B^m, a_B^o, a_B^p \neq 0$

These arithmetic operations maintain the triangular shape of fuzzy numbers and enable their application in fuzzy mathematical programming, thereby facilitating systematic management of uncertainty in model parameters.

- **Triangular Intuitionistic Fuzzy Number**

Triangular Intuitionistic Fuzzy Numbers (TIFNs) extend the concept of Triangular Fuzzy Numbers (TFNs) by incorporating intuitionistic fuzzy sets, first introduced by Atanassov in 1983. Unlike conventional fuzzy sets, which rely solely on membership function, intuitionistic fuzzy sets also include a non-membership function, allowing for a more comprehensive representation of uncertainty. In TIFNs, each element is identified by both its degree of membership and non-membership, effectively capturing hesitation and ambiguity in decision-making scenarios. This dual-function approach makes TIFNs particularly valuable for modeling complex and imprecise conditions where uncertainty plays a significant role.

A TIFN is mathematically defined by two functions: the membership function (μ(x)), representing the degree of belonging of an element to the set, and the non-membership function (v(x)), quantifying the degree of non-belonging. These functions adhere to the condition $0 \leq \mu(x) + v(x) \leq 1$, ensuring a balanced representation of uncertainty (Dymova & Sevastjanov, 2010; Husain et al., 2012). The triangular distribution of a TIFN is determined by three key points for both the membership and non-membership functions, denoted as (b^o, b^m, b^p) and $(\overline{b^o}, b^m, \overline{b^p})$, respectively. In the context of minimization, the peak value (b^m) represents the most likely value, while the lower (b^o) and upper (b^p) bounds define the range of uncertainty, where $b^o \leq b^m \leq b^p$ Conversely, for maximization problems, the order of these parameters is reversed. The non-membership function complements this by quantifying the extent to which elements do not belong to the fuzzy set, offering deeper insights into the inherent vagueness of real-world problems (Burillo & Bustince, 1996).

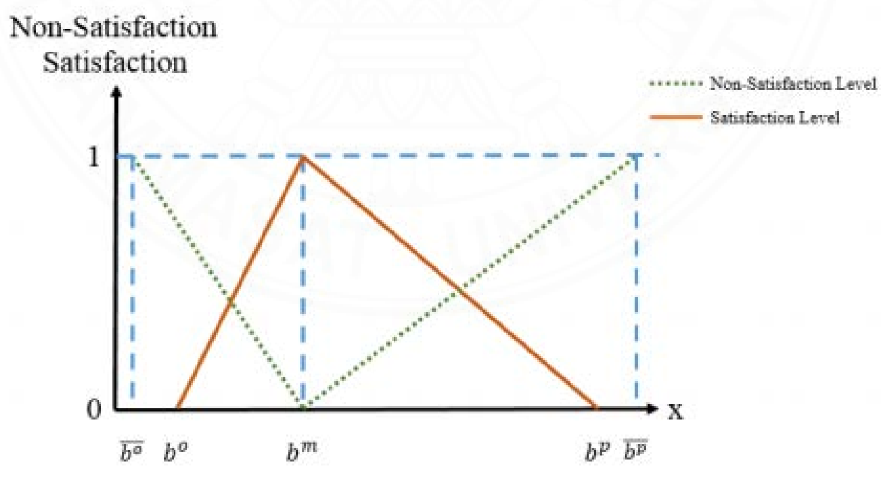

Figure 2.3 Triangular Intuitionistic Distribution.

The six parameters can be described as follows:

1. b^o is an optimistic value that represents the best case of membership function (for minimization context) and the worst case of membership function (for maximization context). It has a very low likelihood or possibility degree equal to 0.
2. b^m is a most likely value that represents the normal case of membership function. It has a very high likelihood or possibility degree equal to 1.
3. b^p is a pessimistic value that represents the worst case of membership function (for minimization context) and the best case of membership function (for maximization context). It has a very low likelihood or possibility degree equal to 0.
4. $\overline{b^o}$ is an optimistic value that represents the best case of non-membership function (for minimization context) and the worst case of non-membership function (for maximization context). It has a very high likelihood or possibility degree equal to 1.
5. b^m is a most likely value that represents the normal case of non-membership function. It has a very low likelihood or possibility degree equal to 0.
6. $\overline{b^p}$ is a pessimistic value that represents the worst case of non-membership function (for minimization context) and the best case of non-membership function (for maximization context). It has a very high likelihood or possibility degree equal to 1.

TIFNs concept is introduced where its perception is analyzed as an unconventional approach to specify a fuzzy set. It holds the concept of the triangular distribution whereas the hesitation allowance is incorporated to provide an acceptable fuzzy set to decision makers. To provide an acceptable fuzzy set to decision makers, TIFNs is applied to (*A*, *R*)-cut approach. This approach can be used to generate the acceptable TIFNs under the controlling percentage of the acceptance level (*A*) and the percentage of the rejection level (*R*) as presented in Figure 2.4. The formulation of (*A*, *R*)-cut approach can be presented as follows:

Let $\tilde{B} = (b^o, b^m, b^p)(\overline{b^o}, b^m, \overline{b^p})$ where $\overline{b^o}, \leq b^o \leq b^m \leq b^p \leq \overline{b^p}$ (2.2)

$$B^o = max\{b^o + A(b^m - b^o), b^m - R(b^m - \overline{b^o})\} \quad (2.3)$$

$$B^m = \frac{B^o + B^p}{2} \quad (2.4)$$

$$B^p = min\{b^p - A(b^p - b^m), b^m + R(\overline{b^p} - b^m)\} \quad (2.5)$$

where $b^o, b^m, and\ b^p$ denote the three key points defining the membership function, corresponding to optimistic, most likely, and pessimistic scenarios. Similarly, $\overline{b^o}, b^m, and\ \overline{b^p}$ refer to the respective data points in the non-membership function. The parameters A and R indicate the degrees of possibility for acceptance and rejection, respectively.

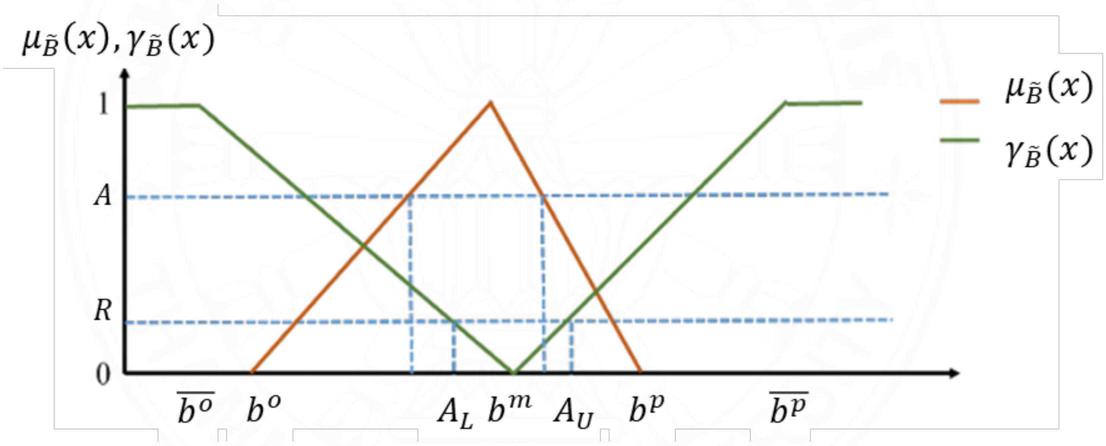

Figure 2.4 (A, R)-Cut Approach.

Mathematically, the degree of membership is a linear function between the bounds b^o and b^p, with a peak at b^m. The non-membership function captures the "degree of doubt" and decreases accordingly as x moves closer to the mode value b^m. The specific formulas are as follows:

$$\mu_{TIFN}(x) = \begin{cases} \frac{x - b^o}{b^m - b^o}, & if\ b^o \leq x \leq b^m \\ \frac{b^p - x}{b^p - b^m}, & if\ b^m \leq x \leq b^p \\ 0, & otherwise \end{cases} \quad (2.6)$$

where $b^m - b^o$ and $b^p - b^m$ must more than zero.

$$v_{TIFN}(x) = 1 - \mu_{TIFN}(x) \quad (2.7)$$

The degree of membership and non-membership are constrained so that their sum is no greater than one: $\mu(x) + v(x) \leq 1$. This reflects the intuitive idea that, for any given value x, it cannot belong to both the fuzzy set and its complement simultaneously at full strength.

2.3.2 Skewness Degree

The skewness degree is a statistical metric that quantifies the asymmetry degree in a probability distribution relative to its mean (Adcock & Shutes (2005); Arnold & Groeneveld (2012)) as shown in Figure 2.5. It indicates whether the data points in a dataset are distributed symmetrically or if they lean more heavily toward one side of the mean. When a distribution has zero skewness, it is perfectly symmetrical, often taking the shape of a bell curve. A positive skewness means the distribution has an extended tail on the right (right-skewed), whereas a negative skewness reflects a longer tail on the left side (left-skewed). The skewness degree provides insights into the shape of the data, which can be critical for understanding underlying trends, patterns, and anomalies in various analytical contexts.

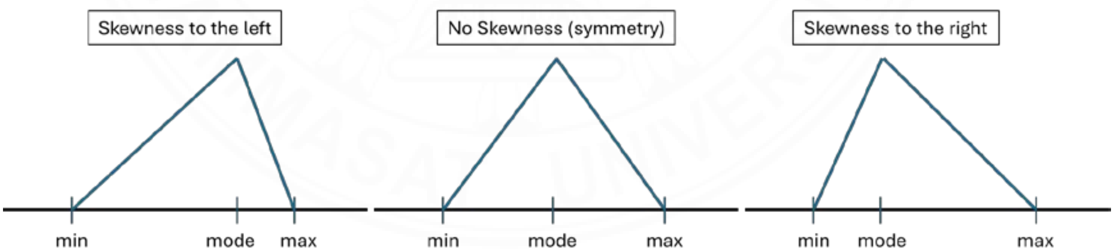

Figure 2.5 Types of Skewness Degree.

The function of the skewness degree lies in its ability to describe and interpret the shape and balance of a dataset. It is especially valuable in fields such as finance, economics, and quality control, where comprehending the characteristics of data distributions is crucial. For instance, in financial risk analysis, skewness helps identify the likelihood of extreme losses or gains by analyzing the distribution of returns. Similarly, in quality control, skewness can signal deviations from expected performance metrics, prompting further investigation. By quantifying asymmetry, the skewness degree complements other statistical metrics like mean and standard deviation, offering a more comprehensive view of data characteristics and enabling better decision-making.

2.4 Mathematical Modeling

Modeling refers to the creation of simplified depictions of real systems aimed at understanding their dynamics and predicting potential results under diverse conditions. (Blomhøj (2004)). These models, which can be physical, conceptual, mathematical, or computational, serve as tools to simulate complex systems that may be difficult to study directly due to factors like scale, uncertainty, or cost (Edmonds (2017)). Depending on the complexity of the system, available data, and analysis goals, different types of models are employed. Physical models represent tangible objects, while conceptual models focus on relationships and structure, mathematical models use equations, and computational models rely on simulations and algorithms.

In optimization, modeling is essential for defining objectives, constraints, and controllable variables, enabling decision-makers to find optimal solutions (Sarker & Newton (2007); Singh (2012)). Models not only support decision-making but also offer a structured approach to testing theories, making predictions, and evaluating decisions without real-world consequences. They are invaluable tools for improving efficiency, minimizing risks, and ensuring systems operate effectively. Additionally, models can be updated with new data, allowing for continuous improvement and learning, which helps refine strategies and solutions over time, particularly in dynamic fields like supply chain management, finance, and resource allocation.

- **Fuzzy Linear Programming Model**

Fuzzy Linear Programming (FLP) extends conventional linear programming by incorporating fuzzy logic to address uncertainty and ambiguity in decision-making. Unlike classical LP, which assumes precise values for parameters, FLP allows for the representation of parameters as fuzzy sets, providing flexibility in modeling problems where data is uncertain or imprecise (Negoita (1981)). Rooted in fuzzy set theory, FLP applies membership functions to the objective function and constraints, describing the satisfaction degree or feasibility of solutions within a specified range. This approach makes FLP especially useful in real-world applications where exact data is difficult to obtain or subject to variation over time.

An FLP model consists of the following components (Delgado et al. (1989)):

- **Fuzzy Objective Function**: In FLP, the objective function is a linear expression that includes fuzzy coefficients, aiming to maximize or minimize the function while considering the fuzziness of the involved parameters.
- **Fuzzy Constraints**: FLP incorporates fuzzy constraints, allowing for the representation of constraints within a range, as opposed to strict equality or inequality constraints.
- **Fuzzy Decision Variables**: Decision variables in FLP are represented as fuzzy numbers, meaning they exist within a range, allowing flexibility and uncertainty in the solution. These variables may have membership functions that define their potential values.

Fuzzy Linear Programming (FLP) offers several advantages over conventional linear programming, particularly its ability to handle uncertainty and vagueness, which makes it well-suited for real-world problems with imprecise data. By directly modeling uncertainty, FLP provides more flexible and robust solutions that can response to changing conditions, making it an effective tool for problems involving subjective judgments, approximations, or estimates. However, FLP also has limitations, such as the defuzzification process, potentially leading to a reduction in information accuracy when converting fuzzy solutions into crisp values. Additionally, the use of fuzzy numbers requires careful interpretation of membership functions, and varying methods of fuzzification and defuzzification can lead to different results, introducing subjectivity. Moreover, solving FLP problems can be more computationally intensive as a consequence of the added complexity of fuzzy numbers, requiring specialized algorithms and techniques for optimization (Buckley & Feuring, 2000; Figueroa–García et al., 2022; Ghanbari et al., 2020).

- **Intuitionistic Fuzzy Linear Programming Model**

Intuitionistic Fuzzy Linear Programming (IFLP) is a sophisticated optimization technique that extends the principles of conventional linear programming to address the challenges of uncertainty and ambiguity in decision-making processes. Conventional linear programming assumes precise numerical data and deterministic models, which are often inadequate for real-world problems where ambiguity and imprecision prevail. IFLP overcomes these limitations by incorporating the concept of intuitionistic fuzzy sets, offering a comprehensive framework for modeling uncertainty through the simultaneous consideration of membership and non-membership (Parvathi (2012); Parvathi & Malathi (2012)).

The foundation of IFLP lies in intuitionistic fuzzy sets, a concept introduced by Atanassov in 1986. Unlike conventional fuzzy sets, which only consider the degree of membership of an element to a set, intuitionistic fuzzy sets introduce additional parameter: non-membership (the indeterminate portion that reflects the lack of knowledge). These parameters enable a richer representation of uncertainty, accommodating the real-world scenarios where precise information is often unavailable. In IFLP models, constraints and objective functions are expressed using intuitionistic fuzzy numbers, effectively capturing the vagueness and imprecision inherent in problem data (Kabiraj et al. (2019)).

IFLP provides a binary perspective in decision-making that flexible for complex systems with incomplete or ambiguous information. Decision variables, constraints, and objective functions are no longer rigidly defined but instead exist within a spectrum of possibilities. This adaptability makes IFLP a powerful tool for addressing problems in fields such as finance, engineering, and supply chain management.

2.5 Defuzzification Approach

Defuzzification is the process of transforming fuzzy quantities, represented by fuzzy sets, into precise, actionable outputs, making them suitable for decision-making. This step is essential in fuzzy logic systems, as it transforms the ambiguous and imprecise results of fuzzy computations into clear, usable information for real-world applications. While fuzzification allows for flexible and nuanced analysis by converting precise inputs into fuzzy sets, defuzzification extracts a single value from the fuzzy set that best represents its overall meaning. This process involves selecting a crisp value from a range of possible values with varying degrees of membership, ensuring that the fuzzy output can be interpreted and applied effectively in decision-making scenarios (Rondeau et al., 1997; Roychowdhury & Pedrycz, 2001; Chakraverty et al., 2019; Leewijck & Kerre, 1999).

- **Defuzzification Approach for Objective Function**

In the context of fuzzy objective functions, these defuzzification approaches are applied to convert fuzzy representations into crisp values, enabling effective optimization and decision-making. By transforming fuzzy objectives into crisp values through defuzzification, decision-makers can better interpret and utilize the results of fuzzy optimization models (Ahmed et al. (2017); Karimi et al. (2022)). This process ensures that the inherent uncertainty in the data is appropriately accounted for, while also enabling concrete, actionable insights to be drawn from the fuzzy analysis. This makes defuzzification an essential step in the practical application of fuzzy set theory in optimization and decision-making processes.

The fuzzy coefficients used in objective function are generally represented as follows:

$$Maximize \quad \tilde{\boldsymbol{c}}\, x \tag{2.8}$$

Here, the triangular distribution is used to explain the type of fuzzy numbers $\tilde{c} = (c^o, c^m, c^p)$. In the context of minimization, $c^o, c^m,$ and c^p are the optimistic, the most likely, and the pessimistic values of $\tilde{c}$, respectively. Conversely, for maximization problems, the order of these parameters is reversed.

- **Defuzzification Approach for Constraints**

In the context of fuzzy constraints, defuzzification techniques are crucial for converting fuzzy representations into crisp values, allowing for effective optimization and decision-making. Fuzzy constraints, which contain inherent vagueness and imprecision, must be transformed into specific, actionable values for practical use. Defuzzification facilitates this process, ensuring that fuzzy constraints become precise values that decision-makers can easily interpret and apply (Runkler & Glesner, 1994; Saletic & Popovic, 2006). By addressing the uncertainty in the data, defuzzification enables optimization models to reflect real-world conditions more accurately, providing clearer insights for decision-makers. This capability to translate fuzzy data into actionable intelligence underscores the importance of defuzzification in the practical application of fuzzy set theory, enhancing the reliability of decision-making in uncertain environments (Verstraete et al., 2024).

The standard structure of fuzzy constraints on the left-hand side of the equations is presented below:

$$Subject\ to:\ (\tilde{\boldsymbol{A}}x) \leq\ b, and\ \ x\ \geq\ 0 \tag{2.9}$$

Here, the triangular distribution is used to explain the type of fuzzy numbers $\tilde{A} = (A^o, A^m, and\ A^p)$, where $A^o, A^m, and\ A^p$ are the optimistic, the most likely, and the pessimistic values of $\tilde{A}$, respectively.

The standard structure of fuzzy constraints on the right-hand side of the equations is presented below:

$$Subject\ to:\ (\tilde{\boldsymbol{A}}x) \leq\ \tilde{\boldsymbol{b}}, and\ \ x\ \geq\ 0 \tag{2.10}$$

Here, the triangular distribution is used to explain the type of fuzzy numbers $\tilde{b} = (b^o, b^m, and\ b^p)$. In the context of minimization, $b^o, b^m, and\ b^p$ are the optimistic, the most likely, and the pessimistic values of $\tilde{b}$, respectively. Conversely, for maximization problems, the order of these parameters is reversed.

2.6 Conflicting Objectives

Conflicting objectives arise when two or more goals within a decision-making or optimization problem cannot be simultaneously achieved. This phenomenon is common in real-world scenarios where resources, priorities, or outcomes are constrained, leading to trade-offs between competing objectives (Raiffa & Keeney (1975)).

In multi-objective optimization, conflicting objectives are formally addressed through mathematical models that account for the trade-offs between goals. The presence of conflicting objectives adds complexity to decision-making as it requires prioritization, negotiation, and sometimes compromise. It also highlights the importance of stakeholder involvement, as different stakeholders may place varying levels of importance on each objective. For example, a company's management might prioritize profitability, while its customers value sustainability and product quality. Addressing such conflicts requires clear communication and a shared understanding of the overarching goals.

Identifying and comprehending conflicting objectives is essential for making well-informed decisions and efficiently allocating resources. It allows organizations and decision-makers to anticipate challenges and devise strategies that align with overarching goals while accommodating trade-offs. By systematically addressing conflicts, decision-makers can achieve solutions that balance competing priorities, leading to more sustainable and practical outcomes in various fields, such as logistics, healthcare, and environmental management (Bell et al. (1977)).

To address conflicting objectives, mathematical frameworks like multi-objective optimization are employed. Multi-objective optimization models aim to find solutions that provide the best possible trade-offs among competing goals. These solutions are represented as a Pareto front, a set of non-dominated solutions where improving one objective would result in worsening another. Decision-makers can then select a solution from the Pareto front based on their preferences and the specific context of the problem (Purshouse & Fleming (2007)).

2.7 Pareto Optimal Solution

A Pareto optimal solution, also known as a Pareto efficient or Pareto frontier solution, is a key concept in multi-objective optimization that represents a state where no objective can be enhanced without compromising at least one other objective. Widely applied in fields such as economics, engineering, and decision-making, a solution is assessed Pareto optimal if it is impossible to reallocate resources or adjust variables to improve one objective without negatively affecting another (Jiménez & Bilbao, 2009; Kovalenko et al., 2020). In multi-objective optimization, the set of Pareto optimal solutions, known as the Pareto front or Pareto frontier, offers decision-makers a range of alternatives that balance competing objectives in various ways. Each solution on the Pareto front represents a different trade-off, helping to understand the relationship between objectives. A solution is Pareto optimal if it is not dominated by any other solution, meaning no other solution is better in all objectives and strictly better in at least one (Deb & Gupta, 2005; Wang & Rangaiah, 2016).

Symbolically, the formulation of Pareto Optimal Solution is articulated as follows:

$$\begin{aligned}&\text{Minimize } \left[\psi_1(x), \psi_2(x), \dots, \psi_j(x)\right]\\&\text{subject to: } x \in X\end{aligned} \quad (2.11)$$

Here, a point $\hat{x} \in X$ is called:

- A **dominated solution** if there exist $x \in X$ such that $\psi_j(x) \leq \psi_j(\hat{x})\ \forall i$, with at least one strict inequality holding.
- A **weak Pareto Optimal Solution** if and only if there does not exist $x \in X$ such that $\psi_j(x) < \psi_j(\hat{x})\ \forall i$
- A **strong Pareto Optimal Solution** if and only if there does not exist another solution $x \in X$ such that $\psi_j(x) \leq \psi_j(\hat{x})\ \forall i$, with at least one strict inequality.

One of the key advantages of using Pareto optimal solutions in multi-objective optimization is the valuable insight they provide into the trade-offs between objectives. By analyzing the Pareto front, decision-makers can better understand how improvements in one objective affect others, leading to more informed and balanced decisions. Additionally, the Pareto optimal concept supports stakeholder negotiations and consensus-building by presenting a set of optimal trade-off solutions. This allows stakeholders with differing priorities to identify mutually acceptable compromises, ensuring that the final decision reflects a balanced consideration of all objectives and results in more sustainable outcomes. However, the epsilon-constrained approach sometimes leaves decision-makers unable to clearly select the most appropriate solution, or without clear preferences for specific objectives. To address this, methods like the linear normalization max method, introduced by Jafaryeganeh et al. (2020), have been developed. This method normalizes attribute values relative to the maximum value for each criterion, enabling comprehensive comparison by summing the normalized ratios of all objectives (Zargaryan et al., 2020).

CHAPTER 3
RESEARCH METHODOLOGIES AND CASE STUDIES

This chapter explores various research methodologies used to address optimization problems under uncertainty, with a particular emphasize on fuzzy optimization approaches. These methodologies play a significant role in advancing optimization practices, especially in environments marked by uncertainty and vagueness. By examining these approaches through the case studies, this chapter highlights their contributions to improving decision-making and optimizing outcomes under uncertain.

3.1 A Conventional Specific Fuzzy Optimization Approach

In this study, a conventional specific fuzzy optimization approach is utilized as a benchmark to appraise the performance of the developed fuzzy optimization approach. By using the conventional method as a reference, the efficiency of the proposed approach can be assessed, particularly in its capacity to address the complexities of uncertainty and conflicting objectives in supply chain planning. The following section outlines the key steps and processes of the conventional specific fuzzy optimization approach, which have been historically employed to tackle supply chain optimization challenges under uncertain circumstances.

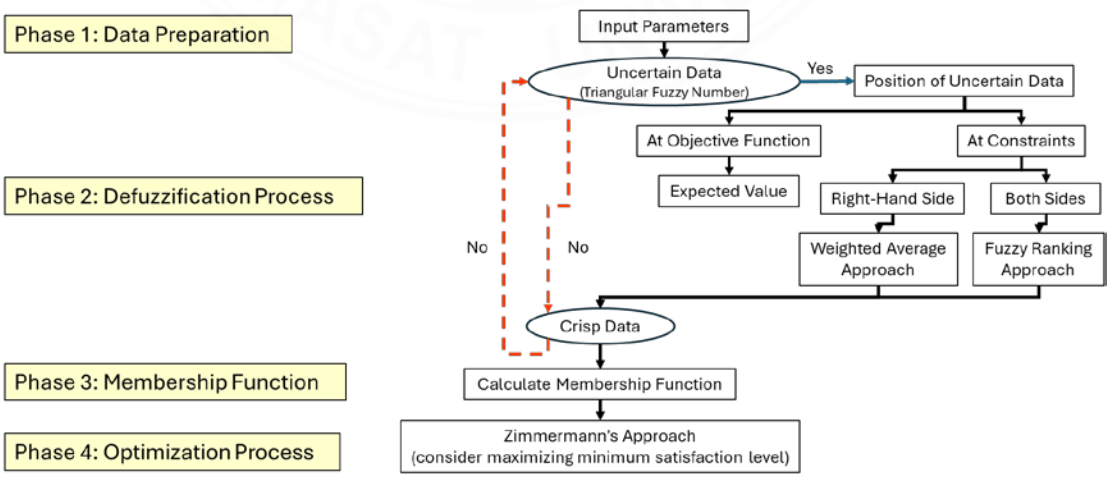

Figure 3.1 Methodology of a conventional specific fuzzy optimization approach.

Phase 1: Data Preparation: Data can typically be classified into two categories: crisp and imprecise. Crisp data are clearly defined and precisely known, making them easy to gather, whereas imprecise data are challenging to collect and manage. To address this, Triangular Fuzzy Numbers (TFNs) are often employed to represent uncertain parameters in a conventional specific fuzzy linear programming models. However, this approach does not adequately capture data hesitation.

Phase 2: Defuzzification Process: This process can convert uncertain data into crisp data. Model fuzziness can be segmented into two primary types based on its position within the model: fuzziness in objective functions and fuzziness in constraints.

- **Defuzzification Approach at the objective functions:** Expected Value (EV) serves as a conventional technique for defuzzifying objective functions by emphasizing their average performance, as shown in the following (Heilpem (1992)).

$$EV(\tilde{Z}) = \frac{\frac{Z^o+Z^m}{2}+\frac{Z^m+Z^p}{2}}{2} = \frac{Z^o+2Z^m+Z^p}{4} \tag{3.1}$$

  where Z^o, Z^m, and Z^p represent the objective function values under optimistic, most likely, and pessimistic scenarios, respectively.

- **Defuzzification Approach at the right-hand side of the constraint:** The Weighted Average (WA) is a mathematical technique used to combine multiple values into a single representative value, with each component assigned a weight based on its relative importance or contribution. It is particularly useful in decision-making and optimization processes, especially when dealing with uncertain or imprecise data. By assigning different weights to individual values, the weighted average ensures that more significant factors have a greater influence on the result. Mathematically, it is calculated by multiplying each value by its corresponding weight and summing the products, providing a flexible approach to aggregating information and reflecting the relative importance of each component in the overall outcome. However, it is important to note that the WA approach does not account for managing risk violations.

$$w^o b^o + w^m b^m + w^p b^p \tag{3.2}$$

$$w^o + w^m + w^p = 1 \tag{3.3}$$

where b^o, b^m, and b^p represent the available resources under optimistic, most likely, and pessimistic conditions, respectively, while w^o, w^m, and w^p correspond to the weights allocated to each of these scenarios.

- **Defuzzification Approach at both sides of the constraint:** The Fuzzy Ranking (FR) approach is a sophisticated defuzzification technique used to handle fuzzy constraints, particularly in scenarios with imprecise values on both sides of an equation. This method addresses uncertainty by decomposing the original fuzzy equation into three sub-equations, each representing optimistic, most likely, and pessimistic cases. These sub-equations align to the lower bound, central value, and upper bound of the fuzzy sets, respectively. By analyzing these cases separately, the FR approach enables the transformation of fuzzy relationships into clear, crisp insights, allowing for a structured evaluation of potential outcomes. However, it is significant to note that the FR approach does not address the management of risk violations.

$$A^o x \leq b^o \tag{3.4}$$

$$A^m x \leq b^m \tag{3.5}$$

$$A^p x \leq b^p \tag{3.6}$$

where A^o, A^m, and A^p, along with b^o, b^m, and b^p, correspond to the values associated with the optimistic, most probable, and pessimistic scenarios, respectively.

Phase 3: Membership Function: The procedure normalizes the differing units of several objective functions onto a standardized range, typically from 0.0 to 1.0, indicating satisfaction levels, as shown below.

- **Membership Function for Minimizing the Objective Function**

$$\mu_{Z_i} = \begin{cases} 1, & Z_i = Z_i^{PIS} \\ \frac{Z_i^{NIS} - Z_i}{Z_i^{NIS} - Z_i^{PIS}}, & Z_i^{PIS} \le Z_i \le Z_i^{NIS} \\ 0, & Z_i = Z_i^{NIS} \end{cases} \tag{3.7}$$

- **Membership Function for Maximizing the Objective Function**

$$\mu_{Z_i} = \begin{cases} 1, & Z_i = Z_i^{PIS} \\ \frac{Z_i - Z_i^{NIS}}{Z_i^{PIS} - Z_i^{NIS}}, & Z_i^{NIS} \le Z_i \le Z_i^{PIS} \\ 0, & Z_i = Z_i^{NIS} \end{cases} \tag{3.8}$$

where Z_i^{NIS} is the maximum value of the *i*th objective function among the solutions of individual minimization problems or the minimum value of the *i*th objective function among the solutions of individual maximization problems. Z_i^{PIS} is the minimum value of the *i*th objective function among the solutions of individual minimization problems or the maximum value of the *i*th objective function among the solutions of individual maximization problems.

Phase 4: Optimization Process: Zimmermann's approach is a foundational methodology in fuzzy optimization, designed to tackle multi-objective decision-making under uncertainty. Developed by Hans-Jürgen Zimmermann in 1978, it combines fuzzy set theory with optimization techniques to address conflicting objectives and imprecise data. This approach allows decision-makers to model uncertainty and achieve a balance between multiple objectives by using fuzzy membership functions, which represent the satisfaction level for each objective. Unlike a conventional optimization, which assumes rigid, well-defined functions, Zimmermann's method accommodates vagueness by expressing objectives as fuzzy goals, where solutions are evaluated based on their proximity to a satisfactory level. The membership functions are aggregated using a max-min operator, aiming to maximize the minimum satisfaction level across all objectives, ensuring a fair balance without extreme trade-offs.

$$\begin{aligned} &Maximize\ \mu_Z \\ &Subjected\ to: x \in F(x) \\ &\qquad\qquad \mu_Z \leq \mu_{Z_i}, \quad i = 1, 2, \dots, I \end{aligned} \tag{3.9}$$

where μ_Z indicates the lowest satisfaction level across all objective functions, and μ_{Z_i} refers to the satisfaction level of each specific objective function.

Numerical example for conventional specific fuzzy optimization

AB manufacturing is the company that makes a line of high qualities Glasses, Bottles, and Cups. It has three plants; Plant1, Plant2, and Plant3, that are used to produce high qualities Glasses, Bottles, and Cups. To produce a Glass, the production time is 2 hours/unit at Plant1. The available production capacity of Plant1 varies according to a triangular distribution with a minimum available production capacity of 8 hours, a most likely available production capacity of 16 hours, and a maximum available production capacity of 24 hours. To produce a Bottle, the production time at Plant2 varies according to a triangular distribution with a minimum production time of 1.5 hours/unit, a most likely production time of 2.5 hours/unit, and a maximum production time of 3.5 hours/unit. The available production capacity of Plant2 varies according to a triangular distribution with a minimum available production capacity of 12 hours, a most likely available production capacity of 24 hours, and a maximum available production capacity of 36 hours. To produce a Cup, the production time is 1.5 hours/unit at Plant3. The available production capacity of Plant3 is 36 hours. Profits of a Glass, a Bottle and a Cup are calculated as ($15, $20, $25), ($25, $30, $35), and ($35, $40, $45), respectively. The AB manufacturing attempts to find out not only how many units of Glasses, Bottles and Cups that should be produced to maximize total profit but also minimize total amount of pollution. For simplicity, the amount of pollution follows a linear function resulting from three decision variables X_1, X_2 and X_3.

$$2X_1 + 3X_2 + 4X_2$$

where X_1, X_2 and X_3 denote decision variables representing numbers of produced Glass, Bottle, and Cup, respectively.

Table 3.1 Parameters relate to production of Glass, Bottle, and Cup.

	Glass	Bottle	Cup	Available Production Capacity
Production Time (Plant1)	2 hours/unit	-		(8 hours, 16 hours, 24 hours)
Production Time (Plant2)	-	(1.5 hours/unit, 2.5 hours/unit, 3.5 hours/unit)		(12 hours, 24 hours, 36 hours)
Production Time (Plant3)	-	-	1.5 hours/unit	36 hours
Profit	($15/unit, $20/unit, $25/unit)	($25/unit, $30/unit, $35/unit)	($35/unit, $40/unit, $45/unit)	-

<u>Mathematical Formulation</u>

- **Objective Functions**

  1.Maximize total profits (defuzzify by Expected Value)

  $Maximize\ Z_1 = \frac{(15+(2*20)+25)}{4}X_1 + \frac{(25+(2*30)+35)}{4}X_2 + \frac{(35+(2*40)+45)}{4}X_3$

  2.Minimize total amount of pollution

  $Minimize\ Z_2 = 2X_1 + 3X_2 + 4X_2$

- **Uncertain Constraints**

  1.Defuzzify by Weighted Average (equally weights assigned (33%))

  Subject to: $2X_1 \leq (0.33 \times 8) + (0.33 \times 16) + (0.33 \times 24)$

  2.Defuzzify by Weighted Average (equally weights assigned (33%))

  Subject to: $1.5X_2 \leq 12$

  $2.5X_2 \leq 24$

  $3.5X_2 \leq 36$

- **Crisp Constraint**

  Subject to: $1.5X_2 \leq 36$

- **Non-negativity Constraint**

  Subject to: $X_1, X_2, X_3 \geq 0$

- **Membership Functions**

  1.Membership Function for Maximization of the Objective Function (Maximize total profits)

  $\mu_{Z_1} = \frac{Z_1 - Z_1^{NIS}}{Z_1^{PIS} - Z_1^{NIS}}$

  Z_1^{PIS} can be calculated as follows:

  $Maxmize\ Z_1 = \frac{(15+(2*20)+25)}{4}X_1 + \frac{(25+(2*30)+35)}{4}X_2 + \frac{(35+(2*40)+45)}{4}X_3$

  Subject to: $2X_1 \leq (0.33 \times 8) + (0.33 \times 16) + (0.33 \times 24)$

  $1.5X_2 \leq 12$

  $2.5X_2 \leq 24$

  $3.5X_2 \leq 36$

  $1.5X_2 \leq 36$

  $X_1, X_2, X_3 \geq 0$

  Z_1^{NIS} can be calculated as follows:

  $Minimize\ Z_1 = \frac{(15+(2*20)+25)}{4}X_1 + \frac{(25+(2*30)+35)}{4}X_2 + \frac{(35+(2*40)+45)}{4}X_3$

  Subject to: $2X_1 \leq (0.33 \times 8) + (0.33 \times 16) + (0.33 \times 24)$

  $1.5X_2 \leq 12$

  $2.5X_2 \leq 24$

  $3.5X_2 \leq 36$

  $1.5X_2 \leq 36$

  $X_1, X_2, X_3 \geq 0$

2.Membership Function for Minimizing the Objective Function (Minimize total amount of pollution)

$\mu_{Z_2} = \frac{Z_2^{NIS} - Z_2}{Z_2^{NIS} - Z_2^{PIS}}$

Z_2^{PIS} can be calculated as follows:

$Minimize\ Z_2 = 2X_1 + 3X_2 + 4X_2$

Subject to: $2X_1 \leq (0.33 \times 8) + (0.33 \times 16) + (0.33 \times 24)$

$1.5X_2 \leq 12$

$2.5X_2 \leq 24$

$3.5X_2 \leq 36$

$1.5X_2 \leq 36$

$X_1, X_2, X_3 \geq 0$

Z_2^{NIS} can be calculated as follows:

$Maximize\ Z_2 = 2X_1 + 3X_2 + 4X_2$

Subject to: $2X_1 \leq (0.33 \times 8) + (0.33 \times 16) + (0.33 \times 24)$

$1.5X_2 \leq 12$

$2.5X_2 \leq 24$

$3.5X_2 \leq 36$

$1.5X_2 \leq 36$

$X_1, X_2, X_3 \geq 0$

- **Optimization Process by Zimmermann's Approach**

$Minimize\ \mu_Z$

Subject to: $2X_1 \leq (0.33 \times 8) + (0.33 \times 16) + (0.33 \times 24)$

$1.5X_2 \leq 12$

$2.5X_2 \leq 24$

$3.5X_2 \leq 36$

$1.5X_2 \leq 36$

$X_1, X_2, X_3 \geq 0$

$\mu_Z \leq \mu_{Z_1} = \frac{Z_1 - Z_1^{NIS}}{Z_1^{PIS} - Z_1^{NIS}}$

$\mu_Z \leq \mu_{Z_2} = \frac{Z_2^{NIS} - Z_2}{Z_2^{NIS} - Z_2^{PIS}}$

3.2 A Five-Phase Hybrid Fuzzy Optimization Approach

The critical problem addressed in this study lies in the limitations of a conventional specific fuzzy optimization approaches in Supply Chain Aggregate Production Planning (SCAPP), which often fail to adequately address real-world complexities. These limitations include issues such as hesitation in decision-making, insufficient robustness to handle uncertainty, an inability to account for non-satisfaction levels, and challenges in achieving Pareto optimality. Existing methods are often ill-equipped to manage the dynamic, multifaceted nature of modern supply chains, where multiple conflicting objectives and uncertainties must be considered. The proposed five-phase hybrid fuzzy optimization approach aims to overcome these deficiencies by systematically improving upon conventional models, offering a more robust and effective framework for decision-makers. This study underscores the necessity of an advanced methodology capable of addressing these shortcomings, providing a more comprehensive solution to the complex challenges faced in SCAPP. A five-phase hybrid fuzzy optimization approach is proposed to assist decision-makers in addressing real-world challenges in Supply Chain Production Planning (SCPP) while overcoming the limitations of the conventional specific fuzzy optimization approach. The proposed algorithm is structured into five distinct phases, each carefully designed to address specific shortcomings of the conventional specific fuzzy optimization approach, with detailed explanations of each phase provided below, as demonstrated in Figure 3.2.

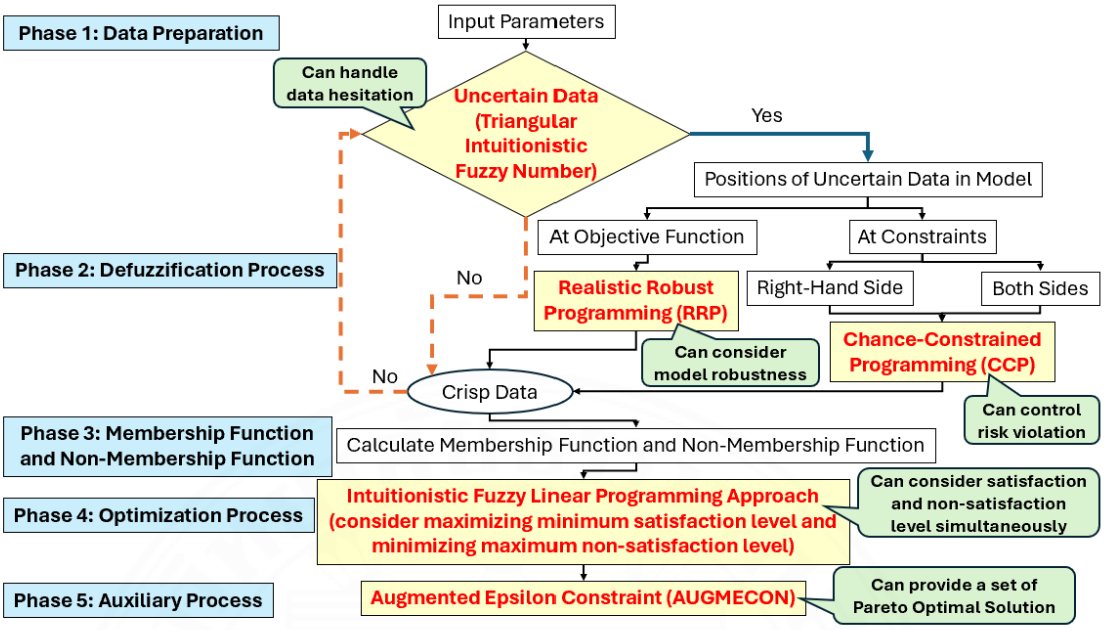

Figure 3.2 Methodology of a five-phase hybrid fuzzy optimization approach.

Phase 1: Data Preparation: Hesitation often arises during the data collection process, and a triangular fuzzy number alone is insufficient to address this uncertainty. To overcome this limitation, the triangular intuitionistic fuzzy number is introduced as an unconventional extension of the fuzzy set concept that can handle data hesitation. This approach retains the essence of the triangular distribution while incorporating hesitation to create a more flexible and representative fuzzy set for decision-makers.

To enhance decision-making, the triangular intuitionistic fuzzy number is applied using the (A, R)-cut approach. This approach enables the generation of an acceptable triangular fuzzy number by controlling the percentage of acceptance level (A) and rejection level (R). The formulation of the (A, R)-cut approach is as follows:

$$Let\ \tilde{B} = (b^o, b^m, b^p)\left(\bar{b}^o, b^m, \bar{b}^p\right)\ where\ \bar{b}^o \le b^o \le b^m \le b^p \le \bar{b}^p$$

$$B^o = max\left\{\bar{b}^o + A(b^m - b^o), b^m - R(b^m - b^o)\right\} \tag{3.10}$$

$$B^m = \frac{B^o + B^p}{2} \tag{3.11}$$

$$B^p = min\{\bar{b}^p - A(b^p - b^m), b^m + R(b^p - b^m)\} \tag{3.12}$$

where b^o, b^m, and b^p represent three key data points of the membership function for the optimistic, most likely, and pessimistic situations, respectively and $\bar{b}^o$, b^m, and $\bar{b}^p$ correspond to the three key data points of the non-membership function for these same situations. The A and R values represent possibility degrees of the acceptable level and rejection level, respectively.

Phase 2: Defuzzification Process: At this stage, all fuzzy data are converted into crisp data, and the risk violations of constraints can also be effectively controlled.

- **Defuzzification Approach at the objective functions:** The concept of model robustness in optimization, as introduced by Mulvey et al. (1995), is divided into two main forms: optimality robustness, which focuses on keeping the solution close to the best possible outcome under uncertainty, and feasibility robustness, which ensures that constraints are met despite uncertain factors. To effectively manage both types of robustness, the Robust Programming (RP) approach was developed, allowing decision-makers to balance the trade-offs between optimality and feasibility under uncertainty. Pishvaee et al. (2012) proposed a comprehensive classification of RP into three distinct classifications: Hard Worst Robust Programming (HWRP), Soft Worst Robust Programming (SWRP), and Realistic Robust Programming (RRP). HWRP is the most conservative approach, designed to minimize the impact of the worst-case scenario under uncertainty. It guarantees that the solution retains feasibility and delivers satisfactory performance even in the worst-case scenarios. This method is especially appropriate for critical situations where failure cannot be afforded. SWRP introduces flexibility by relaxing the strict robustness of HWRP. It allows for a controlled level of risk by tolerating some constraint violations or performance degradation, making it more adaptable to real-world situations where absolute robustness may be unnecessarily rigid or costly. RRP aims to strike a practical balance between conservatism and flexibility. It incorporates more realistic assumptions about nature and the likelihood of uncertainty, often using probabilistic or scenario-based methods. This approach seeks to improve solution quality while maintaining a reasonable level of robustness, thereby enhancing its applicability in practical decision-making contexts. For this research, RRP is selected as the most suitable approach for making robustness of the model, as it offers a practical compromise between optimal performance and feasible solutions, making it ideal for real-world business decision-making. This approach considers three key aspects: the average total performance of the objectives, optimality robustness, and feasibility robustness, as shown below.

$$Minimize\ EV(\tilde{Z}) + Optimality\ Robustness + Feasibility\ Robustness$$

$$Minimize\ \frac{Z^o + 2Z^m + Z^p}{4} + \rho(Z_{max} - Z_{min})$$
$$+((\sigma(d^p - (1-\gamma)d^m - \gamma d^p) + \delta(\gamma B^o + (1-\gamma)B^m - B^o))$$
$$Subject\ to: Gx \geq (1-\gamma)d^m + \gamma d^p$$
$$(\gamma B^o + (1-\gamma)B^m - B^o)y \geq Hx \qquad (3.13)$$

The first term of the objective function targets to minimize the average total performance of the objectives under consideration. The second term represents the difference between the two extreme possible values of Z, where Z_{max} and Z_{min} are determined as follows:

$$Z_{max} = f^p y + c^p x \qquad (3.14)$$

$$Z_{min} = f^o y + c^o x \qquad (3.15)$$

where f^p and c^p represent the worst-case values of the objective function coefficients, while f^o and c^o represent best-case values. x and y are the decision variables associated with the constraints and G and H are crisp coefficients within the constraint structure. In the minimization context, Z^o, Z^m, and Z^p correspond to the optimistic, most likely, and pessimistic values of the objective function, respectively. The parameters d^m and d^p are imprecise right-hand side coefficients of the constraints, representing the most likely and pessimistic values. Similarly, B^o and B^m are imprecise left-hand side coefficients representing the optimistic and most likely values. The parameter ρ denotes the weight assigned to the second term in the objective function, which governs optimality robustness by minimizing the maximum deviation above and below the expected optimal value. The third term addresses feasibility robustness by evaluating the deviation of each constraint, where σ and δ are penalty values associated with potential violations. Additionally, γ represents the confidence level percentage used to manage the risk of constraint violations.

- **Defuzzification Approach at the constraints:** Chance-Constrained Programming (CCP) was originally introduced by Charnes and Cooper (1959) as part of the stochastic programming framework, where constraints involving random variables need to conform to a certain probability level. This approach is widely used in operations research and mathematical programming to manage probabilistic uncertainty. However, in recent decades, this concept has been extended to the fuzzy optimization domain, where the underlying uncertainty is not stochastic but linguistic, imprecise, or fuzzy in nature. In such contexts, some researchers (Liu and Iwamura, 1998; Chakraborty, 2002) have employed the term chance-constrained programming to describe models where fuzzy constraints must comply with a certain possibility level or necessity level. In this interpretation, the term "chance" reflects the possibility measure rather than the probability measure. In this study, the term chance-constrained programming is used in the latter sense, referring to a fuzzy-based generalization of the classical CCP model. Specifically, this study models uncertain parameters as fuzzy numbers and interprets the constraint satisfaction in terms of possibility theory rather than probability theory.

  Chance-Constrained Programming (CCP) is a robust optimization methodology designed to manage uncertainty by integrating fuzzy measurements, specifically credibility, into the optimization model. This technique allows for the handling of fuzzy data while ensuring that constraints are satisfied with a certain level of confidence. The credibility metric, represented by a possibility degree (γ), quantifies the likelihood that fuzzy constraints will be met, with higher values indicating greater confidence. This probabilistic approach provides a more realistic framework for decision-making compared to a conventional deterministic method, accounting for real-world variability. By incorporating credibility into the model, CCP allows decision-makers to adjust the confidence level (control risk violation) according to the importance and nature of the constraints, making the solution more reliable but potentially more conservative.

- **For imprecise right-hand side of constraints**

$$Cr\{\sum_{j=1}^{n} a_{ij}x_j \leq \tilde{b}_i\} \geq \gamma \quad if\ and\ only\ if$$
$$when\ (0 \leq \gamma \leq 0.5): ax \leq (2\gamma)b^m + (1-2\gamma)b^p$$
$$when\ (0.5 < \gamma \leq 1): ax \leq (2\gamma-1)b^o + (2-2\gamma)b^m \qquad (3.16)$$

where b^o, b^m, and b^p correspond to the resource availability under optimistic, most probable, and pessimistic conditions, respectively.

- **For imprecise left-hand side and right-hand side of constraints**

$$Cr\{\sum_{j=1}^{n} \widetilde{a_{ij}}x_j \leq \tilde{b}_i\} \geq \gamma \quad if\ and\ only\ if$$
$$when\ (0 \leq \gamma \leq 0.5): (2\gamma)a^m + (1-2\gamma)a^p x \leq (2\gamma)b^m + (1-2\gamma)b^p$$
$$when\ (0.5 < \gamma \leq 1): (2\gamma-1)a^o + (2-2\gamma)a^m x \leq (2\gamma-1)b^o + (2-2\gamma)b^m$$
$$(3.17)$$

where a^o, a^m, and a^p are the coefficients values in optimistic, most likely, and pessimistic situations, respectively. b^o, b^m, and b^p are values of available resource in optimistic, most likely, and pessimistic situations, respectively.

Phase 3: Membership Function and Non-Membership Function: This process is used to normalize the different units of multiple objective functions to a common scale (0.0–1.0), referred to as the satisfaction level, as shown in Equations (3.7) and (3.8). Additionally, the proposed approach enables decision makers to consider both satisfaction and non-satisfaction levels simultaneously. Consequently, Equations (3.18) and (3.19) can be applied to calculate the non-membership function.

- **Non-Membership Function for Minimizing the Objective Function**

$$\tau_{Z_i} = \begin{Bmatrix} 1, & Z_i = Z_i^{NIS} \\ \frac{Z_i - Z_i^{NIS}}{Z_i^{PIS} - Z_i^{NIS}}, & Z_i^{NIS} \leq Z_i \leq Z_i^{PIS} \\ 0, & Z_i = Z_i^{PIS} \end{Bmatrix} \tag{3.18}$$

- **Non-Membership Function for Maximizing the Objective Function**

$$\tau_{Z_i} = \begin{Bmatrix} 1, & Z_i = Z_i^{NIS} \\ \frac{Z_i^{NIS} - Z_i}{Z_i^{NIS} - Z_i^{PIS}}, & Z_i^{PIS} \leq Z_i \leq Z_i^{NIS} \\ 0, & Z_i = Z_i^{PIS} \end{Bmatrix} \tag{3.19}$$

where Z_i^{NIS} represents the highest value of the i^{th} objective function from individual maximization problem solutions, or the lowest value from individual minimization problem solutions. Conversely, Z_i^{PIS} denotes the lowest value of the i^{th} objective function among maximization solutions, or the highest value among minimization solutions.

Phase 4: Optimization Process: The Intuitionistic Fuzzy Linear Programming (IFLP) approach extends conventional linear programming to address decision-making problems characterized by uncertainty and imprecision. Unlike conventional fuzzy set theory, which only uses membership functions to represent fuzzy data, IFLP incorporates both membership and non-membership functions, offering a more comprehensive modeling of uncertainty. This dual representation allows decision-makers to factor in different levels of uncertainty, making IFLP particularly useful in scenarios where data is imprecise. One of its key advantages is its ability to integrate both optimism and pessimism in decision-making, offering a more flexible and powerful tool compared to conventional specific fuzzy linear programming, especially in capturing a broader range of uncertainty. This is achieved by simultaneously maximizing the minimum satisfaction level and minimizing the maximum non-satisfaction level of the multiple objective functions, as shown below.

$$\begin{aligned} & Maximize\ \mu_Z - \tau_Z \\ & Subjected\ to{:}\ x \in F(x) \\ & \qquad \mu_Z \leq \mu_{Z_i}, \quad i = 1, 2, \dots, I \\ & \qquad \tau_Z \geq \tau_{Z_i}, \quad i = 1, 2, \dots, I \end{aligned} \tag{3.20}$$

where μ_{Z_i} and τ_{Z_i} represent the membership and non-membership functions corresponding to each objective function, respectively.

Phase 5: Auxiliary Process: The Augmented Epsilon Constrained (AUGMECON) method is an optimization technique used in multi-objective programming to convert a problem with conflicting objectives into a series of single-objective problems. By systematically adjusting constraints on secondary objectives with specified epsilon (ε) values, the method generates a set of Pareto optimal solutions, offering a spectrum of trade-offs for decision-makers to evaluate. In AUGMECON, one objective is chosen as the primary goal, while the others are treated as constraints. The iterative process of varying ε values allows for the exploration of different balance points between competing objectives. This approach simplifies the problem by focusing on one objective at a time, providing a clearer understanding of how changes in one objective affect others. Additionally, this approach supports decision-making by generating diverse solutions, each with its own trade-offs, enabling decision-makers to identify the solution that most effectively corresponds to their objectives and priorities.

$$Maximize\ f_1(x) + (eps \times \left(\frac{S_2}{r_2} + \cdots + \frac{S_i}{r_i}\right))$$
$$Subject\ to: f_2(x) - S_2 = \varepsilon_2$$
$$f_i(x) - S_i = \varepsilon_i \tag{3.21}$$

where $eps \in [10^{-6}, 10^{-3}]$. $S_2, \ldots, S_i$ are surplus variables of respective constraints. $r_2, \ldots, r_i$ are ranges of each objective function. Parameters $\varepsilon_2, \ldots, \varepsilon_i$ represent the right-hand side values for a given iteration, selected from the grid points corresponding to each objective function.

The following steps describe the AUGMECON approach:

Step 1: Determine the range between minimum and maximum values of each objective function (r_i)

S**tep 2**: Divide the range between minimum and maximum values of each objective function into equal portions (p_i) and then, the total grid points ($p_i + 1$) are utilized from varying the epsilon values of each objective function.

Step 3: Calculate discretization step for the respective objective function as follows:

$$Step_i = \left(\frac{r_i}{p_i}\right) \tag{3.22}$$

Step 4: Compute the epsilon parameters for the relevant constraint during the h^{th} iteration within a given objective function as shown below:

$$e_i = \omega_i^{min} + (h \times Step_i)\ where\ h = 0, \ldots, p_i \tag{3.23}$$

ω_i^{min} is minimum value of i^{th} objective function.

Step 5: Check a surplus variable value (S_i) that corresponds to the innermost objective function.

Step 6: Bypass the redundant iterations by using the bypass coefficient (bp) that can be calculated as follows:

$$bp = int\left(\frac{S_i}{Step_i}\right) \quad (3.24)$$

where $int()$ is a function that is used to return an integer value of a real number.

Step 7: Iterate Steps 4 through 6 until the final iteration is reached.

Numerical example for conventional specific fuzzy optimization

AB manufacturing is the company that makes a line of high qualities Glasses, Bottles, and Cups. It has three plants; Plant1, Plant2, and Plant3, that are used to produce high qualities Glasses, Bottles, and Cups. To produce a Glass, the production time is 2 hours/unit at Plant1. The available production capacity of Plant1 are (8, 16, 24)(4, 16, 30) hours that are varied according to an intuitionistic triangular distribution. To produce a Bottle, the production time at Plant2 are (1.5, 2.5, 3.5)(1, 2.5, 4) hours/unit that are varied according to an intuitionistic triangular distribution. The available production capacity of Plant2 are (12, 24, 36)(6, 24, 42) hours that are varied according to an intuitionistic triangular distribution. To produce a Cup, the production time is 1.5 hours/unit at Plant3. The available production capacity of Plant3 is 36 hours. Profits of a Glass, a Bottle and a Cup are calculated as ($15, $20, $25)($10, $20, $30), ($25, $30, $35)($20, $30, $40), and ($35, $40, $45)($30, $40, $50), respectively. The AB manufacturing attempts to find out not only how many units of Glasses, Bottles and Cups that should be produced to maximize total profit but also minimize total amount of pollution. For simplicity, the amount of pollution follows a linear function resulting from three decision variables X_1, X_2 and X_3.

$$2X_1 + 3X_2 + 4X_2$$

where X_1, X_2 and X_3 denote decision variables representing numbers of produced Glass, Bottle, and Cup, respectively.

- **Data Preparation**

  To enhance decision-making, the triangular intuitionistic fuzzy number is applied using the (A, R)-cut approach. This approach enables the generation of an acceptable triangular fuzzy number by controlling the percentage of acceptance level (A) and rejection level (R). Assume that $A = 80\%$ and $R = 20\%$.

  1.The available production capacity of Plant1

  $\tilde{B} = (b^o, b^m, b^p)(\bar{b}^o, b^m, \bar{b}^p) = (8, 16, 24)(4, 16, 30)$

  $B^o = max\{\bar{b}^o + A(b^m - b^o), b^m - R(b^m - b^o)\}$

  $= max\{4 + 0.8(16 - 8), 16 - 0.2(16 - 8)\}$

  $= max\{10.4, 14.4\} = 14.4$

$$B^m = \frac{B^o + B^p}{2} = \frac{14.4 + 18.8}{2} = 16.6$$

$$B^p = min\{\bar{b}^p - A(b^p - b^m), b^m + R(b^p - b^m)\}$$

$$= min\{30 - 0.8(24 - 26), 16 + 0.2(30 - 16)\}$$

$$= min\{23.6, 18.8\} = 18.8$$

The available production capacity of Plant1 is (14.4, 16.6, 18.8).

2.The production time at Plant2

$$\tilde{B} = (b^o, b^m, b^p)(\bar{b}^o, b^m, \bar{b}^p) = (1.5, 2.5, 3.5)(1, 2.5, 4)$$

$$B^o = max\{\bar{b}^o + A(b^m - b^o), b^m - R(b^m - b^o)\}$$

$$= max\{1 + 0.8(2.5 - 1.5), 2.5 - 0.2(2.5 - 1.5)\}$$

$$= max\{1.8, 2.3\} = 2.3$$

$$B^m = \frac{B^o + B^p}{2} = \frac{2.3 + 2.7}{2} = 2.5$$

$$B^p = min\{\bar{b}^p - A(b^p - b^m), b^m + R(b^p - b^m)\}$$

$$= min\{4 - 0.8(3.5 - 2.5), 2.5 + 0.2(3.5 - 2.5)\}$$

$$= min\{3.2, 2.7\} = 2.7$$

The production time at Plant2 is (2.3, 2.5, 2.7).

3.The available production capacity of Plant2

$$\tilde{B} = (b^o, b^m, b^p)(\bar{b}^o, b^m, \bar{b}^p) = (12, 24, 36)(6, 24, 42)$$

$$B^o = max\{\bar{b}^o + A(b^m - b^o), b^m - R(b^m - b^o)\}$$

$$= max\{6 + 0.8(24 - 12), 24 - 0.2(24 - 12)\}$$

$$= max\{15.6, 21.6\} = 21.6$$

$$B^m = \frac{B^o + B^p}{2} = \frac{21.6 + 26.4}{2} = 24$$

$$B^p = min\{\bar{b}^p - A(b^p - b^m), b^m + R(b^p - b^m)\}$$

$$= min\{42 - 0.8(36 - 24), 24 + 0.2(36 - 24)\}$$

$$= min\{32.4, 26.4\} = 26.4$$

The available production capacity of Plant2 is (21.6, 24, 26.4).

4. Profits of a Glass

$$\tilde{B} = (b^o, b^m, b^p)(\bar{b}^o, b^m, \bar{b}^p) = (\$15, \$20, \$25)(\$10, \$20, \$30)$$

$$B^o = max\{\bar{b}^o + A(b^m - b^o), b^m - R(b^m - b^o)\}$$

$$= max\{10 + 0.8(20 - 15), 20 - 0.2(20 - 15)\}$$

$$= max\{14, 19\} = 19$$

$B^m = \frac{B^o+B^p}{2} = \frac{19+21}{2} = 20$

$B^p = min\{\bar{b}^p - A(b^p - b^m), b^m + R(b^p - b^m)\}$

$= min\{30 - 0.8(25 - 20), 20 + 0.2(25 - 20)\}$

$= min\{26, 21\} = 21$

Profits of a Glass is (19, 20, 21).

5. Profits of a Bottle

$\tilde{B} = (b^o, b^m, b^p)(\bar{b}^o, b^m, \bar{b}^p) = (\$25, \$30, \$35)(\$20, \$30, \$40)$

$B^o = max\{\bar{b}^o + A(b^m - b^o), b^m - R(b^m - b^o)\}$

$= max\{20 + 0.8(30 - 25), 30 - 0.2(30 - 25)\}$

$= max\{24, 29\} = 29$

$B^m = \frac{B^o+B^p}{2} = \frac{29+31}{2} = 30$

$B^p = min\{\bar{b}^p - A(b^p - b^m), b^m + R(b^p - b^m)\}$

$= min\{40 - 0.8(35 - 30), 30 + 0.2(35 - 30)\}$

$= min\{36, 31\} = 31$

Profits of a Bottle is (29, 30, 31).

6. Profits of a Cup

$\tilde{B} = (b^o, b^m, b^p)(\bar{b}^o, b^m, \bar{b}^p) = (\$35, \$40, \$45)(\$30, \$40, \$50)$

$B^o = max\{\bar{b}^o + A(b^m - b^o), b^m - R(b^m - b^o)\}$

$= max\{30 + 0.8(40 - 35), 40 - 0.2(40 - 35)\}$

$= max\{34, 39\} = 39$

$B^m = \frac{B^o+B^p}{2} = \frac{41+21}{2} = 20$

$B^p = min\{\bar{b}^p - A(b^p - b^m), b^m + R(b^p - b^m)\}$

$= min\{50 - 0.8(45 - 40), 40 + 0.2(45 - 40)\}$

$= min\{46, 41\} = 41$

Profits of a Cup is (39, 40, 41).

Table 3.2 Data preparation for production of Glass, Bottle, and Cup.

	Glass	Bottle	Cup	Available Production Capacity
Production Time (Plant1)	2 hours/unit	-		(14.4 hours, 16.6 hours, 18.8 hours)
Production Time (Plant2)	-	(2.3 hours/unit, 2.5 hours/unit, 2.7 hours/unit).		(21.6 hours, 24 hours, 26.4 hours).
Production Time (Plant3)	-	-	1.5 hours/unit	36 hours
Profit	($19/unit, $20/unit, $21/unit).	($29/unit, $30/unit, $31/unit)	($39/unit, $40/unit, $41/unit)	-

<u>**Mathematical Formulation**</u>

- **Objective Functions**

  1.Maximize total profits (defuzzify by Realistic Robust Programming)

  $Maximize\ Z_1 = EV(\tilde{Z}) + Optimality\ Robustness + Feasibility\ Robustness$

- **First term**

$$EV(\tilde{Z}) = \frac{Z^o + 2Z^m + Z^p}{4}$$

1.Z^o can be calculated as follows:

$Maxmize\ Z_1 = 19X_1 + 29X_2 + 39X_3$

Subject to: $2X_1 \leq (2(0.8)-1)(14.4) + (2-2(0.8))(16.6)$

$(2(0.8)-1)(2.3) + (2-2(0.8))(2.5)x \leq (2(0.8)-1)(21.6) + (2-2(0.8))(24)$

$1.5X_2 \leq 36$

$X_1, X_2, X_3 \geq 0$

2.Z^m can be calculated as follows:

$Maxmize\ Z_1 = 20X_1 + 30X_2 + 40X_3$

Subject to: $2X_1 \leq (2(0.8)-1)(14.4) + (2-2(0.8))(16.6)$

$2(0.8)-1)(2.3) + (2-2(0.8))(2.5)x \leq (2(0.8)-1)(21.6) + (2-2(0.8))(24)$

$1.5X_2 \leq 36$

$X_1, X_2, X_3 \geq 0$

3.Z^p can be calculated as follows:

$Maxmize\ Z_1 = 21X_1 + 31X_2 + 41X_3$

Subject to: $2X_1 \leq (2(0.8)-1)(14.4) + (2-2(0.8))(16.6)$

$2(0.8)-1)(2.3) + (2-2(0.8))(2.5)x \leq (2(0.8)-1)(21.6) + (2-2(0.8))(24)$

$1.5X_2 \leq 36$

$X_1, X_2, X_3 \geq 0$

- **Second term**

$$Optimality\ Robustness = \rho(Z_{max} - Z_{min})$$

where ρ is assumed to be 50%.

1.Z_{max} can be calculated as follows:

$Maxmize\ Z_{max} = f^p y + c^p x = 21X_1 + 31X_2 + 41X_3$

Subject to: $2X_1 \leq (2(0.8) - 1)(14.4) + (2 - 2(0.8))(16.6)$

$2(0.8) - 1)(2.3) + (2 - 2(0.8))(2.5)x \leq (2(0.8) - 1)(21.6) + (2 - 2(0.8))(24)$

$1.5X_2 \leq 36$

$X_1, X_2, X_3 \geq 0$

2.Z_{min} can be calculated as follows:

$Maxmize\ Z_{min} = f^o y + c^o x = 19X_1 + 29X_2 + 39X_3$

Subject to: $2X_1 \leq (2(0.8) - 1)(14.4) + (2 - 2(0.8))(16.6)$

$2(0.8) - 1)(2.3) + (2 - 2(0.8))(2.5)x \leq (2(0.8) - 1)(21.6) + (2 - 2(0.8))(24)$

$1.5X_2 \leq 36$

$X_1, X_2, X_3 \geq 0$

- **Third term**

$$Feasibility\ Robustnes = \sigma(d^p - (1-\gamma)d^m - \gamma d^p) + \delta(\gamma B^o + (1-\gamma)B^m - B^o)$$

where σ and δ are assumed to be 50% and γ is assumed to be 80%.

1.The first term $\sigma(d^p - (1-\gamma)d^m - \gamma d^p)$ can be applied to uncertain right-hand side constraints

$$\sigma(d^p - (1-\gamma)d^m - \gamma d^p) = 0.5\big(18.8 - (1-0.8)16.6 - (0.8)(18.8)\big)$$

$$\sigma(d^p - (1-\gamma)d^m - \gamma d^p) = 0.5\big(21.6 - (1-0.8)24 - (0.8)(21.6)\big)$$

2.The second term $\delta(\gamma B^o + (1-\gamma)B^m - B^o)$ can be applied to uncertain left-hand side constraints

$\delta(\gamma B^o + (1-\gamma)B^m - B^o) = 0.5((0.8)(2.3) + (1-0.8)(2.5) - 2.3)$

2.Minimize total amount of pollution

$Minimize\ Z_2 = 2X_1 + 3X_2 + 4X_2$

- **Uncertain Constraints**

1.Defuzzify by Chance-Constrained Programming ($\gamma = 80\%$)

Subject to: $2X_1 \leq (2(0.8) - 1)(14.4) + (2 - 2(0.8))(16.6)$

$2(0.8) - 1)(2.3) + (2 - 2(0.8))(2.5)x \leq (2(0.8) - 1)(21.6) + (2 - 2(0.8))(24)$

- **Crisp Constraint**

Subject to: $1.5X_2 \leq 36$

- **Non-negativity Constraint**

Subject to: $X_1, X_2, X_3 \geq 0$

- **Membership Functions**

1.Membership Function for Maximizing the Objective Function (Maximize total profits)

$$\mu_{Z_1} = \frac{Z_1 - Z_1^{NIS}}{Z_1^{PIS} - Z_1^{NIS}}$$

Z_1^{PIS} can be calculated as follows:

$Maxmize\ Z_1 = EV(\tilde{Z}) + Optimality\ Robustness + Feasibility\ Robustness$

Subject to: $2X_1 \leq (2(0.8) - 1)(14.4) + (2 - 2(0.8))(16.6)$

$2(0.8) - 1)(2.3) + (2 - 2(0.8))(2.5)x \leq (2(0.8) - 1)(21.6) + (2 - 2(0.8))(24)$

$1.5X_2 \leq 36$

$X_1, X_2, X_3 \geq 0$

Z_1^{NIS} can be calculated as follows:

$Minimize\ Z_1 = EV(\tilde{Z}) + Optimality\ Robustness + Feasibility\ Robustness$

Subject to: $2X_1 \leq (2(0.8) - 1)(14.4) + (2 - 2(0.8))(16.6)$

$2(0.8) - 1)(2.3) + (2 - 2(0.8))(2.5)x \leq (2(0.8) - 1)(21.6) + (2 - 2(0.8))(24)$

$1.5X_2 \leq 36$

$X_1, X_2, X_3 \geq 0$

2.Membership Function for Minimizing the Objective Function (Minimize total amount of pollution)

$$\mu_{Z_2} = \frac{Z_2^{NIS} - Z_2}{Z_2^{NIS} - Z_2^{PIS}}$$

Z_2^{PIS} can be calculated as follows:

$Minimize\ Z_2 = 2X_1 + 3X_2 + 4X_2$

Subject to: $2X_1 \leq (2(0.8) - 1)(14.4) + (2 - 2(0.8))(16.6)$

$2(0.8) - 1)(2.3) + (2 - 2(0.8))(2.5)x \leq (2(0.8) - 1)(21.6) + (2 - 2(0.8))(24)$

$1.5X_2 \leq 36$

$X_1, X_2, X_3 \geq 0$

Z_2^{NIS} can be calculated as follows:

$Maximize\ Z_2 = 2X_1 + 3X_2 + 4X_2$

Subject to: $2X_1 \leq (2(0.8) - 1)(14.4) + (2 - 2(0.8))(16.6)$

$2(0.8) - 1)(2.3) + (2 - 2(0.8))(2.5)x \leq (2(0.8) - 1)(21.6) + (2 - 2(0.8))(24)$

$1.5X_2 \leq 36$

$X_1, X_2, X_3 \geq 0$

- **Non-membership Functions**

1.Non-membership Function for Maximizing the Objective Function (Maximize total profits)

$\tau_{Z_1} = \frac{Z_1^{NIS} - Z_1}{Z_1^{NIS} - Z_1^{PIS}}$

Z_1^{PIS} can be calculated as follows:

$Minimize\ Z_1 = \ EV(\tilde{Z}) + Optimality\ Robustness + Feasibility\ Robustness$

Subject to: $2X_1 \leq (2(0.8) - 1)(14.4) + (2 - 2(0.8))(16.6)$

$2(0.8) - 1)(2.3) + (2 - 2(0.8))(2.5)x \leq (2(0.8) - 1)(21.6) + (2 - 2(0.8))(24)$

$1.5X_2 \leq 36$

$X_1, X_2, X_3 \geq 0$

Z_1^{NIS} can be calculated as follows:

$Maximize\ Z_1 = \ EV(\tilde{Z}) + Optimality\ Robustness + Feasibility\ Robustness$

Subject to: $2X_1 \leq (2(0.8) - 1)(14.4) + (2 - 2(0.8))(16.6)$

$2(0.8) - 1)(2.3) + (2 - 2(0.8))(2.5)x \leq (2(0.8) - 1)(21.6) + (2 - 2(0.8))(24)$

$1.5X_2 \leq 36$

$X_1, X_2, X_3 \geq 0$

2.Non-membership Function for Minimizing the Objective Function (Minimize total amount of pollution)

$\tau_{Z_2} = \frac{Z_2 - Z_2^{NIS}}{Z_2^{PIS} - Z_2^{NIS}}$

Z_2^{PIS} can be calculated as follows:

$Maxmize\ Z_2 = 2X_1 + 3X_2 + \ 4X_2$

Subject to: $2X_1 \leq (2(0.8) - 1)(14.4) + (2 - 2(0.8))(16.6)$

$2(0.8) - 1)(2.3) + (2 - 2(0.8))(2.5)x \leq (2(0.8) - 1)(21.6) + (2 - 2(0.8))(24)$

$1.5X_2 \leq 36$

$X_1, X_2, X_3 \geq 0$

Z_2^{NIS} can be calculated as follows:

$Minimize\ Z_2 = 2X_1 + 3X_2 + \ 4X_2$

Subject to: $2X_1 \leq (2(0.8) - 1)(14.4) + (2 - 2(0.8))(16.6)$

$2(0.8) - 1)(2.3) + (2 - 2(0.8))(2.5)x \leq (2(0.8) - 1)(21.6) + (2 - 2(0.8))(24)$

$1.5X_2 \leq 36$

$X_1, X_2, X_3 \geq 0$

- **Optimization Process by Intuitionistic Fuzzy Linear Programming**

$Minimize\ \mu_Z - \tau_Z$

Subject to: $2X_1 \leq (2(0.8) - 1)(14.4) + (2 - 2(0.8))(16.6)$

$2(0.8) - 1)(2.3) + (2 - 2(0.8))(2.5)x \leq (2(0.8) - 1)(21.6) + (2 - 2(0.8))(24)$

$1.5X_2 \leq 36$

$X_1, X_2, X_3 \geq 0$

$\mu_Z \leq \mu_{Z_1} = \frac{Z_1 - Z_1^{NIS}}{Z_1^{PIS} - Z_1^{NIS}}$

$\mu_Z \leq \mu_{Z_2} = \frac{Z_2^{NIS} - Z_2}{Z_2^{NIS} - Z_2^{PIS}}$

$\tau_Z \geq \tau_{Z_1} = \frac{Z_1^{NIS} - Z_1}{Z_1^{NIS} - Z_1^{PIS}}$

$\tau_Z \geq \tau_{Z_2} = \frac{Z_2 - Z_2^{NIS}}{Z_2^{PIS} - Z_2^{NIS}}$

- **Auxiliary Process by Augmented Epsilon Constrained (AUGMECON) Approach**

$Maximize\ Z_1(x) + (eps \times \frac{S_2}{r_2})$

$Subject\ to: Z_2(x) - S_2 = \varepsilon_2$

$2X_1 \leq (2(0.8) - 1)(14.4) + (2 - 2(0.8))(16.6)$

$2(0.8) - 1)(2.3) + (2 - 2(0.8))(2.5)x \leq (2(0.8) - 1)(21.6) + (2 - 2(0.8))(24)$

$1.5X_2 \leq 36$

$X_1, X_2, X_3 \geq 0$

where $eps \in [10^{-6}, 10^{-3}]$. S_2 is surplus variable of 2nd objective function. r_2 is range of each objective function. ε_2 is parameter for the right-hand side for a specific iteration drawn from the grid points of each objective function.

The following steps describe the AUGMECON approach:

Step 1: Determine the range between minimum and maximum values of each objective function (r_i)

$r_1 = Maimum\ value\ of\ total\ profits - Minimum\ \ value\ of\ total\ profits$

$r_2 = Maimum\ value\ of\ total\ amount\ of\ pollutions$
$\quad -Minimum\ \ value\ of\ total\ amount\ of\ pollutions$

Step 2: Divide the range between minimum value and maximum value of each objective function into equal portions (p_i) and then, the total grid points ($p_i + 1$) are utilized from varying the epsilon values of each objective function.

$$p_i = 10$$

p_i can be assumed based on decision makers' experiences or circumstances.

Step 3: Calculate discretization step for the respective objective function as follows:

$$Step_i = \left(\frac{r_i}{p_i}\right)$$

Step 4: Calculate the epsilon values of the respective constraint in the h^{th} iteration in a particular objective function as follows:

$$e_i = \omega_i^{min} + (h \times Step_i) \; where \; h = 0, \dots, p_i$$

ω_i^{min} is minimum value of i^{th} objective function.

Step 5: Check a surplus variable value (S_i) that corresponds to the innermost objective function.

Step 6: Bypass the redundant iterations by using the bypass coefficient (bp) that can be calculated as follows:

$$bp = int\left(\frac{S_i}{Step_i}\right)$$

where $int()$ is a function that is used to return an integer value of a real number.

- **Step 7:** Repeat Steps 4 through 6 until the final iteration is reached.

3.3 A Unified Fairness and Robustness Fuzzy Optimization Approach

The critical problem addressed in this study arises from the limitations of a conventional specific fuzzy optimization approaches, which often fail to effectively manage critical challenges in multi-objective decision-making, such as ensuring Proportional Fairness (PF) among competing objectives and maintaining robustness under uncertainty. Without the integration of fairness, conventional optimization models tend to produce inequitable outcomes, where some objectives are prioritized at the expense of others, leading to suboptimal and biased decision-making. Furthermore, the lack of robustness, particularly in uncertain and ambiguous environments, compromises the reliability and effectiveness of the resulting plans. To resolve these issues, this study proposes a unified optimization model that combines Proportional Fairness (PF) with Robust Chance-Constrained Programming (RCCP) as shown in Figure 3.3, offering a more balanced and resilient decision-making framework. This approach ensures that both fairness and robustness are adequately addressed, ultimately leading to more reliable and equitable solutions in complex decision-making scenarios.

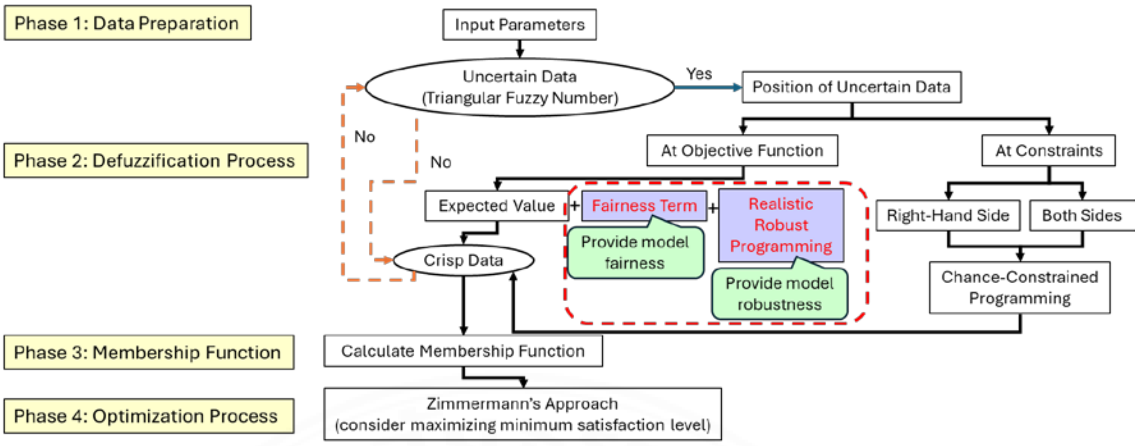

Figure 3.3 Methodology of a unified fairness and robustness fuzzy optimization approach.

Phase 1: Data Preparation: In preparing the data, parameters are grouped into crisp and uncertain categories. Crisp parameters have exact, known values, while uncertain parameters involve ambiguity or imprecision. These uncertainties are modeled using Triangular Fuzzy Numbers (TFNs).

Phase 2: Defuzzification Process: This process converts imprecise data into crisp data. The fuzziness in the model can be classified into two primary types, based on its location: fuzziness in the objective functions and fuzziness in the constraints.

- **Defuzzification Approach at the objective functions:** The Realistic Robust Programming (RRP) approach is utilized here because it is well-adapted to business and profit-centered problems, offering a reasonable trade-off between optimality and feasibility robustness. Accordingly, fairness and the optimality and feasibility elements of RRP are embedded into the EV method to improve the model's robustness and fairness, as outlined in Equations (3.25) – (3.26).

For minimization objectives:

$$\boldsymbol{EV(\tilde{Z}) + Fairness + Optimality\ Robustness + Feasibility\ Robustness}$$

$$= \frac{Z^{o}+2Z^{m}+Z^{p}}{4} + \left(Z_i - Z_i^{PIS}\right) + \rho(Z_{max} - Z_{min}) + \left(\sigma\left(d_j^{p} - (1-\gamma)d_j^{m} - \gamma d_j^{p}\right)\right) \tag{3.25}$$

For maximization objectives:

$$\boldsymbol{EV(\widetilde{Z}) + Fairness + Optimality\ Robustness + Feasibility\ Robustness}$$

$$= \frac{Z^o+2Z^m+Z^p}{4} + (Z_i^{PIS} - Z_i) + \rho(Z_{max} - Z_{min}) + (\sigma(d_j^p - (1-\gamma)d_j^m - \gamma d_j^p)) \quad (3.26)$$

where Z^o, Z^m, and Z^p represent the objective function values under optimistic, most probable, and pessimistic scenarios, respectively. Z_i and Z_i^{PIS} correspond to the value of each individual objective function and its positive ideal solution. The terms Z_{max} and Z_{min} indicate the highest and lowest values of the objective function. For constraint j, d_j^o, d_j^m, and d_j^p denote the optimistic, most likely, and pessimistic estimates of the fuzzy parameters, respectively. The parameter ρ stands for the weighting factor, while σ refers to the penalty imposed for possible constraint violations, both are assigned a value of 50% to maintain fairness. Finally, γ represents the confidence level, which is fixed at 80% in this study.

- **Defuzzification Approach at the constraints:** Similar to the previous approach, the Chance-Constrained Programming (CCP) method is utilized to handle defuzzification of fuzzy constraints, as presented in Equations (3.16) - (3.17).

Phase 3: Membership Function: Similar to the previous approach, the membership function can be computed using Equations (3.7) – (3.8).

Phase 4: Optimization Process: Similar to the previous approach, Zimmermann's method is applied, as demonstrated in Equation (3.9).

Numerical example for conventional specific fuzzy optimization

AB manufacturing is the company that makes a line of high qualities Glasses, Bottles, and Cups. It has three plants; Plant1, Plant2, and Plant3, that are used to produce high qualities Glasses, Bottles, and Cups. To produce a Glass, the production time is 2 hours/unit at Plant1. The available production capacity of Plant1 varies according to a triangular distribution with a minimum available production capacity of 8 hours, a most likely available production capacity of 16 hours, and a maximum available production capacity of 24 hours. To produce a Bottle, the production time at Plant2 varies according to a triangular distribution with a minimum production time of 1.5 hours/unit, a most likely production time of 2.5 hours/unit, and a maximum production time of 3.5 hours/unit. The available production capacity of Plant2 varies according to a triangular distribution with a minimum available production capacity of 12 hours, a most likely available production capacity of 24 hours, and a maximum available production capacity of 36 hours. To produce a Cup, the production time is 1.5 hours/unit at Plant3. The available production capacity of Plant3 is 36 hours. Profits of a Glass, a Bottle and a Cup are calculated as ($15, $20, $25), ($25, $30, $35), and ($35, $40, $45), respectively. The AB manufacturing attempts to find out not only how many units of Glasses, Bottles and Cups that should be produced to maximize total profit but also minimize total amount of pollution. For simplicity, the amount of pollution follows a linear function resulting from three decision variables X_1, X_2 and X_3.

$$2X_1 + 3X_2 + 4X_2$$

where X_1, X_2 and X_3 denote decision variables representing numbers of produced Glass, Bottle, and Cup, respectively.

Table 3.1 Parameters relate to production of Glass, Bottle, and Cup.

	Glass	Bottle	Cup	Available Production Capacity
Production Time (Plant1)	2 hours/unit	-		(8 hours, 16 hours, 24 hours)
Production Time (Plant2)	-	(1.5 hours/unit, 2.5 hours/unit, 3.5 hours/unit)		(12 hours, 24 hours, 36 hours)
Production Time (Plant3)	-	-	1.5 hours/unit	36 hours
Profit	($15/unit, $20/unit, $25/unit)	($25/unit, $30/unit, $35/unit)	($35/unit, $40/unit, $45/unit)	-

<u>Mathematical Formulation</u>

➢ **Objective Functions**

1.Maximize total profits (defuzzify by Realistic Robust Programming and Fairness)

$Maximize\ Z_1 = EV(\tilde{Z}) + Optimality\ Robustness + Feasibility\ Robustness + Fairness$

- **First term**

$$EV(\tilde{Z}) = \frac{Z^o + 2Z^m + Z^p}{4}$$

1.Z^o can be calculated as follows:

$Maxmize\ Z_1 = 15X_1 + 25X_2 + 35X_3$

Subject to: $2X_1 \leq (2(0.8)-1)(8) + (2-2(0.8))(16)$

$(2(0.8)-1)(1.5) + (2-2(0.8))(2.5)x \leq (2(0.8)-1)(12) + (2-2(0.8))(24)$

$1.5X_2 \leq 36$

$X_1, X_2, X_3 \geq 0$

2.Z^m can be calculated as follows:

$Maxmize\ Z_1 = 20X_1 + 30X_2 + 40X_3$

Subject to: $2X_1 \leq (2(0.8)-1)(8) + (2-2(0.8))(16)$

$(2(0.8)-1)(1.5) + (2-2(0.8))(2.5)x \leq (2(0.8)-1)(12) + (2-2(0.8))(24)$

$1.5X_2 \leq 36$

$X_1, X_2, X_3 \geq 0$

3.Z^p can be calculated as follows:

$Maxmize\ Z_1 = 25X_1 + 35X_2 + 45X_3$

Subject to: $2X_1 \leq (2(0.8)-1)(8) + (2-2(0.8))(16)$

$(2(0.8)-1)(1.5) + (2-2(0.8))(2.5)x \leq (2(0.8)-1)(12) + (2-2(0.8))(24)$

$1.5X_2 \leq 36$

$X_1, X_2, X_3 \geq 0$

- **Second term**

$$Optimality\ Robustness = \rho(Z_{max} - Z_{min})$$

where ρ is assumed to be 50%.

1.Z_{max} can be calculated as follows:

Maxmize $Z_{max} = f^p y + c^p x = 25X_1$+$35X_2$+$45X_3$

Subject to: $2X_1 \leq (2(0.8) - 1)(8) + (2 - 2(0.8))(16)$

$(2(0.8) - 1)(1.5) + (2 - 2(0.8))(2.5)x \leq (2(0.8) - 1)(12) + (2 - 2(0.8))(24)$

$1.5X_2 \leq 36$

$X_1, X_2, X_3 \geq 0$

2.Z_{min} can be calculated as follows:

Maxmize $Z_{min} = f^o y + c^o x = 15X_1$+$25X_2$+$35X_3$

Subject to: $2X_1 \leq (2(0.8) - 1)(8) + (2 - 2(0.8))(16)$

$(2(0.8) - 1)(1.5) + (2 - 2(0.8))(2.5)x \leq (2(0.8) - 1)(12) + (2 - 2(0.8))(24)$

$1.5X_2 \leq 36$

$X_1, X_2, X_3 \geq 0$

- **Third term**

$$Feasibility\ Robustnes = \sigma(d^p - (1-\gamma)d^m - \gamma d^p) + \delta(\gamma B^o + (1-\gamma)B^m - B^o)$$

where σ and δ are assumed to be 50% and γ is assumed to be 80%.

1.The first term $\sigma(d^p - (1-\gamma)d^m - \gamma d^p)$ can be applied to uncertain right-hand side constraints

$\sigma(d^p - (1-\gamma)d^m - \gamma d^p) = 0.5\big(24 - (1-0.8)16 - (0.8)(24)\big)$

$\sigma(d^p - (1-\gamma)d^m - \gamma d^p) = 0.5\big(36 - (1-0.8)24 - (0.8)(36)\big)$

2.The second term $\delta(\gamma B^o + (1-\gamma)B^m - B^o)$ can be applied to uncertain left-hand side constraints

$\delta(\gamma B^o + (1-\gamma)B^m - B^o) = 0.5((0.8)(1.5) + (1-0.8)(2.5) - 1.5)$

2.Minimize total amount of pollution

$Minimize\ Z_2 = 2X_1 + 3X_2 + \ 4X_2$

- **Fourth term**

$$Fairness = (Z_i^{PIS} - Z_i)$$

1.For Maximization of the Objective Function (Maximize total profits)

Z_1^{PIS} can be calculated as follows:

$Maxmize\ Z_1 = EV(\tilde{Z}) + Optimality\ Robustness + Feasibility\ Robustness + Fairness$

Subject to: $2X_1 \leq (2(0.8) - 1)(8) + (2 - 2(0.8))(16)$

$(2(0.8) - 1)(1.5) + (2 - 2(0.8))(2.5)x \leq (2(0.8) - 1)(12) + (2 - 2(0.8))(24)$

$1.5X_2 \leq 36$

$X_1, X_2, X_3 \geq 0$

2.For Minimization of the Objective Function (Minimize total amount of pollution)

Z_2^{PIS} can be calculated as follows:

$Minimize\ Z_2 = 2X_1 + 3X_2 + 4X_2$

Subject to: $2X_1 \leq (2(0.8) - 1)(8) + (2 - 2(0.8))(16)$

$(2(0.8) - 1)(1.5) + (2 - 2(0.8))(2.5)x \leq (2(0.8) - 1)(12) + (2 - 2(0.8))(24)$

$1.5X_2 \leq 36$

$X_1, X_2, X_3 \geq 0$

- **Uncertain Constraints**

1.Defuzzify by Chance-Constrained Programming ($\gamma = 80\%$)

Subject to: $2X_1 \leq (2(0.8) - 1)(8) + (2 - 2(0.8))(16)$

$(2(0.8) - 1)(1.5) + (2 - 2(0.8))(2.5)x \leq (2(0.8) - 1)(12) + (2 - 2(0.8))(24)$

- **Crisp Constraint**

Subject to: $1.5X_2 \leq 36$

- **Non-negativity Constraint**

Subject to: $X_1, X_2, X_3 \geq 0$

- **Membership Functions**

1.Membership Function for Maximizing the Objective Function (Maximize total profits)

$$\mu_{Z_1} = \frac{Z_1 - Z_1^{NIS}}{Z_1^{PIS} - Z_1^{NIS}}$$

Z_1^{PIS} can be calculated as follows:

$Maxmize\ Z_1 = EV(\tilde{Z}) + Optimality\ Robustness + Feasibility\ Robustness + Fairness$

Subject to: $2X_1 \leq (2(0.8) - 1)(8) + (2 - 2(0.8))(16)$

$(2(0.8) - 1)(1.5) + (2 - 2(0.8))(2.5)x \leq (2(0.8) - 1)(12) + (2 - 2(0.8))(24)$

$1.5X_2 \leq 36$

$X_1, X_2, X_3 \geq 0$

Z_1^{NIS} can be calculated as follows:

$$Minimize\ Z_1 = EV(\tilde{Z}) + Optimality\ Robustness + Feasibility\ Robustness + Fairness$$

Subject to: $2X_1 \leq (2(0.8)-1)(8) + (2-2(0.8))(16)$

$$(2(0.8)-1)(1.5) + (2-2(0.8))(2.5)x \leq (2(0.8)-1)(12) + (2-2(0.8))(24)$$

$$1.5X_2 \leq 36$$

$$X_1, X_2, X_3 \geq 0$$

2.Membership Function for Minimizing the Objective Function (Minimize total amount of pollution)

$$\mu_{Z_2} = \frac{Z_2^{NIS} - Z_2}{Z_2^{NIS} - Z_2^{PIS}}$$

Z_2^{PIS} can be calculated as follows:

$$Minimize\ Z_2 = 2X_1 + 3X_2 + 4X_2$$

Subject to: $2X_1 \leq (2(0.8)-1)(8) + (2-2(0.8))(16)$

$$(2(0.8)-1)(1.5) + (2-2(0.8))(2.5)x \leq (2(0.8)-1)(12) + (2-2(0.8))(24)$$

$$1.5X_2 \leq 36$$

$$X_1, X_2, X_3 \geq 0$$

- **Optimization Process by Zimmermann's Approach**

$$Minimize\ \mu_Z$$

Subject to: $2X_1 \leq (2(0.8)-1)(8) + (2-2(0.8))(16)$

$$(2(0.8)-1)(1.5) + (2-2(0.8))(2.5)x \leq (2(0.8)-1)(12) + (2-2(0.8))(24)$$

$$1.5X_2 \leq 36$$

$$X_1, X_2, X_3 \geq 0$$

$$\mu_Z \leq \mu_{Z_1} = \frac{Z_1 - Z_1^{NIS}}{Z_1^{PIS} - Z_1^{NIS}}$$

$$\mu_Z \leq \mu_{Z_2} = \frac{Z_2^{NIS} - Z_2}{Z_2^{NIS} - Z_2^{PIS}}$$

3.4 A Downside Risk Mitigation Approach

This study addresses another critical problem of uncertainty within supply chain operations, which are often plagued by imprecise, incomplete, inaccurate, or ambiguous information. These uncertainties pose significant risks to supply chain performance, leading to suboptimal decision-making if not effectively managed Rachev et al. (2011). Conventional fuzzy linear programming approaches typically represent uncertain data using triangular fuzzy numbers (Zhang et al., 2014), assuming symmetrical deviations around a central value. However, this assumption fails to capture the asymmetrical nature of real-world uncertainties, where risks and deviations can differ in magnitude between positive and negative directions. To overcome this limitation, this study introduces the utilization of asymmetrical triangular fuzzy numbers, which better reflect the skewness of real-world data. Additionally, the risk of uncertainty, particularly the downside risk arising from pessimistic and most likely scenarios, is quantified using the Mean Conditional Value at Risk Gap (MCVaRG) (Chiadamrong and Suthamanondh (2024)). This approach, grounded in Conditional Value-at-Risk (CVaR) theory, emphasizes the assessment of the tail end of the outcome distribution beyond a specified threshold, providing a precise evaluation of downside risks. By integrating these advanced risk measures, this study aims to strengthen decision-making in supply chain management under uncertainty, providing a more robust framework for addressing the challenges posed by risk and imprecision.

To minimize the MCVaRG of total supply chain operating costs, the formulation used in this study is presented as follows:

$$MCVaR = \sum_t^T \left[\frac{(1-2\gamma)^2\hat{\sigma}_t}{4(1-\gamma)} + \frac{(1-2\gamma^2)\bar{\sigma}_t}{2(1-\gamma)} + \frac{\breve{\sigma}_t}{4(1-\gamma)}\right] x_t, \quad if\ 0 < \gamma \leq 0.5 \qquad (3.27)$$

$$MCVaR = \sum_t^T [(1-\gamma)\bar{\sigma}_t + \gamma\hat{\sigma}_t]\, x_t, \quad if\ 0.5 < \gamma < 1 \qquad (3.28)$$

Minimize MCVaRG =MCVaR – Total Supply Chain Operation Costs (3.29)

where $\breve{\sigma}_t$, $\bar{\sigma}_t$, and $\hat{\sigma}_t$ represent the optimistic, most likely, and pessimistic supply chain operation cost values, respectively, for each period t, $t = 1, 2, \dots, T$. The variable x_t denotes the decision variables associated with each corresponding period t. Additionally, γ represents the credibility level used to manage uncertainty within the model.

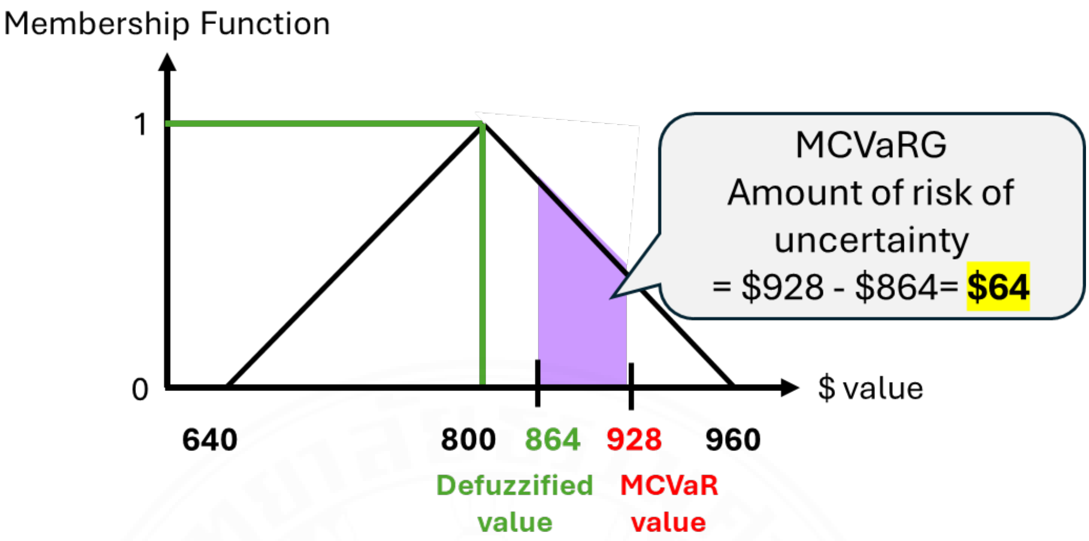

Figure 3.4 Demonstration of MCVaRG calculation.

The process of computing the MCVaRG value is illustrated in Figure 3.4. For instance, the defuzzified value of a fuzzy supply chain operation cost is $864, derived from a Triangular Fuzzy Number with parameters $640 (minimum), $800 (most likely), and $960 (maximum), which represent the range of possible values for the supply chain operation cost.

The defuzzification process is performed employing CCP at a confidence level of 0.8, as specified in Equation (3.16).

$$Cr\{\sum_{j=1}^{n} a_{ij}x_j \leq \tilde{b}_i\} \geq \gamma \quad if\ and\ only\ if$$
$$when\ (0.5 < \gamma \leq 1): ax \leq (2\gamma - 1)b^m + (2 - 2\gamma)b^p$$
$$(2(0.8) - 1)(800) + \big(2 - 2(0.8)\big)(960) = 864$$

Subsequently, the MCVaR is computed using Equation (3.28) with a confidence level of $\gamma = 0.8$, yielding a value of $928.

$$MCVaR = \sum_t^T [(1-\gamma)\bar{\sigma}_t + \gamma\hat{\sigma}_t]\, x_t, \quad if\ 0.5 < \gamma < 1$$
$$MCVaR = (1 - 0.8)(800) + (0.8)(960) = 928$$

The quantified risk associated with supply chain operating costs, represented by MCVaRG of $64, demonstrates the impact of cost uncertainty and the necessity to mitigate this risk factor.

$$MCVaRG = MCVaR - \text{Total Supply Chain Operation Costs} = 928 - 864 = 64$$

3.5 Introduction of Case Studies

The proposed research methodologies are demonstrated through the application of three distinct case studies, each addressing a unique set of challenges. These case studies are designed to showcase the effectiveness of each methodology in tackling specific issues relevant to the research objectives. Each case study presents a different context, supporting an in-depth examination of the approaches and their ability to handle various complexities. The three distinct case studies are presented separately to clearly highlight the advantages and effectiveness of each proposed methodology. This separation ensures that the unique features and contributions of each approach can be thoroughly examined without introducing unnecessary complexity that could obscure the significance of their individual strengths. By isolating the cases, the study maintains analytical clarity, allowing decision-makers to achieve a deeper perception of the specific benefits and applicability of each methodology within the context of supply chain aggregate production planning. This structured presentation also facilitates a more focused evaluation and comparison, enhancing the overall interpretability and practical relevance of the proposed solutions.

The following outlines the key issues and challenges addressed in each case study:

- Case 1: A Five-Phase Hybrid Fuzzy Optimization Approach for Supply Chain Aggregate Production Planning

In the face of increasing competitive market pressures, firms must adopt strategies that allow them to improve performance by addressing multiple objectives simultaneously to secure a competitive advantage. Consequently, there is a need for a practical approach capable of overcoming two major obstacles: conflicting objectives and the uncertainty inherent in supply chain management. This necessitates an effective decision-making framework to assist Decision Makers (DMs) in planning an efficient Supply Chain Aggregate Production Plan (SCAPP). This case study purposes to minimize total supply chain costs, minimize total product shortages, and maximize total purchasing values, all while dealing with imprecise factors such as operating costs, customer demand, defective rates, and service levels. Beyond proposing a solution to these challenges, the study also addresses the weaknesses of conventional specific fuzzy optimization approaches, which often fail to consider hesitation, robustness, non-satisfaction levels, and the generation of Pareto-optimal solutions. To overcome these limitations, a novel five-phase hybrid fuzzy optimization approach is developed, integrating Intuitionistic Fuzzy Linear Programming (IFLP), Realistic Robust Programming (RRP), Chance-Constrained Programming (CCP), and the Augmented Epsilon Constraint (AUGMECON) method. This comprehensive approach enables DMs to obtain the most robust and concrete compromise solution, reflecting their intentions more accurately and ultimately improving the efficiency and effectiveness of SCAPP under uncertain and competitive conditions.

- Contributions and Highlights of Case 1

A key highlight of this case study is the development of a five-phase hybrid fuzzy optimization approach designed to overcome the limitations of a conventional specific fuzzy optimization approach. The proposed methodology integrates several advanced techniques, including Intuitionistic Fuzzy Linear Programming (IFLP), Realistic Robust Programming (RRP), Chance-Constrained Programming (CCP), and Augmented Epsilon Constraint (AUGMECON). These methods address hesitation, enhance robustness, and incorporate both satisfaction and non-satisfaction levels, ultimately producing Pareto-optimal solutions. This integrated approach represents a significant advancement over existing methods and provides a more comprehensive strategy for decision-making in supply chain management, allowing DMs to create more resilient and effective supply chain strategies.

To the best of the authors' understanding, this is the first study to utilize the combined techniques of IFLP, RRP, CCP, and AUGMECON to solve multi-objective SCPP problems in an uncertain environment. This novel integration offers a comprehensive framework to tackle the complex challenges of modern supply chain management, presenting a fresh perspective for managing uncertain data and conflicting objectives.

The feasibility and practicality of the proposed approach are validated through a case study. The outcomes from the case study demonstrate that the methodology effectively minimizes total costs, minimizes total shortages, and maximizes total purchasing value while managing uncertainty and balancing conflicting objectives. These outcomes highlight the practical relevance and applicability of the approach in real-world scenarios, establishing a new benchmark for multi-objective SCAPP under uncertainty.

- Case 2: A Unified Fairness and Robustness Fuzzy Optimization Approach for Supply Chain Aggregate Production Planning

In supply chain management, two critical factors; fairness and robustness, are often overlooked in the context of SCAPP. Failure to account for fairness among multiple objectives can lead to inequitable outcomes due to conflicting priorities across stakeholders. Additionally, neglecting robustness can result in unreliable and non-resilient planning, especially when dealing with uncertain and imprecise data. To address these challenges, this study proposes a unified fairness and robustness fuzzy optimization approach that integrates the principles of Proportional Fairness (PF) and Robust Chance-Constrained Programming (RCCP). By combining these methodologies, the approach aims to achieve a balanced and resilient SCAPP that minimizes total supply chain costs, minimizes total fluctuations in workforce levels, and maximizes total purchasing values under uncertain circumstances. The effectiveness of this approach is demonstrated through a case study, which highlights its ability to improve the fairness and robustness of SCAPP outcomes. Ultimately, the integration of fairness and robustness into SCAPP strengthens the resilience of supply chain operations, ensuring better adaptability in the face of disruptions. It also strengthen corporate reputation through signaling dedication to responsible, reliable, and equitable business practices, thereby promoting long-term sustainability and operational efficiency.

- Contributions and Highlights of Case 2

This case study fills a notable gap in existing literature by introducing the combined principles of proportional fairness and robustness in Aggregate Production Planning (APP) optimization. A conventional specific fuzzy optimization approach often overlooks fairness, leading to the unequal prioritization of objectives, while neglecting robustness can result in vulnerabilities when unexpected disruptions occur. By combining these two principles, the study offers a more comprehensive solution that ensures equitable treatment of all stakeholders within the supply chain while maintaining operational continuity amidst uncertainty. This integration significantly enhances both the efficiency and sustainability of supply chain management practices.

Incorporating fairness into the optimization process ensures that the system prevents bias, fostering positive relationships among diverse stakeholders in the SC. Meanwhile, robustness equips the system to withstand unforeseen challenges and disruptions, ensuring continued operations even in the face of uncertainty. The synergy between fairness and robustness contributes to greater operational stability and bolsters corporate image through the promotion of responsible and dependable practices. This approach positions companies for long-term success while building trust and stability across their supply chain networks.

Furthermore, the proposed approach outperforms conventional specific fuzzy optimization approaches, particularly under conditions with pronounced differences in objective satisfaction levels. In scenarios where one objective’s satisfaction level is disproportionately low, or below the preferences of DMs, a conventional specific fuzzy optimization approach may lead to unfair outcomes. Conversely, when one objective’s satisfaction is excessively high, it can lead to inequitable treatment of other objectives. The proposed approach addresses these imbalances, ensuring a fairer and more balanced optimization process. This feature enhances the model's practical applicability, ensuring that DMs can make more reliable and equitable decisions in multi-objective APP.

- Case 3: A Downside Risk Mitigation Approach for Supply Chain Aggregate Production Planning

As modern business environments become more complex and uncertain, there is a growing need for strong SCAPP strategies that can skillfully navigate interdependencies and ensure seamless coordination across the supply chain hierarchy. Conventional supply chain strategies often overlook critical uncertainties and risks, contributing to inefficiencies and higher operational expenditures. This study addresses these challenges by proposing a business model that integrates open innovation to enhance both resilience and cost performance. Specifically, the study proposes a downside risk mitigation approach aimed at minimizing the probability of adverse outcomes or financial losses caused by fluctuations, unpredictability, and unforeseen events that frequently increase supply chain costs. Through a case study centered on cost and risk minimization, the model employs asymmetrical triangular fuzzy numbers to capture various uncertain factors, including fluctuating costs, demands, and machinery runtime. The results demonstrate the effectiveness in delivering decision makers a comprehensive and optimized SCAPP that enhances operational efficiency, improves reliability, and substantially reduces costs. Furthermore, the model's capability to mitigate the skewness of risks stemming from operational uncertainties provides a strategic advantage in enhancing overall supply chain resilience and ensuring long-term sustainability.

- Contributions and Highlights of Case 3

This case study presents an innovative multi-objective fuzzy linear programming model aimed at optimizing the SCAPP problem. The model simultaneously addresses two key goals: reducing costs and minimizing downside risk, with special emphasis on the Mean-Conditional Value at Risk Gap (MCVaRG). By addressing these dual objectives, the proposed framework offers a strategic SCAPP plan that not only ensures economically viable decisions but also enhances resilience against potential risks. This contribution fills a significant gap in existing literature by providing a comprehensive approach that moves beyond conventional cost-centric models, acknowledging the complexity inherent in modern supply chain management.

The study's innovative approach emphasizes the importance of a framework that balances cost efficiency with risk mitigation. While many conventional SCAPP models primarily focus on minimizing costs, this research broadens the perspective to include risk management as a critical component of supply chain optimization. By simultaneously considering both objectives, the model enables decision-makers to develop plans that are not only economically viable but also resilient to uncertainties, contributing to the long-term sustainability and stability of the SC.

A key highlight of this case study is the introduction of a groundbreaking methodology for addressing the asymmetrical skewness often present in real-world data. Unlike conventional models that assume symmetrical distributions of fuzzy numbers, the study recognizes that data frequently exhibits asymmetry. The inclusion of skewness in triangular fuzzy numbers allows the model to more precisely capture uncertainty and fluctuations in the data. This refined understanding enhances decision-making by providing clearer insight into the potential outcome range and associated probabilities, contributing to stronger risk and cost management.

Incorporating these sophisticated ideas into the model greatly refines strategic planning efforts and enhances the consistency of decisions under challenging conditions. The ability to account for skewed fuzzy numbers enhances the model's precision in managing both risk and cost, making it particularly valuable in uncertain and dynamic environments. Overall, this study advances the field of supply chain management by providing a robust tool for optimizing APP under uncertainty, enhancing both the efficiency and resilience of decision-making in real-world supply chain scenarios.

CHAPTER 4
RESULTS

This chapter concentrates on the practical applications of proposed methodologies and concepts in real-world settings, bridging the gap between theoretical research and industrial practices. It explores how the integration of innovative techniques of fuzzy optimization can enhance decision-making processes in supply chain management. By presenting case studies, this chapter demonstrates how these methodologies address challenges like uncertainty, resource constraints, and conflicting objectives. The goal is to provide actionable insights and frameworks that enable practitioners to optimize operations, reduce risks, and improve overall system performance, making complex theoretical approaches accessible and relevant to industry professionals. This chapter serves as a guide for implementing these tools effectively, highlighting their potential to drive efficiency, cost-effectiveness, and resilience in dynamic and uncertain environments.

4.1 Case 1: A Five-Phase Hybrid Fuzzy Optimization Approach for Supply Chain Aggregate Production Planning

Supply Chain Aggregate Production Planning (SCAPP) plays a crucial role in operational management, directly impacting an organization's performance and competitiveness in the marketplace. In highly competitive environments, firms face the challenge of achieving multiple, often conflicting objectives, all while navigating the uncertainties of supply chain management. Conventional Specific Fuzzy Linear Programming (FLP) approach, often struggle to address these complexities, particularly when dealing with conflicting objectives and imprecise data. To tackle these challenges, this study introduces a five-phase hybrid fuzzy optimization approach that integrates advanced methodologies such as Intuitionistic Fuzzy Linear Programming (IFLP), Realistic Robust Programming (RRP), Chance-Constrained Programming (CCP), and the Augmented Epsilon Constraint (AUGMECON) method. This approach aims to provide a more robust, flexible solution to SCAPP problems. A detailed case study demonstrates how the proposed approach effectively minimizes total supply chain costs, minimizes total product shortages, and maximizes total purchase values under uncertain circumstances, including imprecise operating costs, customer demands, defective rates, and service levels. The results also show that the integrated approach outperforms conventional specific FLP approach, offering enhanced hesitation allowance, robust modeling, and a more comprehensive consideration of satisfaction and non-satisfaction levels. Additionally, it generates a set of strong Pareto-optimal solutions, enabling decision makers to make more informed and effective choices aligned with strategic goals. This study thus provides a valuable tool for enhancing the efficiency and effectiveness of SCAPP in uncertain and competitive environments.

4.1.1 Mathematical Notations and Model

The notations for indexes, parameters, and decision variables are presented in Tables 4.1 to 4.4. Notably, all fuzzy parameters are denoted with a tilde ($\tilde{}$) placed above the corresponding symbols to indicate their fuzzy nature.

Table 4.1 Indexes of SCAPP problem (Case 1).

Indexes	Meaning
r	Raw materials index ($r = 1, \dots, R$)
s	Suppliers index ($s = 1, \dots, S$)
n	Products index (n= $1, \dots, N$)
t	Planning periods index (t= $1, \dots, T$)

Table 4.2 Crisp parameters of SCAPP problem (Case 1).

Crisp Parameters	Meaning
NIL_0	Initial labor force in period *0* (persons)
PL	Productivity of labors ($0 < PL < 1$)
AFV	Acceptable fraction of labor variation (%)
$SCRM_r$	Production site storage limit for raw material *r* (units)
TSS_s	Total evaluation score of supplier *s* with respect to raw material quality (%)
SCF_n	Production site storage limit for final product *n* (units)
PT_n	Manufacturing time per unit of product *n* (min)
ART_t	Regular working time available in period *t* (hours)
AOT_t	Overtime working available in period *t* (hours)
NRM_{rn}	Volume of raw material *r* consumed for each unit of product *n* (units)
$MaxSQ_{nt}$	Maximum allowable subcontracted quantity of product *n* in period *t* (units)
$MaxMC_{nt}$	Maximum machine usage allocated to product *n* in period *t* (machine-hours)
MHU_{nt}	Machine utilization per unit of product *n* in period *t* (machine-hours/unit)
$MaxCRM_{srt}$	Maximum quantity of raw material *r* supplied by supplier *s* in period *t* (units)
ST	Total shortages of products (units)
TVP	Total values of purchasing (units)

Table 4.3 Uncertain parameters of SCAPP problem (Case 1).

Uncertain Parameters	Meaning
$\widetilde{RTC}_t$	Production cost during regular hours under fuzzy conditions in period *t* ($/minute)
$\widetilde{OTC}_t$	Production cost during overtime under fuzziness in period *t* ($/minute)
$\widetilde{SC}_t$	Production cost of subcontracting under fuzziness in period *t* ($/minute)
$\widetilde{SW}_t$	Labor wage under fuzziness in period *t* ($/person)
$\widetilde{HC}_t$	Labor hiring cost under fuzziness in period *t* ($/person)
$\widetilde{FC}_t$	Labor dismissal cost under fuzziness in period *t* ($/person)
$\widetilde{ACSL}_t$	Production plant service level threshold with fuzziness in period *t* (%)
$\widetilde{IRMC}_{rt}$	Inventory holding cost of raw material *r* under fuzziness in period *t* ($/unit)
$\widetilde{TRMC}_{st}$	Logistics cost under fuzziness for raw material *r* from supplier *s* in period *t* ($/unit)
$\widetilde{AVSL}_{st}$	Average fuzzy service quality level of supplier *s* in period *t* (%)
$\widetilde{TFC}_{nt}$	Logistics cost under fuzziness for product *n* shipped from plant to customers in period *t* ($/unit)
$\widetilde{IFC}_{nt}$	Inventory holding cost of product *n* under fuzziness in period *t* ($/unit)
$\widetilde{D}_{nt}$	Customer demand under fuzziness for product *n* in period *t* (units)
$\widetilde{PSC}_{nt}$	Penalty cost under fuzziness for product *n* shortage in period *t* ($/unit)
$\widetilde{ACFRM}_{srt}$	Acceptable fuzzy failure percentage for raw material *r* in the production plant in period *t* (%)
$\widetilde{AVFRM}_{srt}$	Average fuzzy failure percentage of raw material *r* supplied by supplier *s* in period *t* (%)
$\widetilde{PC}_{srt}$	Unit acquisition cost under fuzziness for raw material *r* supplied by supplier *s* in period *t* ($/unit)
$\widetilde{TSC}$	Total supply chain operational costs ($)
$\widetilde{PuC}$	Total purchasing costs ($)
$\widetilde{PrC}$	Total production costs ($)
$\widetilde{WLC}$	Total costs of worker ($)
$\widetilde{IVC}$	Total inventory costs ($)
$\widetilde{SPC}$	Total shipping costs ($)
$\widetilde{STC}$	Total shortage costs ($)

Table 4.4 Decision variables of SCAPP problem (Case 1).

Decision Variables	Meaning
NHW_t	Total labor force employed in period *t* (persons)
NFW_t	Total labor force terminated in period *t* (persons)
IRM_{rt}	Ending inventory of raw material *r* in period *t* (units)
RTQ_{nt}	Production quantity of product *n* within regular working hours in period *t* (units)
OTQ_{nt}	Production quantity of product *n* within overtime hours in period *t* (units)
SQ_{nt}	Units of product *n* manufactured through subcontracting in period *t* (units)
FQ_{nt}	Amount of product *n* delivered to customers in period *t* (units)
IF_{nt}	Ending inventory of product *n* in period *t* (units)
SFO_{nt}	Quantity of product *n* shortage for customers in period *t* (units)
RMQ_{srt}	Amount of raw material *r* delivered by supplier *s* during period *t* (units)

Objective Functions

1. Minimization of Total Supply Chain Costs: This is typically a primary goal when developing an effective supply chain production plan. Total supply chain costs generally include purchasing costs, production costs, labor costs, inventory costs, shipping costs, and shortage costs over a specific period. These costs may be uncertain due to incomplete or unavailable information. The uncertain values of these costs can be represented as triangular fuzzy numbers or triangular intuitionistic fuzzy numbers. The objective function can be expressed as follows:

$$
\begin{aligned}
&Minimize\ \widetilde{TSC} = \widetilde{PuC} + \widetilde{PrC} + \widetilde{WLC} + \widetilde{IVC} + \widetilde{SPC} + \widetilde{STC} \\
&= \textstyle\sum_{s=1}^{S}\sum_{r=1}^{R}\sum_{t=1}^{T} \widetilde{PC}_{srt} \times RMQ_{srt} + \sum_{n=1}^{N}\sum_{t=1}^{T} \widetilde{RTC}_t \times PT_n \times RTQ_{nt} \\
&+ \textstyle\sum_{n=1}^{N}\sum_{t=1}^{T} \widetilde{OTC}_t \times PT_n \times OTQ_{nt} + \sum_{n=1}^{N}\sum_{t=1}^{T} \widetilde{SC}_t \times PT_n \times SQ_{nt} \\
&+ \textstyle\sum_{t=1}^{T} \widetilde{SW}_t \times NW_t + \sum_{t=1}^{T} \widetilde{HC}_t \times NHW_t + \sum_{t=1}^{T} \widetilde{FC}_t \times NFW_t \\
&+ \textstyle\sum_{r=1}^{R}\sum_{t=1}^{T} \widetilde{IRMC}_{rt} \times IRM_{rt} + \sum_{n=1}^{N}\sum_{t=1}^{T} \widetilde{IFC}_{nt} \times IF_{nt} \\
&+ \textstyle\sum_{s=1}^{S}\sum_{r=1}^{R}\sum_{t=1}^{T} \widetilde{TRMC}_{st} \times RMQ_{srt} + \sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{t=1}^{T} \widetilde{TFC}_{mt} \times FQ_{nmt} \\
&+ \textstyle\sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{t=1}^{T} \widetilde{PSC}_{nmt} \times SFO_{nmt}
\end{aligned} \tag{4.1}
$$

Equation (4.1) represents the minimization of total supply chain costs as an economic objective. This includes the total purchasing cost, total production cost, total labor cost, total inventory cost, total shipping cost, and total shortage cost. The total purchasing cost is incurred from acquiring the necessary raw materials from each supplier. The total production cost is the sum of costs for producing products during regular hours, overtime, and subcontracting. The total labor cost includes salary expenses as well as the expenses related to recruitment and termination of labors. The total inventory cost arises from storing raw materials and products. The total shipping cost encompasses the transportation of raw materials from suppliers to manufacturers and the delivery of products from manufacturers to customers. Lastly, the total shortage cost represents the penalty incurred due to product shortages.

2. Minimization of Total Product Shortages: Product shortages occur when volatile customer demand and limited warehouse capacity result in insufficient inventory to meet customer needs. To address this, minimizing total product shortages becomes a critical consideration, helping firms fulfill customer requirements more effectively. This objective is formally expressed by the following mathematical formulation:

$$Minimize\ ST = \sum_{n=1}^{N} \sum_{m=1}^{M} \sum_{t=1}^{T} SFO_{nmt} \quad (4.2)$$

3. Maximization of Total Purchasing Value: Maximizing the total value of purchasing is another essential objective. This goal ensures that an organization procures not only the highest quantity of raw materials but also the highest quality materials, evaluated based on factors such as price, quality, and timely delivery. This objective can be expressed by the following mathematical equation:

$$Maximize\ TVP = \sum_{s=1}^{S} TSS_s \times RMQ_{srt} \quad (4.3)$$

Note: Suppliers can be evaluated and scored based on their performance using the decision-makers' expertise. The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) is a ranking and scoring method that can be employed for supplier evaluation. It provides decision-makers with a comprehensive weighted score for each supplier, aiding in informed decision-making.

Constraints

1. Quality of Raw Materials: The quality of raw materials can be assessed for each supplier during each period. This involves ensuring that the overall average failure rate of the raw materials provided does not exceed the specified acceptable failure rate for each material.

$$\sum_{s=1}^{S} \widetilde{AVFRM}_{sr} \times RMQ_{srt} \leq \widetilde{ACFRM}_{t} \times \sum_{s=1}^{S} RMQ_{srt} \qquad \forall\, r \in R, t \in T \qquad (4.4)$$

2. Suppliers' Capacity: This reflects the greatest volume of raw materials that a supplier can offer within a designated timeframe.

$$RMQ_{srt} \leq MaxCRM_{srt} \qquad \forall\, s \in S, r \in R, t \in T \qquad (4.5)$$

3. Service Level: This measures the performance of each supplier in terms of on-time delivery during each period. Specifically, the overall average service level of suppliers must meet or exceed the specified acceptable service level threshold.

$$\sum_{r=1}^{R} \sum_{s=1}^{S} \widetilde{AVSL}_{s} \times RMQ_{srt} \geq \widetilde{ACSL} \times \sum_{r=1}^{R} \sum_{s=1}^{S} RMQ_{srt} \qquad \forall\, t \in T \qquad (4.6)$$

4. Available Resource of Raw Materials: This constraint ensures that the aggregate raw material demand for the two products, including usage during regular hours, overtime, and subcontracted production, remains within the total raw material supply available from all suppliers in each period.

$$\sum_{n=1}^{N} NRM_{rn} \times (RTQ_{nt} + OTQ_{nt} + SQ_{nt}) \leq \sum_{s=1}^{S} RMQ_{srt} \qquad \forall\, r \in R, t \in T \quad (4.7)$$

5. Product Shortages: This represents the quantity of products that cannot be supplied when customer demand exceeds available inventory. Any product shortages will incur a penalty cost or shortage cost.

$$SFO_{mnt} = SFO_{nm(t-1)} + \widetilde{D}_{nmt} - FQ_{nmt} \qquad \forall\, n \in N, m \in M, t \in T \qquad (4.8)$$

6. Available Production Time: This refers to the constraint on production time, encompassing both regular working hours and overtime, which is determined by the available workforce capacity.

$$NW_t \times PL \times (ART_t + AOT_t) \geq \sum_{n=1}^{N}(RTQ_{nt} + OTQ_{nt}) \times PT_n \qquad \forall\, t \in T \qquad (4.9)$$

7. Subcontracting Quantity Limitation: This constraint sets the maximum allowable production volume for products in the designated subcontracting period.

$$SQ_{nt} \leq MaxSQ_{nt} \qquad \forall\, n \in N, t \in T \qquad (4.10)$$

8. Inventory of Raw Materials: This indicates the residual amount of raw materials available after production for every period.

$$IRM_{rt} = IRM_{r(t-1)} + \sum_{s=1}^{S} RMQ_{srt} - \left((\sum_{n=1}^{N} RTQ_{nt} + OTQ_{nt} + SQ_{nt}) \times NRM_{rn}\right) \qquad \forall\, r \in R, t \in T \qquad (4.11)$$

9. Inventory of Products: This denotes the residual quantity of products remaining after customer demand has been met for each period.

$$IF_{nt} = IF_{n(t-1)} + RTQ_{nt} + OTQ_{nt} + SQ_{nt} - \sum_{m=1}^{M} FQ_{mnt} \qquad \forall\, n \in N, t \in T \quad (4.12)$$

10. Storage Capacity of Raw Materials: This sets the maximum storage capacity for raw materials at the manufacturing facility.

$$\sum_{r=1}^{R} IRM_{rt} \leq SCRM_r \qquad \forall\, t \in T \qquad (4.13)$$

11. Product storage capacity: This refers to the maximum inventory level of products that can be stored at the manufacturer’s facility.

$$\sum_{n=1}^{N} IF_{nt} \leq SCF_n \qquad \forall\, t > 1 \qquad (4.14)$$

12. Setting the initial worker level: This refers to determining the number of workers assigned during the initial period.

$$NW_t = NIL_0 \qquad \forall\, t < 2 \tag{4.15}$$

13. Adjusting workforce levels: This refers to optimizing the workforce allocation in each period to maintain operational balance.

$$NW_t = NW_{(t-1)} + NHW_t - NFW_t \qquad \forall\, t > 1 \tag{4.16}$$

14. Workforce variation proportion: This allows decision-makers to control the extent of workforce variation in each period by specifying the acceptable percentage of workforce fluctuations.

$$NHW_t + NFW_t \le AFV \times NW_{(t-1)} \qquad \forall\, t \in T \tag{4.17}$$

15. Machine capacity: This refers to the maximum capacity of machines available for manufacturing products during both regular hours and overtime in each period.

$$\sum_{n=1}^{N} MHU_{nt} \times (RTQ_{nt} + OTQ_{nt}) \le MaxMC_{nt} \qquad \forall\, t \in T \tag{4.18}$$

16. Non-Negativity: All decision variables are constrained to be non-negative by (4.19)–(4.23), with certain variables mandated as integers.

$$NW_t, NHW_t, NFW_t \ge 0 \; and \; integer \qquad \forall\, t \in T \tag{4.19}$$

$$RTQ_{nt}, OTQ_{nt}, SQ_{nt}, IF_{nt} \ge 0 \; and \; integer \qquad \forall\, n \in N, t \in T \tag{4.20}$$

$$IRM_{rt} \ge 0 \qquad \forall\, r \in R, t \in T \tag{4.21}$$

$$SFO_{nmt}, FQ_{nmt} \ge 0 \qquad \forall\, n \in N, m \in M, t \in T \tag{4.22}$$

$$RMQ_{srt} \ge 0 \qquad \forall\, s \in S, r \in R, t \in T \tag{4.23}$$

4.1.2 Problem Description of Case 1

A numerical case study on a supply chain production problem is performed to illustrate and validate the effectiveness of the proposed five-phase hybrid approach. The case study involves a supply chain network comprising four qualified suppliers supplying three essential raw materials, one manufacturer responsible for producing two types of products, and customers with product demands over a six-month planning horizon, as depicted in Figure 4.1. The analysis focuses on three key objectives: minimizing total supply chain costs, minimizing product shortages, and maximizing total purchasing value. These objectives are addressed within an uncertain environment characterized by variability in failure rates of product, levels of service, customer demand, and costs. The uncertainties are represented using triangular fuzzy numbers in the conventional specific fuzzy linear programming approach and triangular intuitionistic fuzzy numbers in the proposed hybrid approach, highlighting the latter's enhanced capability to handle uncertainty and achieve more robust results.

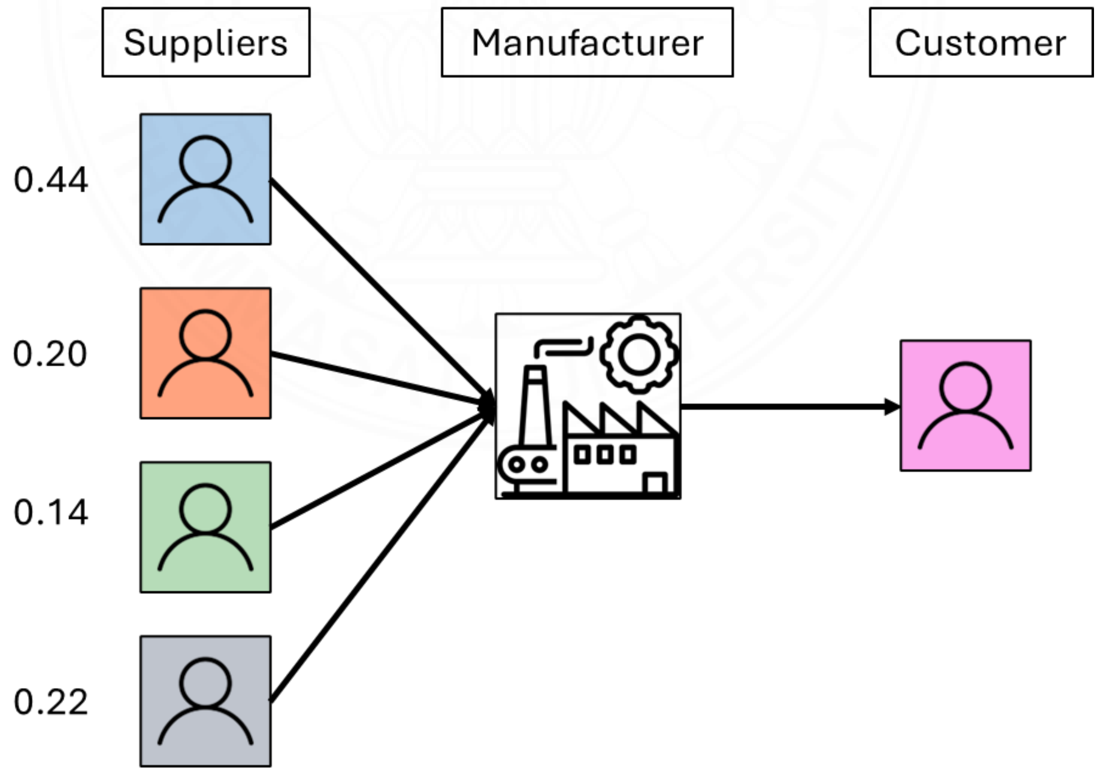

Figure 4.1 The structure of SCAPP.

The assumptions for the SCAPP plan are as follows:

- A list of qualified suppliers is identified, evaluated, and scored according to price, raw material quality, and level of service, as detailed in Table 4.5. Fluctuations in raw material failure rates derives from defects, while variations in the manufacturer's service level are influenced by the timeliness of deliveries.
- Customers are assigned dynamic demand for each product throughout the six-month planning horizon. Demand for each product may be satisfied in full or result in a shortage, with any shortages incurring associated penalty costs.
- All costs related to the supply chain are considered uncertain throughout the planning horizon.
- Delivery lead time is assumed to have no significant impact.
- The initial inventory quantities and worker levels are predefined at the start of the planning horizon.
- Maximum machine capacity and warehouse space at the manufacturer are defined.
- The number of subcontracted product quantities is limited.

To demonstrate the effectiveness of the hybrid methods, the problem is formulated to simultaneously optimize three objectives (total supply chain costs, total product shortages, and total purchasing value) with equal importance (weightless). However, the model can easily accommodate different weight assignments if required by decision makers.

Table 4.5 Performance of suppliers.

Criteria	Supplier (s)			
	S_1	S_2	S_3	S_4
Price	Expensive	Affordable	Affordable	Reasonable
Quality of Raw Material	Premium	Low	Low	Fair
Service Level of Supplier	Intensive	Reliable	Poor	Poor
Weighted Score (TSS_t)	0.44	0.20	0.14	0.22

Table 4.6 The crisp value of input parameters of SCAPP problem.

Parameters	Values					
	t = 1	t = 2	t = 3	t = 4	t = 5	t = 6
ART_t	144	160	168	176	120	192
AOT_t	50	50	50	60	40	60
MHU_{nt}	t = 1	t = 2	t = 3	t = 4	t = 5	t = 6
n = 1	0.3	0.3	0.3	0.3	0.3	0.3
n = 2	0.5	0.5	0.5	0.5	0.5	0.5
$MaxSQ_{nt}$	t = 1	t = 2	t = 3	t = 4	t = 5	t = 6
n = 1	150	150	150	150	150	150
n = 2	170	170	170	170	170	170
$MaxMC_{nt}$	t = 1	t = 2	t = 3	t = 4	t = 5	t = 6
n = 1	264	264	264	264	264	264
n = 2	288	288	288	288	288	288
	n = 1			n = 2		
SCF_n	3,000			3,000		
PT_n	0.2			0.4		
	r = 1		r = 2		r = 3	
$SCRM_r$	10,000		10,000		10,000	
NRM_{rn}	r = 1		r = 2		r = 3	
n = 1	2		3		0	
n = 2	2		3		1	
NIL_0	5					
PL	80					
AFV	20					

Table 4.7 The fuzzy value of input parameters of SCAPP problem.

Parameters	Values					
	t = 1	t = 2	t = 3	t = 4	t = 5	t = 6
$\widetilde{RTC}_t$	0.6	0.6	0.6	0.6	0.6	0.6
$\widetilde{OTC}_t$	1	1	1	1	1	1
$\widetilde{SC}_t$	1.4	1.4	1.4	1.4	1.4	1.4
$\widetilde{SW}_t$	150	150	150	150	150	150
$\widetilde{HC}_t$	50	50	50	50	50	50
$\widetilde{FC}_t$	70	70	70	70	70	70
$\widetilde{ACSL}_t$	0.7	0.7	0.7	0.7	0.7	0.7
$\widetilde{D}_{nt}$	t = 1	t = 2	t = 3	t = 4	t = 5	t = 6
n = 1	2,420	1,210	3,440	1,630	4,360	2,550
n = 2	2,510	4,320	1,630	3,440	1,250	2,460
$\widetilde{TFC}_n$ (*10^{-3})	t = 1	t = 2	t = 3	t = 4	t = 5	t = 6
n = 1	5	5	5	5	5	5
n = 2	7	7	7	7	7	7
$\widetilde{PSC}_{nt}$	t = 1	t = 2	t = 3	t = 4	t = 5	t = 6
n = 1	2.5	2.5	2.5	2.5	2.5	2.5
n = 2	2.8	2.8	2.8	2.8	2.8	2.8
$\widetilde{IRMC}_{rt}$	t = 1	t = 2	t = 3	t = 4	t = 5	t = 6
r = 1	1.8	1.8	1.8	1.8	1.8	1.8
r = 2	1.9	1.9	1.9	1.9	1.9	1.9
r = 3	1.7	1.7	1.7	1.7	1.7	1.7
$\widetilde{AVSL}_{st}$ (*10^{-3})	t = 1	t = 2	t = 3	t = 4	t = 5	t = 6
s = 1	0.8	0.8	0.8	0.8	0.8	0.8
s = 2	0.75	0.75	0.75	0.75	0.75	0.75
s = 3	0.7	0.7	0.7	0.7	0.7	0.7
s = 4	0.7	0.7	0.7	0.7	0.7	0.7

Parameters	Values					
$\widetilde{TRMC}_{st}$ (*10^{-4})	t = 1	t = 2	t = 3	t = 4	t = 5	t = 6
s = 1	1	1	1	1	1	1
s = 2	0.6	0.6	0.6	0.6	0.6	0.6
s = 3	0.3	0.3	0.3	0.3	0.3	0.3
s = 4	0.6	0.6	0.6	0.6	0.6	0.6
$\widetilde{ACFRM}_{srt}$ (*10^{-3})	r = 1		r = 2		r = 3	
s = 1	0.9		1		1.1	
s = 2	1.5		1.7		1.6	
s = 3	1.5		1.7		1.6	
s = 4	1.2		1.4		1.3	
$\widetilde{AVFRM}_{srt}$ (*10^{-3})	r = 1		r = 2		r = 3	
s = 1	0.009		0.01		0.0088	
s = 2	0.015		0.017		0.0128	
s = 3	0.015		0.017		0.0128	
s = 4	0.012		0.014		0.0104	
$\widetilde{PC}_{srt}$ (*10^{-3})	r = 1		r = 2		r = 3	
s = 1	2		3		1	
s = 2	1		2		0.5	
s = 3	0.5		1		0.3	
s = 4	1		2		0.5	
$MaxCRM_{srt}$ (*10^{-3})	r = 1		r = 2		r = 3	
s = 1	3.5		3.5		3.5	
s = 2	3		3		3	
s = 3	3.5		3		4.5	
s = 4	3		3.5		3.5	

Tables 4.6 and 4.7 present the input parameters for the supply chain production planning model, including both crisp and fuzzy values. For this demonstration, three key points of a triangular fuzzy number are generated by adding and subtracting 20% from the most likely value. Similarly, six key points of a triangular intuitionistic fuzzy number are generated by adding and subtracting 20% from the most likely value for the membership function and adding and subtracting 40% from the most likely value for the non-membership function.

4.1.3 Results of Case 1

This section highlights the effectiveness and strengths of the proposed five-phase hybrid fuzzy optimization approach by comparing its results with those of the conventional specific fuzzy optimization approach.

- **Result of A Conventional Specific Fuzzy Optimization Approach**

Table 4.8 Result of a conventional specific fuzzy optimization approach.

	Conventional Specific Fuzzy Optimization Approach
Minimize Total Supply Chain Costs	\$268,520
Minimize Total Shortage of Products	85,871 units
Maximize Total Values of Purchasing	12,644 units
Satisfaction Level of 1st objective	50.004%
Satisfaction Level of 2nd objective	82.959%
Satisfaction Level of 3rd objective	50.000%
Maximize minimum satisfaction value	50.000%

The conventional specific fuzzy optimization approach effectively generates a supply chain production plan with a minimum total cost of \$268,520, a minimum total product shortage of 85,871 units, and a maximum total purchasing value of 12,644 units. The overall satisfaction level achieved is 50%, with the minimum satisfaction level among the objective functions being maximized. The satisfaction levels for the first, second, and third objectives are 50.004%, 82.959%, and 50%, respectively.

- **Result of Five-Phase Hybrid Fuzzy Optimization Approach**

A set of Pareto optimal solutions generated by this approach, allowing decision-makers to select the most preferred solution. In cases where the preference for objectives is not clearly defined, various methods are proposed to assist DMs in making informed decisions. One such method is the linear normalization max method, initially introduced by Jafaryeganeh et al. (2020) and applied in this study. Performance is normalized by scaling each attribute value relative to the maximum value within its criterion, with the overall score derived from the sum of these scaled ratios for all objectives. The linear normalization max method is computed through distinct equations depending on whether the goal is maximization or minimization.

- Normalized ratio of a maximization objective

$$NR_{Z_j} = \frac{Z_j}{Z_j^{PIS}} \qquad (4.24)$$

- Normalized ratio of a minimization objective

$$NR_{Z_j} = (1 - \frac{Z_j}{Z_j^{NIS}}) \qquad (4.25)$$

where Z_j, Z_j^{PIS} and Z_j^{NIS} are objective value and Positive Ideal Solution (PIS) and Negative Ideal Solutions (NIS) values of each objective function.

Then, these aggregated normalized ratios quantify the total deviation from ideal solutions, whereby a greater score signifies a more optimal solution relative to others.

$$\text{Aggregated score} = \sum_{j=1}^{J} normalized\ ratio\ of\ Z_j \qquad (4.26)$$

Table 4.9 A set of pareto optimal solutions of the proposed five-phase hybrid fuzzy optimization approach.

No. Of grid points (GP)	e_2	e_3	Objective Function			Satisfaction Level			Non-Satisfaction Level			Maximize minimum satisfaction level and Minimize maximum non-satisfaction level (%)	Normalized Ratio			Aggregated Score
			Minimize Total Supply Chain Costs ($)	Minimize Total Shortage of Products (units)	Maximize Total Values of Purchasing (units)	μ_{Z_1} (%)	μ_{Z_2} (%)	μ_{Z_3} (%)	τ_{Z_1} (%)	τ_{Z_2} (%)	τ_{Z_3} (%)		NR_{Z_1}	NR_{Z_2}	NR_{Z_3}	
0	78,326	7,784	222,460	76,750	7,784	84.104	100	84.102	15.876	0	15.898	68.2049	0.0332	0.0000	0.6921	0.7254
1	78,248	8,660	222,460	76,829	8,660	84.104	100	84.102	15.876	0	15.898	68.2049	0.0332	0.0010	0.6575	0.6918
2	78,169	9,535	222,460	76,908	9,535	84.104	100	84.102	15.876	0	15.898	68.2049	0.0332	0.0020	0.6229	0.6582
3	78,090	10,410	222,460	76,987	10,410	84.104	100	84.102	15.876	0	15.898	68.2049	0.0332	0.0030	0.5883	0.6246
4	78,011	11,285	222,460	77,066	11,285	84.104	100	84.102	15.876	0	15.898	68.2053	0.0332	0.0041	0.5537	0.5910
5	77,932	12,160	222,460	77,144	12,160	84.104	100	84.102	15.876	0	15.898	68.2053	0.0332	0.0051	0.5191	0.5575
6	77,854	13,035	222,460	77,223	13,035	84.104	100	84.102	15.876	0	15.898	68.2056	0.0332	0.0061	0.4845	0.5239
7	77,775	13,911	222,460	77,302	13,911	84.104	100	84.102	15.876	0	15.898	68.2056	0.0332	0.0071	0.4498	0.4903
8	77,696	14,786	222,460	77,381	14,786	84.104	100	84.102	15.876	0	15.898	68.2056	0.0332	0.0082	0.4152	0.4567
9	77,617	15,661	222,460	77,460	15,661	84.104	100	84.102	15.876	0	15.898	68.2056	0.0332	0.0092	0.3806	0.4231
10	77,538	16,536	222,460	77,538	16,536	84.104	100	84.102	15.876	0	15.898	68.2056	0.0332	0.0102	0.3460	0.3895
11	77,460	17,411	222,460	77,617	17,411	84.104	100	84.102	15.876	0	15.898	68.2056	0.0332	0.0113	0.3114	0.3560
12	77,381	18,286	222,460	77,969	18,286	84.104	100	84.102	15.876	0	15.898	68.2056	0.0332	0.0123	0.2768	0.3224
13	77,302	19,161	222,480	77,775	19,161	84.104	100	84.064	15.915	0	15.936	68.1271	0.0333	0.0133	0.2422	0.2889
14	77,223	20,037	222,480	77,854	20,037	84.065	100	84.064	15.915	0	15.936	68.1271	0.0333	0.0143	0.2076	0.2553
15	77,144	20,912	222,480	77,932	20,912	84.065	100	84.064	15.915	0	15.936	68.1271	0.0333	0.0154	0.1730	0.2217
16	77,066	21,787	222,481	78,011	21,787	84.065	100	84.064	15.915	0	15.936	68.1271	0.0333	0.0164	0.1384	0.1881
17	**76,987**	**22,662**	**223,680**	**78,090**	**22,662**	**81.399**	**100**	**85.004**	**18.581**	**0**	**14.996**	**62.8181**	**0.0389**	**0.0174**	**0.1038**	**0.1601**
18	76,908	23,537	230,440	78,169	23,537	66.386	100	90.003	33.594	0	9.997	32.7924	0.0703	0.0184	0.6921	0.1580
19	76,829	24,412	237,250	78,248	24,412	51.279	100	95.001	48.701	0	4.999	2.5781	0.1019	0.0195	0.3460	0.1560
20	76,750	25,287	237,800	78,326	25,287	50.059	100	100	49.921	0	0	0.1380	0.1045	0.0205	0.0000	0.1250

4.1.3.1 Case 1's Comparison of the Results

This section presents a comparison of the optimal results from the conventional specific fuzzy optimization approach and the proposed five-phase hybrid fuzzy optimization approach, emphasizing the key contributions.

Table 4.10 Results comparison of Case 1.

	Conventional Specific Fuzzy Optimization Approach	Five-Phase Hybrid Fuzzy Optimization Approach
Minimize Total Supply Chain Costs	$268,520	$223,680
Minimize Total Shortage of Products	85,871 units	78,090 units
Maximize Total Values of Purchasing	12,644 units	22,662 units
Satisfaction Level of 1st objective	50.004%	81.399%
Satisfaction Level of 2nd objective	82.959%	100%
Satisfaction Level of 3rd objective	50.000%	85.004%
Non-Satisfaction Level of 1st objective	-	18.581%
Non-Satisfaction Level of 2nd objective	-	0%
Non-Satisfaction Level of 3rd objective	-	14.996%
Maximize minimum satisfaction value	50.000%	-
Maximize Minimum Satisfaction Value and Minimize Maximum Non-Satisfaction Value	-	62.818%

According to Table 4.10, the efficient supply chain production plan derived from the five-phase hybrid fuzzy optimization approach, which simultaneously maximizes the minimum satisfaction value and minimizes the maximum non-satisfaction value, demonstrates clear advantages over the plan based on the conventional specific fuzzy optimization approach, which focuses solely on maximizing the minimum satisfaction value. The superior performance of the hybrid approach is evident not only in the aggregated results but also in the stability of objective function values. Specifically, the hybrid model achieves a minimum total supply chain cost of $223,680, with observed cost fluctuations ranging between $223,680 and $256,310, indicating a more controlled and predictable cost behavior. It also results in a minimum total product shortage of 78,090 units, fluctuating within a narrower band of 78,090 to 82,450 units, demonstrating improved reliability in meeting customer demand. Furthermore, the maximum total purchasing value reaches 22,662 units, varying between 20,310 and 22,662 units, which reflects more consistent procurement planning. In contrast, the conventional approach exhibits wider and more erratic fluctuations, with total supply chain costs ranging from $268,520 up to $301,240, product shortages vary from 85,871 to 93,500 units and purchasing values ranging from as low as 10,020 to a maximum of only 12,644 units. These wider fluctuations indicate less robustness under uncertainty. Therefore, the hybrid approach not only delivers better nominal performance but also improves the stability and resilience of the SC against fluctuating conditions.

4.1.3.2 Case 1's Validation of the Results

Validating the results is a critical step in confirming the robustness and reliability of the proposed five-phase hybrid fuzzy optimization approach. This includes assessing its ability to handle data fuzziness and hesitation, manage data fuzziness by adjusting the confidence level of constraints, and evaluate the model's overall robustness.

- **Test Ability to Handle Data Fuzziness and Data Hesitation**

  To assess its ability to handle data fuzziness and hesitation, the model based on the proposed five-phase hybrid fuzzy optimization approach is tested with different acceptable sets of Triangular Intuitionistic Fuzzy Numbers (TIFN). This is done by varying the acceptable level percentage (A = 50%, 60%, 70%, 80%, 90%) and the rejection level percentage (R = 50%, 40%, 30%, 20%, 10%) to identify the most efficient data set.

Table 4.11 Results of testing ability to handle data fuzziness and data hesitation.

A (%)	*R* (%)	Objective Function			
		Minimize Total Supply Chain Costs ($)	Minimize Total Shortage of Products (units)	Maximize Total Values of Purchasing (units)	Deviation (%)
50	50	262,720	80,262	12,647	0.02598
60	40	261,580	78,272	12,647	0.02261
70	30	260,410	77,834	12,650	0.04748
80	20	259,240	77,375	12,651	0.05215
90	10	257,990	76,960	12,673	2.29106

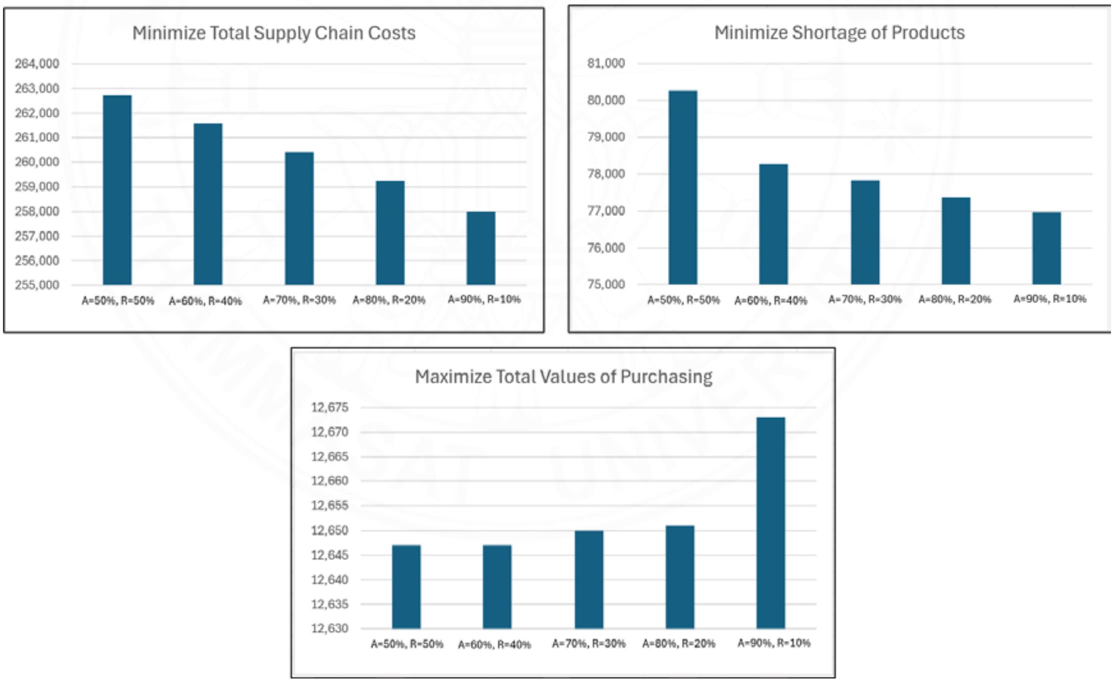

Figure 4.2 Objective function values of testing ability to handle data fuzziness and data hesitation.

As shown in the outcomes in Table 4.11 and Figure 4.2, increasing the acceptable level percentage or decreasing the rejection level percentage leads to improved values for the objective functions. Consequently, this approach enables the generation of efficient outputs, allowing decision-makers to adjust the acceptable level (α) and rejection level (β) percentages according to their experiences or preferences. This flexibility supports the development of effective operational and strategic plans to address future uncertainties.

- **Test Ability to Handle Data Fuzziness by Setting the Confident Level of Constraints**

  To evaluate its capability to handle data fuzziness by adjusting the confidence level of constraints, the model based on the proposed five-phase hybrid fuzzy optimization approach is tested using various confidence level percentages (γ= 50%, 60%, 70%, 80%, 90%, and 100%).

Table 4.12 Result of testing ability to handle data fuzziness by setting the confident level of constraints.

% confident level (γ)	Objective Function		
	Minimize Total Supply Chain Costs ($)	Minimize Total Shortage of Products (units)	Maximize Total Values of Purchasing (units)
50	222,140	75,854	22,775
60	224,550	76,750	21,879
70	226,910	77,626	21,003
80	229,250	78,497	20,132
90	231,610	79,373	19,256
100	233,930	80,231	18,398

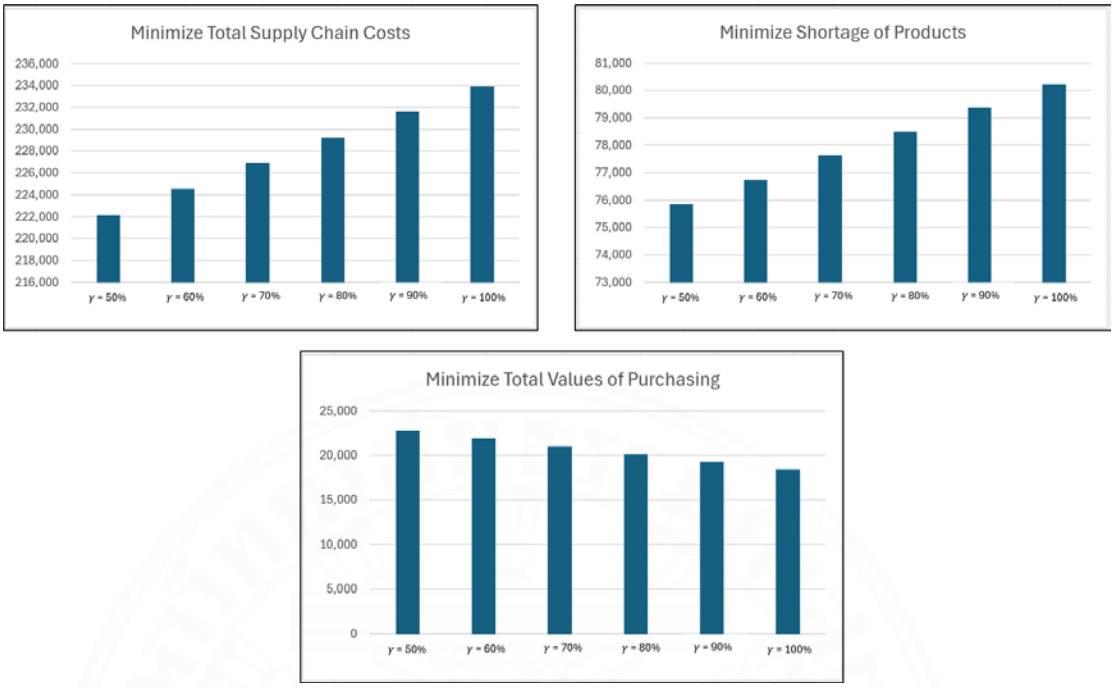

Figure 4.3 Objective function values of testing ability to handle data fuzziness by setting the confident level of constraints.

As shown in Table 4.12 and Figure 4.3, increasing the confidence level percentage results in higher values for all objective functions. This indicates that the objective functions become less desirable due to the trade-off between the confidence level for risk violations and the optimal value of the objective function. Therefore, when the confidence level for risk violation is high (indicating a lower risk of constraint violations), the feasible solution set is reduced, leading to less favorable optimal objective function values.

- **Test Ability of Model Robustness**

  To evaluate the robustness of the model, the outcomes obtained from the RRP approach are compared with those of the conventional approach, which utilizes the EV and CCP methods without incorporating RRP. The comparison is based on two performance metrics: the average value and standard deviation, which reflect the efficiency and reliability of the optimal solution. This evaluation is conducted across 10 scenarios, each generated using a uniform distribution between the pessimistic value and optimistic value of the fuzzy parameters.

Accordingly, only the minimization of total supply chain costs; fuzzy objective function, is tested under these scenarios, as presented in Table 4.13.

Table 4.13 Result of testing ability of model robustness.

No. of Scenarios	EV and CCP Approaches (γ = 0.8)	RRP Approach (γ = 0.8)
1	$207,903.433	$208,213.495
2	$209,885.950	$209,934.020
3	$211,606.324	$211,654.546
4	$213,326.704	$213,375.070
5	$215,047.079	$215,077.055
6	$216,767.454	$216,798.344
7	$218,487.833	$218,517.705
8	$220,208.207	$220,238.185
9	$221,928.585	$221,958.562
10	$223,648.962	$223,678.938
Average	$215,881.053	$215,844.592
Standard Deviation	$5,252.465	$5,042.711

As shown in Table 4.13, the average values of both approaches are similar, but the Standard Deviation (SD) value for the RRP approach is smaller. This clearly indicates that the RRP approach is more effective in handling information distribution. Therefore, the robustness of the model can be confirmed.

4.1.4 Summary

Creating an effective and realistic supply chain production plan requires addressing two critical challenges: data uncertainty and conflicting objectives. This study offers a range of valuable insights and implications to support managerial decision-making in practice.

In real-world applications, it is difficult for companies to separate supply chain operations from production planning. This study provides a valuable example of how these operations can be integrated and optimized simultaneously. Moreover, collected data can be imprecise due to factors such as unavailability, incompleteness, estimation errors, time variation, and DMs' hesitation. An effective approach is essential to manage these uncertainties, enabling companies to develop more robust operational and strategic plans to handle future uncertainties effectively.

A significant challenge in practical applications is the difficulty DMs face in controlling the fuzziness levels of constraints. However, knowing the optimal fuzziness levels can assist DMs in making better decisions for their operations. By employing a credibility level to indicate the likelihood of a fuzzy event, the uncertain parameters can be transformed into crisp values. Adjusting these credibility levels leads to a range of optimal results, from optimistic to pessimistic, providing planners and managers with flexible inputs. This enables them to develop operational and strategic plans that account for different scenarios, allowing them to select the most appropriate plan based on their specific situation.

Another issue in supply chain operations and production planning is the presence of data noise, which cannot be fully controlled. Data noise arises from both the data collection process and calculations. The model robustness addressed in this study helps generate reliable optimal solutions, even in the presence of data noise, enhancing the overall reliability of the decision-making process.

When multiple conflicting objectives are considered simultaneously, this study demonstrates that a set of strong Pareto optimal solutions can be generated. These solutions reflect different compromises between satisfaction and non-satisfaction levels, offering DMs valuable choices in alignment with their policies, where no objectives are drastically worsened or overly sacrificed.

The five-phase hybrid approach proposed in this study outperforms the conventional fuzzy linear programming approach in several ways. It was demonstrated and validated through a multiple-objective SCPP problem that incorporates uncertainty in customer demand and related costs. The SCPP problem integrated procurement, production, and distribution plans, optimizing the minimization of total supply chain costs, the minimization of product shortages, and the maximization of total purchasing values simultaneously.

The proposed approach effectively addresses the weaknesses of the conventional specific FLP model. It uses Triangular Intuitionistic Fuzzy Numbers (TIFN) to represent both imprecise data and data hesitation. The (α, β)-cut approach filters out unacceptable data, while the Realistic Robust Programming (RRP) manages uncertainty in fuzzy objective functions and enhances model robustness. The Chance-Constrained Programming (CCP) approach deals with uncertainty in fuzzy constraints and sets credibility levels. The Intuitionistic Fuzzy Linear Programming (IFLP) approach optimizes multi-objective problems with respect to both satisfaction and non-satisfaction levels. Finally, The AUGMECON approach concludes by generating multiple Pareto optimal solutions, offering flexibility for decision-makers to select an alternative that best fits their objectives or constraints.

In summary, the optimal solutions obtained from the proposed five-phase hybrid approach demonstrate its effectiveness in providing efficient and consistent solutions. It also provides flexibility by generating different efficient solutions, allowing DMs to select the preferred satisfactory solution. Despite these strengths, the study has some limitations. First, there are no restrictions on the amount of fuzziness parameters, which could impact the final solution. Second, all fuzzy parameters were represented by triangular distributions, but other distribution types could also be used. Third, the study could be expanded to include more realistic conditions, such as multiple manufacturers, customers, and distributors, which would increase the complexity and realism of the model. As the model becomes more complex, exploring the use of meta-heuristic algorithms, such as Genetic Algorithms (GA), could offer a near optimal results or approximately optimal results. Additionally, incorporating alternative transportation routes and addressing vehicle routing and lateral transshipment problems could further enhance the model’s applicability.

4.2 Case 2: A Unified Fairness and Robustness Fuzzy Optimization Approach for Supply Chain Aggregate Production Planning

Aggregate Production Planning (APP) in Supply Chain (SC) management is essential for aligning production activities with organizational goals. However, conventional specific fuzzy optimization approach to APP often fails to address two critical challenges: Proportional Fairness (PF) among competing objectives and robustness under uncertainty. The lack of fairness in multi-objective optimization can lead to inequitable outcomes, where certain objectives are prioritized over others based on differing priorities. Similarly, neglecting robustness in APP optimization can result in unreliable and non-resilient plans, particularly when dealing with uncertain or imprecise information. These challenges underscore the need for an innovative approach that integrates both fairness and robustness into the APP process. To address this gap, this study proposes a unified fairness and robustness optimization model that combines Proportional Fairness (PF) and Robust Chance-Constrained Programming (RCCP). This study aims to address the complexities of managing multi-objective APP in uncertain environments. The effectiveness of the proposed approach is demonstrated through a case study that focuses on minimizing total costs, minimizing total workforce level fluctuations, and maximizing total value of purchasing. Comparative analysis shows that the proposed approach outperforms conventional specific fuzzy optimization approach by enhancing both fairness and robustness in APP outcomes. This study provides decision-makers with a comprehensive framework to achieve equitable and resilient APP solutions, contributing to the long-term sustainability and efficiency of supply chain operations.

4.2.1 Mathematical Notations and Model

The notations for indexes, parameters, and decision variables are provided in Tables 4.14 to 4.17. Notably, all fuzzy parameters are represented with a tilde ($\tilde{}$) above the corresponding symbols to indicate their fuzzy nature.

Table 4.14 Indexes of SCAPP problem (Case 2).

Indexes	Meaning
s	List of suppliers ($s = 1, \dots, S$)
t	List of planning horizons (t= $1, \dots, T$)

Table 4.15 Crisp parameters of SCAPP problem (Case 2).

Crisp Parameters	Meaning
IW_0	Initial staffing level (persons)
P	Worker output efficiency (%) ($0 < PL < 1$)
AWV	Allowed variation in staffing (%)
TS_s	Total score of supplier *s* (%)
$ProdT$	Processing time per product at the plant (minutes)
ART_t	Regular time availability in period *t* (hours)
AOT_t	Overtime availability in period *t* (hours)
RPP	Quantity of raw materials required per product (units)
$MaxM_t$	Maximum operational capacity of machines in period *t* (m/c-hours)
MU_t	Machine operating time per unit in period *t* (m/c-hours/unit)
WSR_t	The amount of warehouse capacity reserved for raw materials at the factory in period *t* (m^2/unit)
WSP_t	The amount of warehouse capacity reserved for final products at the factory in period *t* (m^2/unit)
$MaxWS_t$	The upper limit of storage space usable at the factory in period *t* (m^2)
$MaxR_{st}$	The upper limit of raw material available from supplier *s* in period in period *t* (units)

Table 4.16 Uncertain parameters of SCAPP problem (Case 2).

Uncertain Parameters	Meaning
$\widetilde{CRT}_t$	Production cost under fuzziness for regular hours in period *t* ($/minute)
$\widetilde{COT}_t$	Production cost under fuzziness for overtime hours in period *t* ($/minute)
$\widetilde{WS}_t$	Wage cost under fuzziness for workers in period *t* ($/person)
$\widetilde{HC}_t$	Recruitment cost under fuzziness in period *t* ($/person)
$\widetilde{FC}_t$	Termination cost under fuzziness in period *t* ($/person)
$\widetilde{ACSL}_t$	Service level threshold under fuzziness for the production plant during period *t* (%)
$\widetilde{ICR}_t$	Raw material inventory cost under fuzziness in period *t* ($/unit)
$\widetilde{TCR}_{st}$	Transportation expenses under fuzziness for raw materials from supplier *s* in period *t* ($/unit)
$\widetilde{AVSL}_{st}$	Average service performance under fuzziness from supplier *s* in period *t* (%)
$\widetilde{TCP}_t$	Delivery expenses under fuzziness for shipments from the production plant to customers in period *t* ($/unit)
$\widetilde{ICP}_t$	Product inventory carrying cost under fuzziness in period *t* ($/unit)
$\widetilde{PeC}_t$	Penalty cost under fuzziness for product stockouts affecting customers in period *t* ($/unit)
$\widetilde{De}_t$	Product demand under fuzziness from customers in period *t* (units)
$\widetilde{ACFR}$	Tolerable raw material failure rate under fuzziness in the production facility (%)
$\widetilde{AVFR}_s$	Average fuzzy defect rate for raw materials delivered by supplier *s* (%)
$\widetilde{PuC}_{st}$	Acquisition expenses under fuzziness for raw materials from supplier *s* in period *t* ($/unit)

Table 4.17 Decision variables of SCAPP problem (Case 2).

Decision Variables	Meaning
NW_t	Workforce headcount in period *t* (persons)
NHW_t	Total recruited workforce in period *t* (persons)
NFW_t	Total terminated workforce in period *t* (persons)
IR_t	Ending raw material stock for period *t* (units)
RTQ_t	Production volume within regular time in period *t* (units)
OTQ_t	Production volume within overtime in period *t* (units)
PQ_t	Product volume allocated to customers in period *t* (units)
IP_t	Ending product stock for period *t* (units)
SPO_t	Insufficient product availability for customers in period *t* (units)
RQ_{srt}	Raw material volume delivered by supplier *s* in period *t* (units)

<u>**Objective Functions**</u>

1. Minimizing total costs is a fundamental objective in formulating an effective APP strategy within a SC. This objective underscores the importance of cost efficiency in ensuring the overall competitiveness and sustainability of supply chain operations. Typically, the total costs (denoted as $\widetilde{TC}$) in such models are subject to uncertainty, reflecting the inherent variability in supply chain processes. These costs are aggregated as the sum of several critical components, including purchasing costs ($\widetilde{TPuC}$), production costs ($\widetilde{TPrC}$), workers' costs ($\widetilde{TWC}$), inventory costs ($\widetilde{TIC}$), transportation costs ($\widetilde{TTC}$), and shortage costs ($\widetilde{TSQ}$), each of which contributes to the total financial expenditure over a specified planning horizon.

$$
\begin{aligned}
&Minimize\ \widetilde{TSC} = \widetilde{TPuC} + \widetilde{TPrC} + \widetilde{TWLC} + \widetilde{TIC} + \widetilde{TTC} + \widetilde{TSQ} \\
&= \left(\sum_{s=1}^{S}\sum_{t=1}^{T} \widetilde{PuC}_{st} \times RQ_{st}\right) + \left(\sum_{t=1}^{T} \widetilde{CRT}_t \times PT \times RTQ_t\right) \\
&+\left(\sum_{t=1}^{T} \widetilde{COT}_t \times PT \times OTQ_t\right) + \left(\sum_{t=1}^{T} \widetilde{WS}_t \times NW_t\right) + \left(\sum_{t=1}^{T} \widetilde{HC}_t \times NHW_t\right) \\
&+\left(\sum_{t=1}^{T} \widetilde{FC}_t \times NFW_t\right) + \left(\sum_{t=1}^{T} \widetilde{ICR}_t \times IR_t\right) + \left(\sum_{t=1}^{T} \widetilde{ICP}_t \times IP_t\right) \\
&+\left(\sum_{s=1}^{S}\sum_{t=1}^{T} \widetilde{TCR}_{st} \times RQ_{st}\right) + \left(\sum_{t=1}^{T} \widetilde{TCP}_t \times PQ_t\right) + \left(\sum_{t=1}^{T} \widetilde{PeC}_t \times SPQ_t\right) \qquad (4.27)
\end{aligned}
$$

2. Minimizing fluctuations in workforce levels is a vital aspect of effective supply chain and production planning, as maintaining a stable workforce is crucial for operational efficiency and long-term sustainability. Workforce fluctuations, often caused by seasonal demand variations or production uncertainties, pose significant challenges for organizations. Excessive changes in workforce levels can lead to the loss of experienced and skilled workers, whose expertise is vital for maintaining productivity and quality. Furthermore, these fluctuations often result in substantial costs, including recruitment, training, severance, and overtime compensation, which can strain financial resources and reduce overall profitability.

$$Minimize\ TCNW = \sum_{t=1}^{T}(NHW_t - NFW_t) \qquad (4.28)$$

3. Maximizing the total value of purchasing is a critical objective in supply chain production planning, ensuring the company obtains the ideal number of raw materials from top-quality suppliers. This objective emphasizes strategic procurement practices that prioritize suppliers based on their performance in key criteria such as competitive pricing, superior quality, and timely delivery. By focusing on total value of purchasing, organizations can strengthen their supply chain resilience, reduce costs, and enhance the overall efficiency of their production processes.

$$Maximize\ TVP = \sum_{s=1}^{S} TS_s \times RQ_{st} \qquad (4.29)$$

Constraints

1. Raw Material Quality Assessment: This criterion plays a vital role in ensuring the efficiency and reliability of supply chain operations. It serves as a systematic method for assessing the raw materials quality provided by suppliers in each specific period. High-quality raw materials are critical for maintaining product standards, reducing defects, and ensuring efficient production processes. By assessing raw material quality regularly, organizations can ensure that the supplied inputs meet predefined specifications and performance requirements.

$$\sum_{s=1}^{S} \widetilde{AVFR}_s \times RQ_{st} \le \widetilde{ACFR}_t \times \sum_{s=1}^{S} RMQ_{srt} \qquad \forall\ t \in T \qquad (4.30)$$

2. Supplier Capacity: Supplier capacity denotes the highest quantity of raw materials that a supplier can consistently provide within a designated time frame. This metric is critical for effective supply chain planning, as it directly impacts the ability of the organization to meet production schedules and customer demand. Understanding supplier capacity enables companies to allocate resources more effectively, balance supply with demand, and avoid potential bottlenecks in the production process.

$$RQ_{st} \leq MaxR_{st} \qquad \forall\, s \in S, t \in T \tag{4.31}$$

3. Supplier Service Level: The supplier service level is an important measure that evaluates suppliers' reliability and effectiveness, especially regarding their punctuality in delivering raw materials within each designated period. This metric is essential for maintaining a smooth and uninterrupted supply chain, as timely deliveries are crucial for meeting production schedules, fulfilling customer demands, and avoiding costly delays.

$$\sum_{s=1}^{S} \widetilde{AVSL}_s \times RQ_{st} \geq \widetilde{ACSL} \times \sum_{s=1}^{S} RQ_{st} \qquad \forall\, t \in T \tag{4.32}$$

4. Raw Material Availability: Ensuring raw material availability is a cornerstone of effective supply chain and production planning. This criterion represents the combined total resources supplied by all vendors within a given period, ensuring that the overall raw material demand for production during that time is fully satisfied.

$$RPP \times (RTQ_{nt} + OTQ_{nt}) \leq \sum_{s=1}^{S} RQ_{st} \qquad \forall\, t \in T \tag{4.33}$$

5. Raw Material Inventory: Raw material inventory represents the leftover quantity of raw materials available at the close of each production period, following the fulfillment of manufacturing needs for that timeframe. This remaining stock is crucial for maintaining smooth operations and reducing the risks linked to supply chain interruptions.

$$IR_t = IR_{(t-1)} + \sum_{s=1}^{S} RQ_{st} - (RTQ_t + OTQ_t) \times RPP \qquad \forall\, t \in T \tag{4.34}$$

6. Product Shortages: Product shortages occur when the available quantity of products is insufficient to meet customer demand within a specified period. This metric highlights the gap between the required quantity of products and the actual amount that can be supplied to customers, indicating instances where demand exceeds production capacity or supply availability. A product shortage may occur due to various factors, such as production delays, supply chain disruptions, insufficient raw materials, or inaccurate demand forecasting.

$$SPQ_t = SPQ_{(t-1)} + \widetilde{De}_t - PQ_t \qquad \forall\, t \in T \tag{4.35}$$

7. Production Time Availability: Production time availability is a critical constraint that governs the total hours available for production activities within a given period. It includes both regular working hours and overtime hours, which are subject to limitations based on workforce levels and operational capacities. This constraint is essential for ensuring that the production process aligns with the required output to meet customer demand while adhering to workforce availability and scheduling restrictions.

$$NW_t \times P \times (RT_t + OT_t) \geq (RTQ_{nt} + OTQ_{nt}) \times PT \qquad \forall\, t \in\, T \tag{4.36}$$

8. Product Inventory: Product inventory refers to the remaining stock of finished products after fulfilling customer demand within a specific period. It functions as a key measure of the company's effectiveness in overseeing production and distribution operations, ensuring that customer orders are fulfilled promptly. The level of product inventory directly reflects the balance between production output, customer demand, and the effectiveness of inventory management strategies.

$$IP_t = IP_{(t-1)} + RTQ_t + OTQ_t - PQ_t \qquad \forall\, t \in T \tag{4.37}$$

9. Warehouse Space Limitation: Warehouse space limitation refers to the confined storage capacity at the manufacturing facility available for storing raw materials and products within each period. It is a critical operational constraint that directly impacts the efficiency of both production processes and inventory management. The limited availability of warehouse space forces companies to optimize the storage and movement of materials and goods, ensuring that space is utilized efficiently to avoid bottlenecks, storage inefficiencies, and potential disruptions in production schedules.

$$(WSP_t \times IP_t) + (WSR_t \times IR_t) \leq MaxWS_t \qquad \forall\, t \in T \tag{4.38}$$

10. Workforce Balancing: Workforce balancing is a critical operational strategy used to allocate the number of workers across various periods in a way that ensures equitable distribution based on production needs, skill requirements, and other operational factors. This equation helps to achieve a workforce allocation that supports optimal productivity while minimizing disruptions caused by fluctuations in workforce levels. Proper workforce balancing enables companies to maintain an optimal number of employees at the appropriate times, leading to more streamlined production, lower labor expenses, and enhanced overall productivity.

$$NW_t = NW_{(t-1)} + NHW_t - NFW_t \qquad \forall\, t > 1 \tag{4.39}$$

11. Workforce Level Variation Proportion: The concept of workforce level variation proportion is crucial in managing the fluctuations in workforce size over time, ensuring that these variations are controlled within acceptable limits. This equation is used to control the extent of changes or fluctuations in workforce levels between consecutive periods. By effectively controlling workforce variation, organizations can mitigate the risks associated with extreme fluctuations, such as labor shortages or excesses, which could adversely impact production efficiency, labor costs, and employee morale.

$$NHW_t + NFW_t \leq AWV \times NW_{(t-1)} \qquad \forall\, t \in T \tag{4.40}$$

12. Machine Capacity: Machine capacity is defined as the highest volume of output a machine can produce within a given period, usually measured in units per hour, day, or shift. This value is a critical parameter in manufacturing planning and optimization, as it directly influences production efficiency, throughput, and the overall capacity of the production facility to meet demand. Machine capacity plays a significant role in determining how well a company can balance supply with demand, as any limitation in machine capacity can lead to production delays, inefficiencies, or even an inability to meet customer expectations.

$$MU_t \times (RTQ_{nt} + OTQ_{nt}) \leq MaxM_t \qquad \forall\, t \in T \tag{4.41}$$

13. Non-Negativity: Constraints (4.42) – (4.45) ensure that all decision variable values are non-negative, with certain values required to be integers.

$$NW_t, NHW_t, NFW_t \geq 0 \; and \; integer \qquad \forall\, t \in T \tag{4.42}$$

$$RTQ_t, OTQ_t, SQ_t, IP_t \geq 0 \; and \; integer \qquad \forall\, t \in T \tag{4.43}$$

$$IR_t, SPQ_t, PQ_t \geq 0 \qquad \forall\, t \in T \tag{4.44}$$

$$RMQ_{st} \geq 0 \qquad \forall\, s \in S, t \in T \tag{4.45}$$

4.2.2 Problem Description of Case 2

The supply chain optimization for the APP problem includes four authorized suppliers supplying raw materials, one manufacturing facility managing production, and the customer, as illustrated in Figure 4.4. The planning period covers six months. This optimization model focuses on three main objectives: (1) minimizing total costs, which include expenses related to raw material procurement, production, and other associated costs; (2) minimizing workforce variability to ensure stable and efficient labor management; and (3) maximizing the total purchasing value to enhance raw material acquisition. The optimization takes place in an uncertain setting, where key elements such as customer demand, product failure rates, service quality, and costs vary and are modeled using Triangular Fuzzy Numbers (TFNs). Given the problem’s inherent complexity, a thorough methodology is necessary to effectively manage the multiple objectives and uncertainties present in supply chain operations.

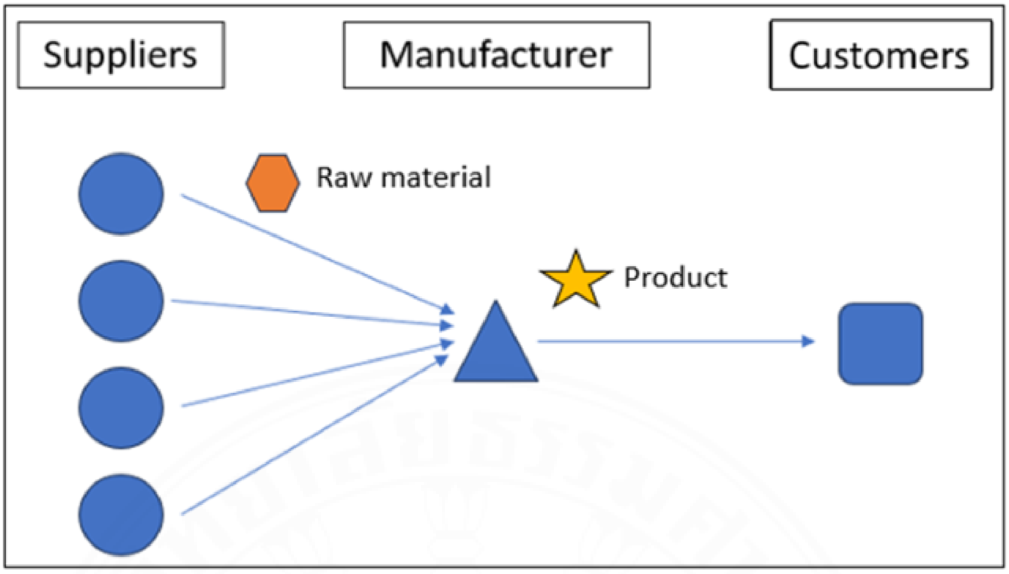

Figure 4.4 The structure of supply chain.

The evaluation involved a group of certified suppliers whose performance was rated according to price, quality of raw materials, and supplier service levels, as summarized in Table 4.18.

Table 4.18 Performance of suppliers.

Criteria	Supplier (s)			
	S_1	S_2	S_3	S_4
Price of Raw Material	Expensive	Standard	Cheap	Standard
Quality of Raw Material	Top-tier	Weak	Weak	Good
Service Level of Supplier	Superior	Satisfactory	Substandard	Substandard
Weighted Score (TSS_t)	0.44	0.20	0.14	0.22

The assumptions for the SCPP plan are as follows:

- The uncertainty in raw material failure rates arises from possible material defects, while variability in the manufacturer's service level is linked to inconsistencies in delivery punctuality.
- Customer demand for products fluctuates over the six-month planning horizon, and all associated supply chain costs are affected by uncertainty.

- Meeting customer demand can result in either full fulfillment or shortages.
- Any shortage results in a penalty, represented by associated shortage costs.
- Lead time is considered insignificant.

Tables 4.19 and 4.20 display the input parameters for the APP in the SC model, including both precise and fuzzy data. In this case, the three points defining the Triangular Fuzzy Numbers (TFNs) are calculated by applying a ±20% deviation from the most likely value.

Table 4.19 Precise parameters.

<table>
<tr><td colspan="2">Parameters</td><td colspan="2">Values</td><td colspan="2">Parameters</td><td colspan="2">Values</td></tr>
<tr><td colspan="2">IW_0</td><td colspan="2">10 persons</td><td colspan="2">$CapP$</td><td colspan="2">3,000 units</td></tr>
<tr><td colspan="2">P</td><td colspan="2">65%</td><td colspan="2">$ProdT$</td><td colspan="2">0.4 minutes</td></tr>
<tr><td colspan="2">AWV</td><td colspan="2">15%</td><td colspan="2">RPP</td><td colspan="2">5 units</td></tr>
<tr><td colspan="2">$CapR$</td><td colspan="2">10,000 units</td><td colspan="4"></td></tr>
<tr><td colspan="2"></td><td>t=1</td><td>t=2</td><td>t=3</td><td>t=4</td><td>t=5</td><td>t=6</td></tr>
<tr><td colspan="2">ART_t (hours)</td><td>144</td><td>160</td><td>168</td><td>176</td><td>120</td><td>192</td></tr>
<tr><td colspan="2">AOT_t (hours)</td><td>50</td><td>50</td><td>50</td><td>60</td><td>40</td><td>60</td></tr>
<tr><td colspan="2">$MaxM_t$ (m/c-hours)</td><td>250</td><td>250</td><td>250</td><td>250</td><td>250</td><td>250</td></tr>
<tr><td colspan="2">MU_t (m/c-hours/unit)</td><td>0.5</td><td>0.5</td><td>0.5</td><td>0.5</td><td>0.5</td><td>0.5</td></tr>
<tr><td colspan="2">WSR_t (m^2/unit)</td><td>7</td><td>7</td><td>7</td><td>7</td><td>7</td><td>7</td></tr>
<tr><td colspan="2">WSP_t (m^2/unit)</td><td>3.5</td><td>3.5</td><td>3.5</td><td>3.5</td><td>3.5</td><td>3.5</td></tr>
<tr><td colspan="2">$MaxWS_t$ (m^2)</td><td>5,000</td><td>5,000</td><td>5,000</td><td>5,000</td><td>5,000</td><td>5,000</td></tr>
<tr><td rowspan="4">$MaxR_{st}$ (units)</td><td>s=1</td><td>3,500</td><td>3,500</td><td>3,500</td><td>3,500</td><td>3,500</td><td>3,500</td></tr>
<tr><td>s=2</td><td>3,000</td><td>3,000</td><td>3,000</td><td>3,000</td><td>3,000</td><td>3,000</td></tr>
<tr><td>s=3</td><td>3,500</td><td>3,500</td><td>3,500</td><td>3,500</td><td>3,500</td><td>3,500</td></tr>
<tr><td>s=4</td><td>3,000</td><td>3,000</td><td>3,000</td><td>3,000</td><td>3,000</td><td>3,000</td></tr>
</table>

Table 4.20 Fuzzy Parameters (most likely value).

Parameters							
Parameters				Values			
$\widetilde{ACE}$				1.2%			
		t=1	t=2	t=3	t=4	t=5	t=6
$\widetilde{CRT}_t$ (\$)		0.6	0.6	0.6	0.6	0.6	0.6
$\widetilde{COT}_t$ (\$)		1.2	1.2	1.2	1.2	1.2	1.2
$\widetilde{WS}_t$ (\$)		150	150	150	150	150	150
$\widetilde{HC}_t$ (\$)		50	50	50	50	50	50
$\widetilde{FC}_t$ (\$)		70	70	70	70	70	70
$\widetilde{ACSL}_t$ (%)		0.7	0.7	0.7	0.7	0.7	0.7
$\widetilde{ICR}_t$ (\$)		1.8	1.8	1.8	1.8	1.8	1.8
$\widetilde{ICP}_t$ (\$)		4.59	4.59	4.59	4.59	4.59	4.59
$\widetilde{TCP}_t$ (\$)		8.4	8.4	8.4	8.4	8.4	8.4
$\widetilde{PeC}_t$ (\$)		2.8	2.8	2.8	2.8	2.8	2.8
$\widetilde{De}_t$ (units)		2,510	4,320	1,630	3,440	1,250	2,460
$\widetilde{TCR}_{st}$ (\$)	s=1	1	1	1	1	1	1
	s=2	0.6	0.6	0.6	0.6	0.6	0.6
	s=3	0.3	0.3	0.3	0.3	0.3	0.3
	s=4	0.6	0.6	0.6	0.6	0.6	0.6
$\widetilde{PuC}_{st}$ (\$)	s=1	2	2	2	2	2	2
	s=2	1	1	1	1	1	1
	s=3	0.5	0.5	0.5	0.5	0.5	0.5
	s=4	1	1	1	1	1	1
				s=1	s=2	s=3	s=4
$\widetilde{AVSL}_s$ (%)				0.8	0.75	0.7	0.7
$\widetilde{AVFR}_s$ (%)				0.009	0.015	0.015	0.015

4.2.3 Results of Case 2

The obtained results from conventional specific fuzzy optimization approach and a unified fairness and robustness fuzzy optimization approach are presented and compared to evaluate their effectiveness and advantages. By comparing the performance of both approaches, this study provides insights into which methodology best aligns with the objectives of minimizing total costs, minimizing total workforce levels, and maximizing the total value of purchasing while maintaining a robust and fair solution in the face of uncertainty.

- **Result of Conventional Specific Fuzzy Optimization Approach**

Table 4.21 Result of conventional specific fuzzy optimization approach.

	Minimize Total Supply Chain Costs	Minimize Fluctuation in Workforce Levels	Maximize Total Values of Purchasing
Conventional Specific Fuzzy Optimization Approach	$129,640	4 persons	1,202 units
Satisfaction Level (Membership Function)	39.997%	42.857%	85.007%

It should be noted that 39.997% represents the lowest satisfaction level achieved when maximizing the minimum satisfaction level.

According to Table 4.21, the conventional specific fuzzy optimization approach delivers optimal results, achieving a minimum total cost of $129,640, a minimal total workforce fluctuation of 4 persons, and a maximum total purchasing value of 1,202 units. The overall satisfaction level is calculated at 39.997%, reflecting the focus on maximizing the minimum satisfaction level across the objective functions. However, it is important to highlight that the highest satisfaction level in the purchasing maximization objective could introduce fairness concerns. Specifically, this emphasis may lead to an imbalance in how stakeholders' objectives are prioritized, creating challenges in ensuring an equitable distribution of benefits across all parties involved.

- **Result of Fuzzy Optimization with Proportional Fairness Approach**

Table 4.22 Result of fuzzy optimization with proportional fairness approach.

	Minimize Total Supply Chain Costs	Minimize Fluctuation in Workforce Levels	Maximize Total Values of Purchasing
Fuzzy Optimization with Proportional Fairness Approach	$121,740	4 persons	849 units
Satisfaction Level (Membership Function)	49.997%	42.857%	49.975%

It should be noted that 42.857% represents the lowest satisfaction level achieved when maximizing the minimum satisfaction level.

As illustrated in Table 4.22, the fuzzy optimization using the proportional fairness method achieves a minimum total cost of $121,740, a workforce fluctuation as low as 4 employees, and a maximum purchasing quantity of 849 units. The overall satisfaction level reaches 42.857%, highlighting the focus on maximizing the lowest satisfaction level across the objective functions.

- **Result of Unified Fairness and Robustness Fuzzy Optimization Approach**

Table 4.23 Result of unified fairness and robustness fuzzy optimization approach.

	Minimize Total Supply Chain Costs	Minimize Fluctuation in Workforce Levels	Maximize Total Values of Purchasing
Unified Proportional Fairness and Robustness Fuzzy Optimization Approach	$118,650	5 persons	768 units
Satisfaction Level (Membership Function)	54.868%	35.143%	42.561%

It should be noted that 35.143% represents the lowest satisfaction level achieved when maximizing the minimum satisfaction level.

As detailed in Table 4.23, the unified fairness and robustness fuzzy optimization method achieves the lowest total cost of $118,650, the smallest workforce fluctuation of 5 employees, and the greatest total purchasing volume of 768 units. The overall satisfaction level stands at 35.143%, indicating a focus on maximizing the minimum satisfaction level among the objectives. Specifically, the satisfaction levels for minimizing total supply chain costs, reducing workforce fluctuations, and maximizing purchasing values are 54.868%, 35.143%, and 42.561%, respectively.

4.2.3.1 Case 2's Comparison of the Results

- **Result Comparison between Conventional Specific Fuzzy Optimization Approach and Fuzzy Optimization with Proportional Fairness Approach**

Table 4.24 Result comparison between conventional specific fuzzy optimization approach and fuzzy optimization with proportional fairness approach.

		Minimize Total Supply Chain Costs	Minimize Fluctuation in Workforce Levels	Maximize Total Values of Purchasing
Conventional Specific Fuzzy Optimization Approach	Objective Values	$129,640	4 persons	1,202 units
	Satisfaction Level (Membership Function)	39.997%	42.857%	85.007%
	% Fairness	14.398%	42.857%	15.053%
Fuzzy Optimization with Proportional Fairness Approach	Objective Values	$121,740	4 persons	849 units
	Satisfaction Level (Membership Function)	49.997%	42.857%	49.975%
	% Fairness	39.653%	42.857%	40.000%

It is important to note that the minimum satisfaction levels are 39.997% for the Conventional Specific Fuzzy Optimization Approach and 42.857% for the Fuzzy Optimization Approach with Proportional Fairness, both achieved by maximizing the minimum satisfaction level.

Table 4.24 provides a comparative analysis of the outcomes obtained from the fuzzy optimization with the proportional fairness approach and the conventional specific fuzzy optimization approach, with emphasis on two key dimensions: satisfaction level and fairness level.

- **Comparing in terms of satisfaction levels and objective values**

Figure 4.5 The satisfaction level and objective value comparison**.**

As illustrated in Figure 4.5, introducing the fairness term into the model results in notable changes. The minimum total cost drops from $129,640 to $121,740, while the minimum workforce fluctuation stays steady at 4 employees. Meanwhile, the maximum total purchasing volume decreases from 1,202 units to 849 units. The satisfaction level for minimizing total supply chain costs rises from 39.99% to 49.99%, the satisfaction for minimizing workforce fluctuations remains at 42.86%, and the satisfaction level for maximizing total purchasing value falls from 85.01% to 49.98%.

- **Comparing in terms of fairness level**

This study uses proportional fairness to assess the model's equity, guaranteeing that no single objective is given undue preference over others. A fairness score of 0% indicates that the objective is either insignificant or entirely overlooked, while a score of 100% reflects that the objective is fully prioritized as the main focus. The fairness percentage is determined by the following formula:

$$\frac{x_i^{NIS} - x_i}{x_i} \tag{4.46}$$

where x_i represents the obtained solution for each objective function, and x_i^{NIS} corresponds to the Negative Ideal Solution (NIS) for each objective function.

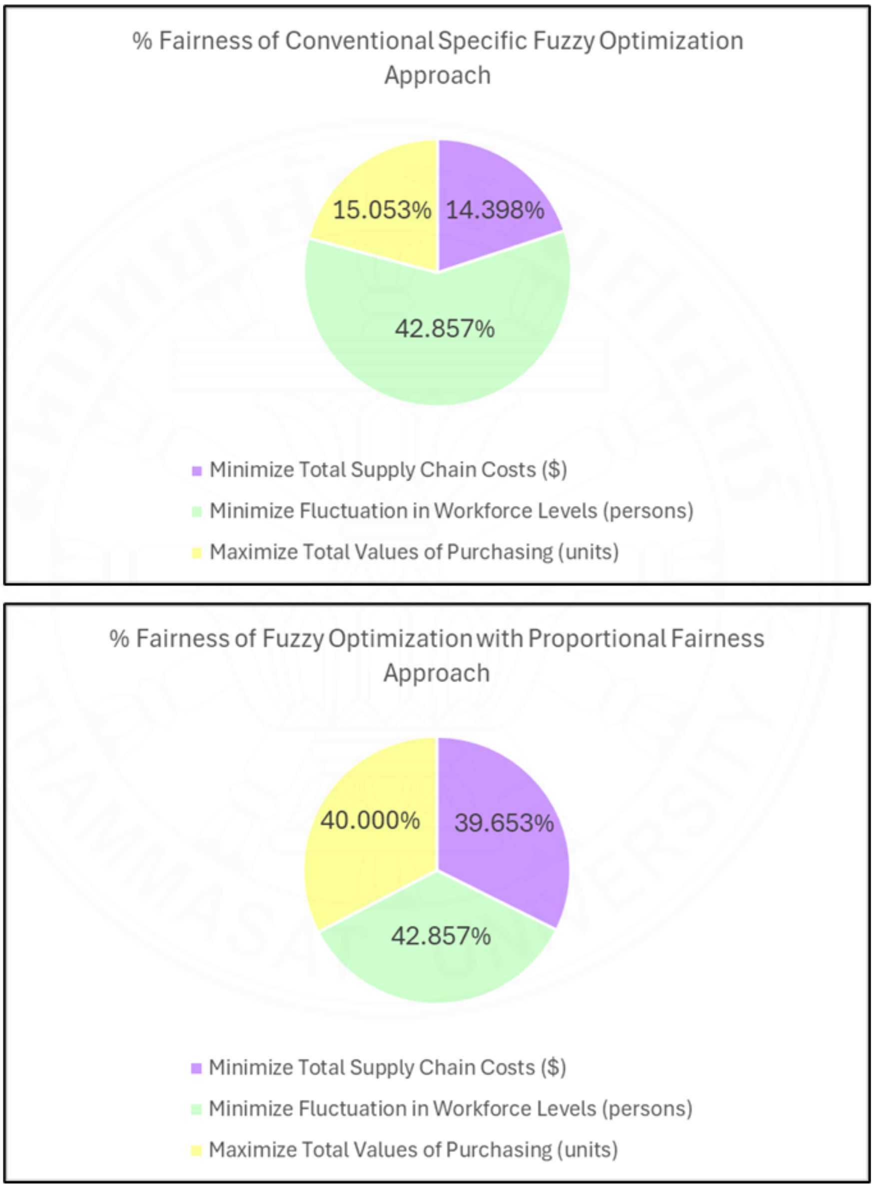

Figure 4.6 The fairness level comparison.

As illustrated in Figure 4.6, the inclusion of the fairness term in the model results in fairness values for all objective functions that are more closely aligned, contrasting with the imbalanced fairness values observed in the conventional specific fuzzy optimization approach. This signifies that the trade-off solutions across all objective functions are now managed with improved fairness and balance. Consequently, the fairness of the model is effectively substantiated.

- **Result Comparison between Conventional Specific Fuzzy Optimization Approach, Fuzzy Optimization with Proportional Fairness Approach, and Unified Fairness and Robustness Fuzzy Optimization Approach**

Table 4.25 Result comparison between conventional specific fuzzy optimization approach, fuzzy optimization with proportional fairness approach, and unified fairness and robustness fuzzy optimization approach.

		Minimize Total Supply Chain Costs	Minimize Fluctuation in Workforce Levels	Maximize Total Values of Purchasing
Conventional Specific Fuzzy Optimization Approach	Objective Values	$129,640	4 persons	1,202 units
	Level of Satisfaction (Membership Function)	39.997%	42.857%	85.007%
	Percentage of Fairness	14.398%	42.857%	85.007%
Fuzzy Optimization with Proportional Fairness Approach	Objective Values	$121,740	4 persons	849 units
	Level of Satisfaction (Membership Function)	49.997%	42.857%	49.975%
	Percentage of Fairness	39.653%	42.857%	40.000%
Unified Fairness and Robustness Fuzzy Optimization Approach	Objective Values	$118,650	5 persons	768 units
	Level of Satisfaction (Membership Function)	54.868%	35.143%	42.561%
	Percentage of Fairness	42.185%	57.143%	45.724%

It is important to note that the minimum satisfaction levels are 39.997% for the Conventional Specific Fuzzy Optimization Approach and 42.857% for the Fuzzy Optimization Approach with Proportional Fairness, both achieved by maximizing the minimum satisfaction level.

Table 4.25 compares the outcomes of the unified fairness and robustness fuzzy optimization approach with those of the conventional specific fuzzy optimization approach and the fuzzy optimization with proportional fairness approach across three key aspects:

- **Comparing in terms of satisfaction levels and objective values**

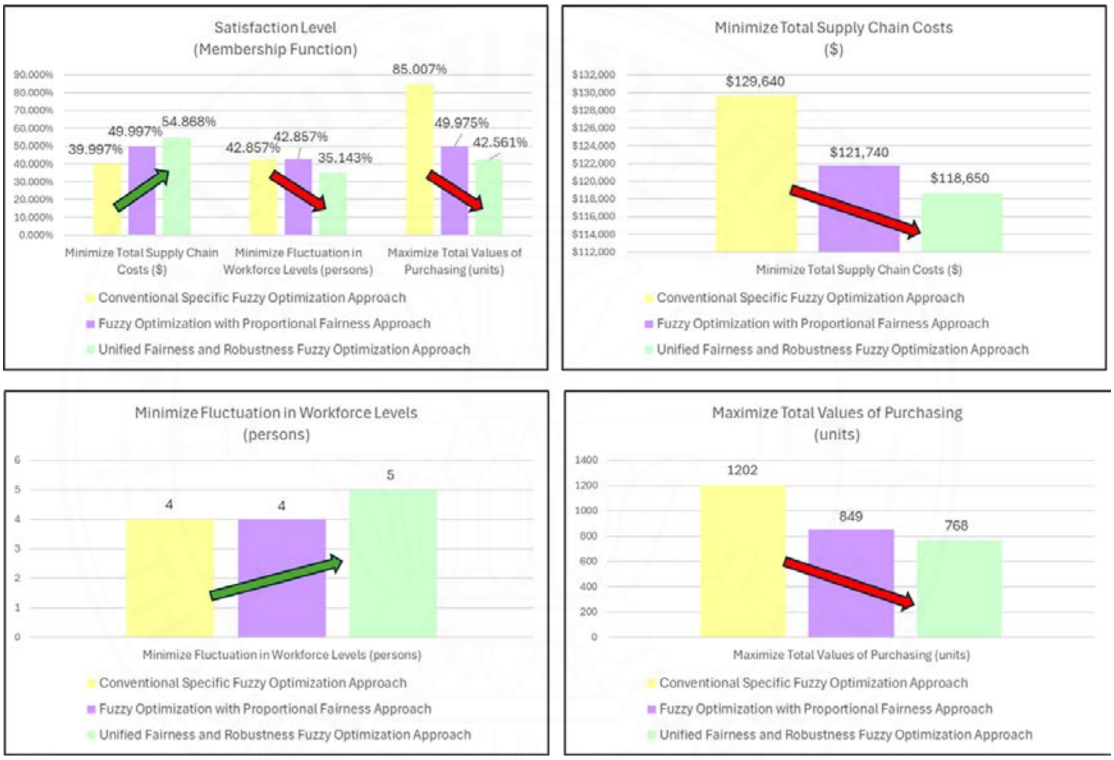

Figure 4.7 The satisfaction level and objective value comparison**.**

As depicted in Figure 4.7, applying the unified fairness and robustness fuzzy optimization method results in a reduction of the minimum total cost from $129,640 to $118,650, a slight increase in the minimum workforce fluctuation from 4 to 5 employees, and a decrease in the maximum total purchasing volume from 1,202 to 768 units. The satisfaction level for minimizing total supply chain costs rises from 39.99% to 54.87%, while the satisfaction for reducing workforce fluctuations falls from 42.86% to 35.14%, and the satisfaction level for maximizing purchasing values drops from 85.01% to 42.56%.

- **Comparing in terms of fairness level**

Figure 4.8 The fairness level comparison**.**

Figure 4.8 displays the outcomes of integrating both proportional fairness and robustness within the model. The results reveal that the fairness percentage achieved by this combined approach exceeds those of the conventional specific fuzzy optimization method and the fuzzy optimization using proportional fairness. Additionally, the unified fairness and robustness fuzzy optimization approach maintains a more balanced fairness percentage across the various objective functions.

- **Comparing in terms of robustness level**

To evaluate the model's robustness, the results from the unified fairness and robustness fuzzy optimization approach are compared against those from the conventional specific fuzzy optimization method and the fuzzy optimization with proportional fairness. The comparison uses the average value and standard deviation as primary indicators to measure the effectiveness and consistency of the optimal solutions. This analysis performed over 10 scenarios, where fuzzy parameters are randomly and uniformly varied within their pessimistic and optimistic limits. As a result, only the fuzzy objective function related to minimizing total supply chain costs is examined in these scenarios, as presented in Table 4.26.

As presented in Table 4.26, the average values obtained from the three fuzzy optimization approaches are closely comparable. Nevertheless, the unified fairness and robustness fuzzy optimization approach exhibits the lowest Coefficient of Variation (CV), indicating its superior capability in managing data variability. This underscores the approach's effectiveness in controlling input data fluctuations, thereby enhancing the overall robustness of the model.

Table 4.26 Result comparison of robustness level.

	Scenario				Average	Standard Deviation	Coefficient of Variation
Conventional Specific Fuzzy Optimization Approach	1	2	3	4	$121,912.52	$14,459.86	0.11861
	$99,043.20	$104,532.28	$109,297.84	$114,467.32			
	5	6	7	8			
	$119,132.88	$124,378.84	$129,867.92	$134,333.48			
	9	10					
	$139,802.96	$144,268.52					
	Scenario				Average	Standard Deviation	Coefficient of Variation
Fuzzy Optimization with Proportional Fairness Approach	1	2	3	4	$118,972.99	$14113.38	0.11863
	$96,865.60	$101,749.44	$106,813.12	$111,780.16			
	5	6	7	8			
	$116,043.84	$121,590.72	$126,474.56	$131,038.24			
	9	10					
	$136,205.28	$141,168.96					
	Scenario				Average	Standard Deviation	Coefficient of Variation
Unified Fairness and Robustness Fuzzy Optimization Approach	1	2	3	4	$111,594.16	$11,632.97	0.10424
	$93,688.00	$97,191.20	$101,677.60	$105,266.80			
	5	6	7	8			
	$109,453.20	$113,725.60	$117,428.80	$121,615.20			
	9	10					
	$125,904.40	$129,990.80					

4.2.3.2 Case 2's Sensitivity Analysis

The proposed approach integrates three critical parameters: the possibility degree of confidence level (γ), ρ, representing the penalty associated with potential violations of the objective function, and σ, denoting the penalty for possible violations of individual constraints. These parameters can be adjusted according to the decision maker's preferences and may affect the resulting plans and their comparative outcomes. Accordingly, a sensitivity analysis of these parameters will be conducted as described below.

- **Sensitivity Analysis of the Percentage of Credibility**

As previously noted, credibility denotes the degree of trustworthiness or reliability. A higher possibility degree of confidence signifies increased assurance that the fuzzy event will occur, thereby reducing the risk of violation.

According to Table 4.27, it can be concluded as follows:

- As the possibility degree of confidence level (γ) varies from 0% to 100%, the minimum total cost rises from $94,040 to $124,080, while the minimum total workforce fluctuation remains steady at 5 persons. Furthermore, the maximum total purchasing value increases from 697 units to 1,001 units.
- Regarding satisfaction percentages, the results indicate that as the possibility degree of confidence level (γ) increases, corresponding to a lower risk of violation, the satisfaction percentage for minimizing total supply chain costs decreases. The satisfaction percentage for minimizing total workforce fluctuations remains constant, while the satisfaction percentage for maximizing total purchasing values also declines. This occurs because, as satisfaction levels rise, the value of minimization objectives decreases, whereas the value of maximization objectives increases.

- Regarding the fairness percentages, the outcomes demonstrate that as the possibility degree of confidence level (γ) increases, indicating a lower risk of violation, the fairness percentage for minimizing total supply chain costs decreases, while the fairness percentage for minimizing total workforce fluctuations remains unchanged. Furthermore, the fairness percentage for maximizing total purchasing values also declines. This trend arises because a higher confidence level, corresponding to a reduced risk of violation, leads to a decrease in the fairness percentages.

Table 4.27 Results of sensitivity analysis of the percentage of credibility.

	$\gamma = 0$	$\gamma = 10$	$\gamma = 20$	$\gamma = 30$	$\gamma = 40$	$\gamma = 50$	$\gamma = 60$	$\gamma = 70$	$\gamma = 80$	$\gamma = 90$	$\gamma = 100$
Minimizing Total Supply Chain Cost ($)	94,040	97,010	100,100	103,190	106,280	109,560	112,450	115,540	**118,650**	121,710	124,080
Minimizing Fluctuation in Workforce Levels (persons)	5	5	5	5	5	5	5	5	**5**	5	5
Maximizing Total Values of Purchasing (units)	1,001	989	966	948	905	872	837	794	**768**	723	697
% Satisfaction of Minimizing Total Supply Chain Cost	62.761	61.653	60.839	59.619	58.922	57.713	56.814	55.547	**54.868**	53.234	52.146
% Satisfaction of Minimizing Fluctuation in Workforce Levels	35.143	35.143	35.143	35.143	35.143	35.143	35.143	35.143	**35.143**	35.143	35.143
% Satisfaction of Maximizing Total Values of Purchasing	50.544	49.832	48.456	47.981	46.683	45.167	44.859	43.742	**42.561**	41.754	40.826
% Fairness of Minimizing Total Supply Chain Cost	50.90	49.82	48.52	47.06	46.31	45.45	44.82	43.39	**42.18**	40.14	39.72
% Fairness of Minimizing Fluctuation in Workforce Levels	57.14	57.14	57.14	57.14	57.14	57.14	57.14	57.14	**57.14**	57.14	57.14
% Fairness of Maximizing Total Values of Purchasing	53.21	52.06	51.68	50.96%	49.58	48.81	47.27	46.90	**45.72**	44.50	43.27

The highlighted cell shows the results obtained by applying γ at 80%, as utilized in the case study.

- **Sensitivity Analysis on the Penalty Value of a Possible Violation of Objective Function and the Penalty Value of a Possible Violation of each Constraint**

As previously noted, ρ denotes the penalty value associated with potential violations of the objective function, while σ represents the penalty value for potential violations of individual constraints, with their sum constrained to equal 1. Tables 4.28 and 4.29 present a sensitivity analysis exploring variations in the proportions of ρ and σ, assessing the model's fairness and robustness.

As presented in Table 4.28, variations in the penalty values for potential violations of the objective function (ρ) and individual constraints (σ) do not affect the model's robustness. The average and Standard Deviation (SD) values across all models remain consistent with prior results, with the unified fairness and robustness approach maintaining the lowest Coefficient of Variation (CV). Consequently, the robustness of the model is affirmed.

Table 4.28 Result of sensitivity analysis on the penalty value of a possible violation of objective function and the penalty value of a possible violation of each constraint for testing model robustness.

	Scenario				Average	Standard Deviation	Coefficient of Variation
Conventional Specific Fuzzy Optimization Approach	ρ=0, σ=100	ρ=10, σ=90	ρ=20, σ=80	ρ=30, σ=70	$129,529.55	$12,398.51	0.09572
	$109,764	$113,968	$117,968	$121,836			
	ρ=40, σ=60	ρ=50, σ=50	ρ=60, σ=40	ρ=70, σ=30			
	$125,232	**$129,640**	$133,187	$137,480			
	ρ=80, σ=20	ρ=90, σ=10	ρ=100, σ=0				
	$141,560	$145,050	$149,140				
	Scenario				Average	Standard Deviation	Coefficient of Variation
Fuzzy Optimization with Proportional Fairness Approach	ρ=0, σ=100	ρ=10, σ=90	ρ=20, σ=80	ρ=30, σ=70	$127,522.64	$12,328.82	0.09668
	$107,788	$111,991	$115,878	$119,767			
	ρ=40, σ=60	ρ=50, σ=50	ρ=60, σ=40	ρ=70, σ=30			
	$123,953	**$127,803**	$131,029	$135,115			
	ρ=80, σ=20	ρ=90, σ=10	ρ=100, σ=0				
	$139,184	$143,191	$147,050				
	Scenario				Average	Standard Deviation	Coefficient of Variation
Unified Fairness and Robustness Fuzzy Optimization Approach	ρ=0, σ=100	ρ=10, σ=90	ρ=20, σ=80	ρ=30, σ=70	$118,540.09	$9,641.40	0.08133
	$103,296	$106,257	$109,534	$112,171			
	ρ=40, σ=60	ρ=50, σ=50	ρ=60, σ=40	ρ=70, σ=30			
	$115,432	**$118,650**	$121,843	$124,681			
	ρ=80, σ=20	ρ=90, σ=10	ρ=100, σ=0				
	$127,736	$130,914	$133,427				

The highlighted cell displays the results of applying ρ and σ at 50%, as implemented in the case study.

Table 4.29 Result of sensitivity analysis on the penalty value of a possible violation of objective function and the penalty value of a possible violation of each constraint.

	ρ=0, σ=100	ρ=10, σ=90	ρ=20, σ=80	ρ=30, σ=70	ρ=40, σ=60	ρ=50, σ=50	ρ=60, σ=40	ρ=70, σ=30	ρ=80, σ=20	ρ=90, σ=10	ρ=100, σ=0
Minimizing Total Supply Chain Cost	$103,296	$106,257	$109,534	$112,871	$115,432	**$118,650**	$121,543	$124,681	$127,436	$130,214	$133,427
Minimizing Fluctuation in Workforce Levels	5 persons	5 persons	5 persons	5 persons	5 persons	**5 persons**	5 persons	5 persons	5 persons	5 persons	5 persons
Maximizing Total Values of Purchasing	881 units	862 units	836 units	811 units	789 units	**768 units**	747 units	725 units	701 units	683 units	664 units
% Satisfaction of Minimizing Total Supply Chain Cost	59.87%	58.85%	57.46%	56.55%	55.67%	**54.87%**	53.46%	51.94%	50.51%	49.47%	48.36%
% Satisfaction of Minimizing Fluctuation in Workforce Levels	35.14%	35.14%	35.14%	35.14%	35.14%	**35.14%**	35.14%	35.14%	35.14%	35.14%	35.14%
% Satisfaction of Maximizing Total Values of Purchasing	45.64%	45.02%	44.85%	44.68%	43.48%	**42.56%**	42.05%	41.86%	41.23%	40.75%	40.43%
% Fairness of Minimizing Total Supply Chain Cost	47.54%	46.42%	45.36%	45.07%	43.63%	**42.18%**	40.72%	39.16%	38.22%	37.34%	36.19%
% Fairness of Minimizing Fluctuation in Workforce Levels	57.14%	57.14%	57.14%	57.14%	57.14%	**57.14%**	57.14%	57.14%	57.14%	57.14%	57.14%
% Fairness of Maximizing Total Values of Purchasing	49.18%	48.53%	47.25%	47.09%	46.78%	**45.72%**	44.84%	44.55%	43.31%	43.12%	42.85%

The highlighted cell displays the results of applying ρ and σ at 50%, as implemented in the case study.

According to Table 4.29, it can be concluded as follows:

- When the penalty value for potential violations of the objective function (ρ) is varied from 0% to 100%, or equivalently, the penalty value for potential violations of individual constraints (σ) is adjusted from 100% to 0%, the minimum total cost increases from $103,296 to $133,427. The minimum workforce fluctuations remain constant at 5 persons, while the maximum total purchasing value decreases from 881 units to 664 units.
- Regarding satisfaction percentages, the results indicate that an increase in the penalty value for potential violations of the objective function (ρ), or a corresponding decrease in the penalty value for violations of individual constraints (σ), leads to a decline in the satisfaction percentage for minimizing total supply chain costs. The satisfaction percentage for minimizing workforce fluctuations remains unchanged, while the satisfaction percentage for maximizing total purchasing values also decreases. This behavior arises because a higher satisfaction level corresponds to a lower value for minimization objectives and a higher value for maximization objectives.
- Regarding the fairness percentages, the results show that an increase in the penalty value for potential violations of the objective function (ρ), accompanied by a corresponding decrease in the penalty value for violations of individual constraints (σ), leads to a decline in the fairness percentage for minimizing total supply chain costs. The fairness percentage for minimizing workforce fluctuations remains unchanged, while the fairness percentage for maximizing total purchasing values also decreases. This effect arises because elevating the penalty associated with objective function violations (ρ) increases the optimality term, thereby narrowing the gap between Z_{max} (the maximum value of the objective function) and Z_{min} (the minimum value of the objective function), which enhances model robustness. However, this adjustment causes the obtained solution to deviate further from the positive ideal solution, resulting in less favorable objective values. Consequently, both satisfaction levels and fairness percentages decline.

4.2.4 Summary

This study provides managerial insights for decision-makers in supply chain aggregate production planning under uncertainty. One key takeaway is the advantage of incorporating multiple objectives into APP. Unlike single-objective approaches, a multi-objective strategy allows for greater flexibility and resilience in dynamic environments, helping decision-makers address challenges like supply network disruptions or demand shifts. This approach also fosters the creation of robust risk mitigation plans, ensuring long-term stability and sustainability in the SC.

This study further explores the integration of chance constraint programming into APP, which introduces a probabilistic element to conventional models. By considering the likelihood of different outcomes, this method enhances decision-making, improves resilience, and supports more effective risk management. Additionally, the incorporation of fairness into the APP framework helps maintain stable relationships among stakeholders, ensuring equitable treatment of all parties and fostering trust within the supply chain. This is crucial for mitigating risks and ensuring a more resilient and collaborative supply chain ecosystem.

The concept of robustness in APP is also central to the study's findings. A robust APP enables organizations to maintain stability and operational efficiency in the face of dynamic changes and disruptions. By proactively identifying and addressing potential risks, a robust system ensures that resources are allocated efficiently and that the organization remains adaptable to uncertainties. The study underscores the importance of integrating fairness and robustness into a fuzzy optimization approach, making the supply chain more resilient and better suited to handle the complexities of real-world challenges.

Furthermore, this study demonstrates the superiority of the proposed unified fairness and robustness fuzzy optimization approach compared to conventional methods. By concurrently optimizing multiple conflicting objectives, namely, minimizing supply chain costs, stabilizing workforce levels, and maximizing purchasing values under uncertainty, the proposed approach proves its effectiveness in practical applications. The incorporation of triangular fuzzy numbers to model imprecise data, combined with the introduction of a fairness term and Realistic Robust

Programming (RRP), substantially enhances both the fairness and robustness of the optimization process.

The optimal solutions obtained through this approach highlight its ability to resolve complex optimization challenges, particularly in scenarios with conflicting objectives. However, the study acknowledges certain limitations, including the absence of constraints on the degree of fuzziness and the opportunity for future research to investigate different distribution models for the fuzzy parameters. This study suggests that future work could refine the model by incorporating advanced meta-heuristic algorithms for even better optimization outcomes in more complex scenarios.

4.3 Case 3: A Downside Risk Mitigation Approach for Supply Chain Aggregate Production Planning

In today's dynamic and unpredictable business landscape, formulating effective strategies for Supply Chain Aggregate Production Planning (SCAPP) presents considerable challenges for decision-makers. Conventional fuzzy optimization methods often prove inadequate in handling the uncertainties and risks that are intrinsic to supply chain operations, resulting in less-than-optimal outcomes and increased operational expenses. These shortcomings become especially apparent when coordinating activities across various levels of the supply chain, where disruptions, fluctuating demand, and unexpected events can significantly raise costs and impair efficiency. As a result, there is a growing demand for more resilient and comprehensive approaches capable of managing these uncertainties while enhancing supply chain performance. This research proposes an innovative business model that integrates open innovation principles to improve both cost efficiency and resilience within the supply chain. To address uncertainty-related risks, particularly those associated with adverse outcomes, the Mean-Conditional Value at Risk Gap (MCVaRG) is employed. Additionally, the model leverages asymmetrical triangular fuzzy numbers to reflect the inherent ambiguity and variability in critical supply chain elements such as costs, customer requirements, and machine operating times.

4.3.1 Mathematical Notations and Model

The notations for indexes, parameters, and decision variables are detailed in Tables 4.30 to 4.34. Notably, all fuzzy parameters are distinguished by a tilde ($\tilde{}$) placed above their respective symbols to signify their fuzzy characteristics.

Table 4.30 Indexes of SCAPP problem (Case 3).

Indexes	Meaning
s	Suppliers' array ($s = 1, \dots, S$)
r	Retailers' array ($r = 1, \dots, R$)
d	Planning periods' array ($d = 1, \dots, D$)

Table 4.31 Crisp parameters of SCAPP problem (Case 3).

Crisp Parameters	Meaning
LH_d	Labor time allocated per product unit at the plant for period *d* (person-hours/unit)
$MaxPCRT_d$	Regular-time production limit of the plant in period *d* (units)
$MaxPCOT_d$	Overtime production limit of the plant in period *d* (units)
$MaxSupCap_{sd}$	Maximum amount of raw materials available from supplier *s* in period *d* (units)
RMQ_{sd}	Raw material units supplied by supplier *s* in period *d* (units)
RTQ_d	Total units produced within the regular time in period *d* (units)
OTQ_d	Total units produced within the overtime in period *d* (units)
$ShortPPQ_d$	Product deficit at the plant in period *d* (units)
$IRMQ_d$	Inventory level of raw materials at the plant in period *d* (units)
IPQ_d	Inventory level of products at the plant in period *d* (units)
HL_d	Total workforce employed in period *d* (persons)
FL_d	Total workforce fired in period *d* (persons)
L_d	Total of workforce in period *d* (persons)
$TranQ_{rd}$	Product shipment volume directed to retailer *r* in period *d* (units)
IRQ_{rd}	Inventory of products held by retailer *r* in period *d* (units)
$ShortPRQ_{rd}$	Product deficit experienced by retailer *r* in period *d* (units)

Table 4.32 Uncertain parameters of SCAPP problem (Case 3).

Uncertain Parameters	Meaning
$\widetilde{RMC}_{sd}$	Uncertain cost of raw material delivered by supplier *s* in period *d* ($/unit)
$\widetilde{RC}_d$	Uncertain cost associated with regular-time production per product unit in period *d* ($/unit)
$\widetilde{OC}_d$	Uncertain cost associated with overtime production per product unit in period *d* ($/unit)
$\widetilde{ShortPC}_d$	Uncertain cost per unit related to shortages at the plant in period *d* ($/unit)
$\widetilde{InCRM}_d$	Uncertain cost associated with storing one unit of raw materials at the plant in period *d* ($/unit)
$\widetilde{InCPM}_d$	Uncertain cost associated with holding one unit of product inventory at the plant in period *d* ($/unit)
$\widetilde{HC}_d$	Uncertain cost associated with labor hiring in period *d* ($/person)
$\widetilde{FC}_d$	Uncertain cost associated with labor firing in period *d* ($/person)
$\widetilde{TranC}_{rd}$	Uncertain transportation cost per unit of product delivered to retailer *r* during period *d* ($/unit)
$\widetilde{InCPR}_{rd}$	Uncertain cost associated with storing one unit of product inventory at retailer *r* during period *d* ($/unit)
$\widetilde{PCLS}_{rd}$	Uncertain cost penalty per unit due to lost sales at retailer *r* during period *d* ($/unit)
$\widetilde{De}_{rd}$	Uncertain demand quantity of products at retailer *r* during period *d* (units)

Table 4.33 Decision variables of SCAPP problem (Case 3).

Decision Variables	Meaning
RMQ_{sd}	Total raw materials furnished by supplier *s* in period *d* (units)
RTQ_d	Production volume during regular working hours in period *d* (units)
OTQ_d	Production volume during overtime working hours in period *d* (units)
$ShortPPQ_d$	Amount of unmet product demand at the plant in period *d* (units)
$IRMQ_d$	Inventory level of raw materials stored at the plant in period *d* (units)
IPQ_d	Inventory level of products stored at the plant in period *d* (units)
HL_d	Amount of hired labors in period *d* (persons)
FL_d	Amount of fired labors fired in period *d* (persons)
L_d	Amount of overall labors in period *d* (persons)
$TranQ_{rd}$	Volume of products transported to retailer *r* in period *d* (units)
IRQ_{rd}	Amount of products kept in inventory at retailer *r* in period *d* (units)
$ShortPRQ_{rd}$	Shortfall in product availability at retailer *r* in period *d* (units)

Table 4.34 Related notations of SCAPP problem.

Notations	Meaning
γ	Credibility level
$\widetilde{TSCNC}$	Total supply chain network costs
$\widetilde{ProCC}$	Total procurement costs
$\widetilde{ProDC}$	Total production costs
$\widetilde{DisTC}$	Total distribution costs

<u>Objective Functions</u>

1. Minimizing total supply chain operation costs is widely considered a fundamental objective when developing an efficient supply chain system. The total supply chain network costs ($\widetilde{TSCNC}$) are typically subject to uncertainty and consist of the combined procurement costs ($\widetilde{ProCC}$), production costs ($\widetilde{ProDC}$), and distribution costs ($\widetilde{DisTC}$) over a specified period. Procurement costs include the expenses associated with purchasing raw materials, while production costs account for regular production costs, overtime production costs, product shortage costs, raw material and product inventory holding costs, and costs related to labor hiring and firing. Distribution costs encompass expenditures at the retail level, such as transportation costs, inventory holding costs, and penalty costs incurred from lost sales at retail locations.

$$\begin{aligned} &Minimize\ (\widetilde{TSCNC}) = \widetilde{ProCC} + \widetilde{ProDC} + \widetilde{DisTC} \\ &= \textstyle\sum_s^S \sum_d^D \widetilde{RMC}_{sd} \times RMQ_{sd} + \left(\sum_d^D \widetilde{RC}_d \times RTQ_d\right) + \left(\sum_d^D \widetilde{OC}_d \times OTQ_d\right) \\ &+ \left(\textstyle\sum_d^D \widetilde{ShortPC}_d \times ShortPPQ_d\right) + \left(\sum_d^D \widetilde{InCRM}_d \times \frac{IRMQ_{d-1}+IRMQ_d}{2}\right) \\ &+ \left(\textstyle\sum_d^D \widetilde{InCPM}_d \times \frac{IPQ_{d-1}+IPQ_d}{2}\right) + \left(\sum_d^D \widetilde{HC}_d \times HL_d\right) + \left(\sum_d^D \widetilde{FC}_d \times FL_d\right) \\ &+\left(\textstyle\sum_r^R \sum_d^D \widetilde{TranC}_{rd} \times TranQ_{rd}\right) + \left(\sum_r^R \sum_d^D \widetilde{InCPR}_{rd} \times \frac{IRQ_{rd-1}+IRQ_{rd}}{2}\right) \\ &+ \left(\textstyle\sum_r^R \sum_d^D \widetilde{PCLS}_{rd} \times ShortPRQ_{rd}\right) \end{aligned} \tag{4.47}$$

2. Minimizing the Mean-Conditional Value at Risk Gap (MCVaRG) of total supply chain operation costs plays an essential role in building a resilient supply chain. It supports decision-makers in mitigating uncertainties in costs, particularly focusing on reducing the risk of adverse outcomes. Downside risk represents the likelihood of incurring costs that exceed expected levels, ensuring that the supply chain remains cost-efficient even under adverse conditions.

$$Minimize\ MCVaRG = (MCVaR(\widetilde{ProCC}) + MCVaR(\widetilde{ProDC}) + MCVaR(\widetilde{DisTC})) - \widetilde{TSCNC}$$
$$= \sum_s^S \sum_d^D [(1-\gamma)RMC_{sd}^m + (\gamma)RMC_{sd}^p] \times RMQ_{sd} + \left(\sum_d^D [(1-\gamma)RC_d^m + (\gamma)RC_d^p] \times RTQ_d\right)$$
$$+ \left(\sum_d^D [(1-\gamma)OC_d^m + (\gamma)OC_d^p] \times OTQ_d\right) + \left(\sum_d^D [(1-\gamma)ShortPC_d^m + (\gamma)ShortPC_d^p] \times ShortPPQ_d\right)$$
$$+ \left(\sum_d^D [(1-\gamma)InCRM_d^m + (\gamma)InCRM_d^p] \times \frac{IRMQ_{d-1}+IRMQ_d}{2}\right) + \left(\sum_d^D [(1-\gamma)HC_d^m + (\gamma)HC_d^p] \times HL_d\right)$$
$$+ \left(\sum_d^D [(1-\gamma)InCPM_d^m + (\gamma)InCPM_d^p] \times \frac{IPQ_{d-1}+IPQ_d}{2}\right) + \left(\sum_d^D [(1-\gamma)FC_d^m + (\gamma)FC_d^p] \times FL_d\right)$$
$$+ \left(\sum_r^R \sum_d^D [(1-\gamma)TranC_{rd}^m + (\gamma)TranC_{rd}^p] \times TranQ_{rd}\right)$$
$$+ \left(\sum_r^R \sum_d^D [(1-\gamma)InCPR_{rd}^m + (\gamma)InCPR_{rd}^p] \times \frac{IRQ_{rd-1}+IRQ_{rd}}{2}\right)$$
$$+ \left(\sum_r^R \sum_d^D [(1-\gamma)PCLS_{rd}^m + (\gamma)PCLS_{rd}^p] \times ShortPRQ_{rd}\right) \quad (4.48)$$

The parameter γ denotes the credibility level, reflecting the extent of trustworthiness. For this study, γ is assigned a value of 80%.

<u>Constraints</u>

1. Suppliers' Capacity for Providing Raw Materials: The maximum quantity of raw materials that suppliers can deliver within a specific period, reflecting their production and logistical capabilities.

$$RMQ_{sd} \leq MaxSupCap_{sd} \qquad \forall\, s, d \quad (4.49)$$

2. Raw Material Availability: The extent to which required raw materials are accessible from suppliers, considering factors like supply chain disruptions, inventory levels, and lead times for procurement.

$$\sum_{s=1}^{S} RMQ_{sd} \geq (RTQ_d + OTQ_d) \qquad \forall\, d \quad (4.50)$$

3. Product Shortages at the Plant: It occurs when the production facility lacks sufficient raw materials or components to meet the planned production targets, potentially causing delays, increased costs, or missed delivery deadlines.

$$ShortPPQ_d = IPQ_{d-1} - ShortPPQ_{d-1} + RTQ_d + OTQ_d - IPQ_d - \widetilde{De}_{rd} \quad \forall\, r, d \quad (4.51)$$

4. Labor Capacity: It refers to the availability and ability of the workforce to meet production demands. It includes the number of workers, their skills, working hours, and productivity levels, ensuring that the production plant can operate efficiently without shortages or excessive overtime.

$$LH_d \times RTQ_d \leq L_d * 9{,}600 \qquad \forall\, d \tag{4.52}$$

5. Workforce Balancing: It refers to the strategic allocation of labor resources to match production needs and workloads. It involves adjusting staffing levels across different shifts or production stages to ensure efficient operations, minimize downtime, and avoid overworking employees while maintaining optimal productivity.

$$L_d = L_{(d-1)} + HL_d - FL_d \qquad \forall\, d \tag{4.53}$$

6. Limitation of Regular Time Production: This indicates the greatest production volume that can be reached within regular shift times, without requiring overtime. This limitation is typically determined by factors such as available labor, equipment capacity, and operational hours, and plays a key role in managing production schedules and costs.

$$RTQ_d \leq 28{,}000 \qquad \forall\, d \tag{4.54}$$

7. Limitation of Overtime Production: Indicates the greatest additional production capacity available by utilizing labor during overtime periods. This limitation is often constrained by factors such as labor laws, employee availability, and increased labor costs, and must be carefully managed to optimize production while controlling costs.

$$OTQ_d \leq 7{,}000 \qquad \forall\, d \tag{4.55}$$

8. Raw Material Inventory: Refers to the stock of raw materials held by a company for production purposes. It ensures that the production process can continue smoothly without interruptions due to shortages. The inventory level is managed to balance the cost of holding materials with the need to meet production demands, while also accounting for factors like lead time, demand fluctuations, and storage costs.

$$IRMQ_d = IRMQ_{(d-1)} + \sum_{s=1}^{S} RMQ_{sd} - (RTQ_d + OTQ_d) \qquad \forall\, d \tag{4.56}$$

9. Limitation of Transferring Products to Retailers: This refers to constraints in the ability to deliver products from the production facility to retail locations. These limitations can include factors like transportation capacity, logistical challenges, delivery schedules, or regulatory restrictions. Efficient management of these constraints ensures that retailers receive products on time, preventing stockouts or delays that could negatively impact sales and customer satisfaction.

$$\sum_{r}^{R} TranQ_{rd} \leq RTQ_d + OTQ_d \qquad \forall\, d \tag{4.57}$$

10. Minimum Retailer Service Level for Satisfying Demand: This refers to the minimum level of product availability that retailers must maintain to meet customer demand. It ensures that retailers have enough stock to avoid stockouts, aiming to satisfy customers' needs consistently. Meeting this service level is vital for maintaining customer satisfaction, loyalty, and competitive advantage in the market.

$$TranQ_{rd} \geq 0.8 \times \widetilde{D}e_{rd} \qquad \forall\, r, d \tag{4.58}$$

11. Product Shortages at Retailers: This refers to the situation where a retailer does not have enough stock of a product to meet customer demand. This usually causes sales losses, decreases customer satisfaction, and may negatively impact the retailer’s image. Proper management of product shortages is vital to maintaining supply chain efficiency and fulfilling customer requirements on schedule.

$$ShortPQR_{rd} = IRQ_{rd-1} - ShortPQR_{rd-1} + TranQ_{rd} - IRQ_{rd} - \widetilde{D}e_{rd} \qquad \forall\, r, d \tag{4.59}$$

12. Non-Negativity: Constraints (4.60) – (4.63) ensure that all decision variable values are non-negative, with certain values required to be integers.

$$L_d, HL_d, FL_d, IRMQ_d \geq 0 \text{ and Integer} \qquad \forall\, d \tag{4.60}$$

$$RTQ_d,\ OTQ_d, ShortPPQ_d, IRMQ_d,\ IPQ_d \geq 0 \text{ and Integer} \qquad \forall\, d \tag{4.61}$$

$$RMQ_{sd} \geq 0 \text{ and Integer} \qquad \forall\, s, d \tag{4.62}$$

$$TranQ_{rd},\ IRQ_{rd},\ ShortPRQ_{rd} \geq 0 \text{ and Integer} \qquad \forall\, r, d \tag{4.63}$$

4.3.2 Problem Description of Case 3

This study features a case study of a small Thai manufacturer specializing in plastic bottles to demonstrate and evaluate the proposed fuzzy multi-criteria decision-making model. The production process begins with melting Polyethylene Terephthalate (PET) resin, which is then shaped into a tubular form, known as a parison, by extrusion through a circular die. This parison is inserted into a mold cavity within a blow molding machine and expanded with high-pressure air to conform to the mold's shape. Afterward, the product is cooled to solidify, and excess material is removed to ensure a clean finish. Final steps include quality checks, labeling, and packaging to maintain consistency in producing standardized plastic bottles for distribution. In the SC, four certified suppliers offer PET resin at varying prices, depending on resin quality and pricing flexibility. The production plant is limited by its manufacturing capacity, while six retailers, located in different regions, generate diverse demand levels, as depicted in Figure 4.9. The SCAPP planning period for this case extends over six months.

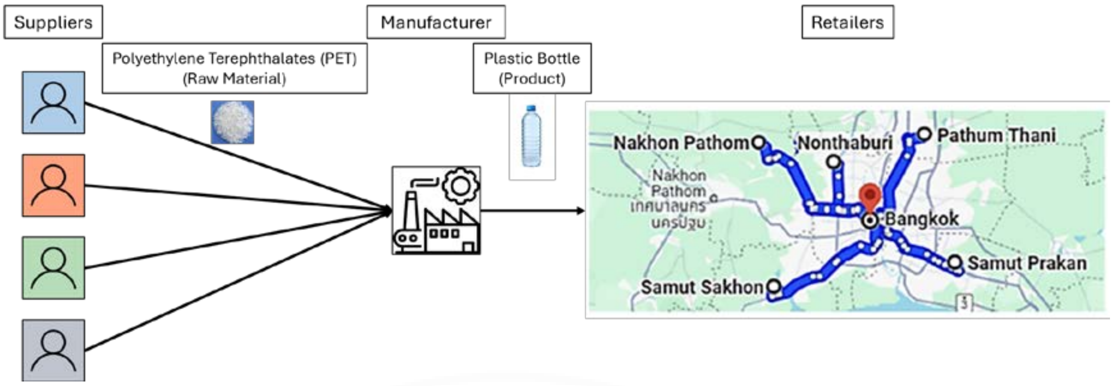

Figure 4.9 Supply network design along with retailer site locations.

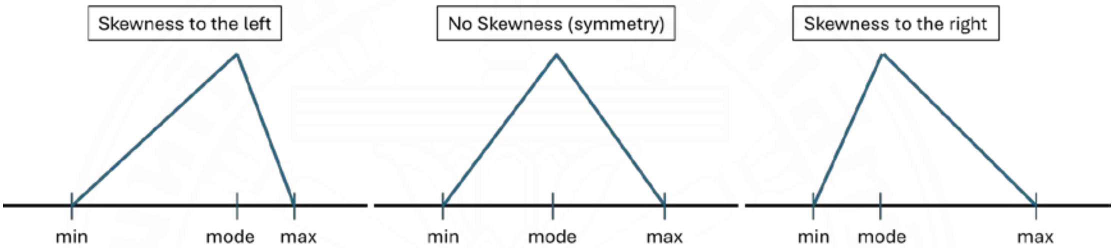

Figure 4.10 Skewed configurations within triangular fuzzy sets.

Figure 4.10 demonstrates the asymmetrical skewness of risks, highlighting three distinct forms of skewness in triangular fuzzy numbers: left-skewed, symmetric, and right-skewed. In terms of cost and risk, left-skewness indicates a higher likelihood of achieving lower costs and reduced risk of uncertainty. Symmetry suggests an equal chance of either obtaining lower costs with lower risk or higher costs with higher risk. Right-skewness, conversely, indicates a higher probability of encountering higher costs and greater risk of uncertainty.

Table 4.35 Raw material pricing represented under fuzzy conditions.

$\widetilde{RMC}_{sd}$	Triangular Fuzzy Number			Skewness Type
	Optimistic	Most Likely	Pessimistic	
Supplier 1	5.23	9.50	13.78	Symmetry (±45%)
Supplier 2	6.50	10.00	13.50	Symmetry (±35%)
Supplier 3	8.44	11.25	14.06	Symmetry (±25%)
Supplier 4	10.84	12.75	14.66	Symmetry (±15%)

Four qualified suppliers offer PET resin at varying uncertain prices, which are modeled using symmetrical Triangular Fuzzy Numbers (TFNs) as shown in Table 4.35. The raw material prices depend on both the quality of the resin and the supplier's ability to maintain price stability. For instance, Supplier 1 provides the lowest quality resin with limited reliability, resulting in the lowest price that can vary by ±45% around its most probable value. Conversely, Supplier 4 supplies the highest quality resin with strong reliability, commanding the highest price with fluctuations confined to ±15% of its most likely price. All suppliers have an equal maximum supply capacity ($MaxSupCap_{st}$) of 25,000 units per period.

Table 4.36 Uncertain parameters associated with the manufacturing facility.

Most Likely	$\widetilde{RC}_d$	$\widetilde{OC}_d$	$\widetilde{ShortPC}_d$	$\widetilde{InCRM}_d$
	\$12.50/ unit	\$18.75/ unit	\$37.50/ unit	\$0.10/ unit
	$\widetilde{InCPM}_d$	$\widetilde{HC}_d$	$\widetilde{FC}_d$	
	\$0.30/ unit	\$160/ man	\$280/ man	

The production plant's fuzzy parameters are represented using symmetrical Triangular Fuzzy Numbers (TFNs) with a variability of ±20% around their most probable values, as detailed in Table 4.36. The plant must fulfill a minimum of 80% of the demand for each retailer, which could lead to some lost sales at certain locations, incurring penalty costs accordingly. This study assumes delivery lead times to be negligible and does not account for any subcontracting. Additionally, Table 4.37 lists other deterministic parameters applied in this case study.

Table 4.37 Crisp parameters.

Values	LH_d	$MaxPCRT_d$	$MaxPCOT_d$
	0.016 man-hours/unit	28,000 units	7,000 units

Table 4.38 Imprecise six-month retailers' demand.

Period (d)	$\widetilde{De_{rd}}$ (units)					
	Retailer 1	Retailer 2	Retailer 3	Retailer 4	Retailer 5	Retailer 6
d = 1	5,796	4,849	3,967	7,904	9,716	7,809
d = 2	4,971	3,996	2,505	7,016	8,902	6,520
d = 3	4,200	2,646	1,510	6,222	7,462	5,037
d = 4	3,711	2,546	1,293	6,370	7,251	5,528
d = 5	5,004	4,418	2,728	7,386	8,305	6,453
d = 6	6,287	5,362	3,779	8,865	9,396	7,328

The demand for plastic bottles experiences seasonal variations, driven by changes in water consumption throughout the year. The supply chain comprises six retailers, each located in different areas and exhibiting unique seasonal demand patterns. These demands are modeled using symmetrical Triangular Fuzzy Numbers (TFNs) with a ±20% variability around their most probable values, as shown in Table 4.38. Both the production plant and retailers are situated relatively close to each other in central Thailand. The production facility is located in Bangkok, while the retailers operate across Bangkok, Nonthaburi, Pathum Thani, Nakhon Pathom, Samut Sakhon, and Samut Prakan. This geographical distribution leads to differences in transportation, inventory holding, and penalty costs associated with lost sales, which are detailed in Figure 4.11 and Table 4.39.

Table 4.39 Costs related to retailers.

	$\widetilde{TranC}_{rd}$			Skewness Type	Risk of Uncertainty
	Optimistic	Most Likely	Pessimistic		
Retailer 1	11.70	13.00	14.30	Symmetry	Low
Retailer 2	6.00	10.00	11.00	Left skew	Low
Retailer 3	3.45	11.50	16.10	Left skew	Medium
Retailer 4	14.40	16.00	22.40	Right skew	Medium
Retailer 5	10.50	17.50	29.75	Right skew	High
Retailer 6	4.35	14.50	24.65	Symmetry	High
	$\widetilde{InCPR}_{rd}$			Skewness Type	Risk of Uncertainty
	Optimistic	Most Likely	Pessimistic		
Retailer 1	6.30	7.00	7.70	Symmetry	Low
Retailer 2	2.40	4.00	4.40	Left skew	Low
Retailer 3	1.65	5.50	7.70	Left skew	Medium
Retailer 4	9.00	10.00	14.00	Right skew	Medium
Retailer 5	6.90	11.50	19.55	Right skew	High
Retailer 6	2.55	8.50	14.45	Symmetry	High
	$\widetilde{PCLS}_{rd}$			Skewness Type	Risk of Uncertainty
	Optimistic	Most Likely	Pessimistic		
Retailer 1	6.90	23.00	36.80	Symmetry	High
Retailer 2	6.00	20.00	28.00	Left skew	Medium
Retailer 3	12.90	21.50	23.65	Left skew	Low
Retailer 4	15.60	26.00	44.20	Right skew	High
Retailer 5	24.75	27.50	38.50	Right skew	Medium
Retailer 6	22.05	24.50	26.95	Symmetry	Low

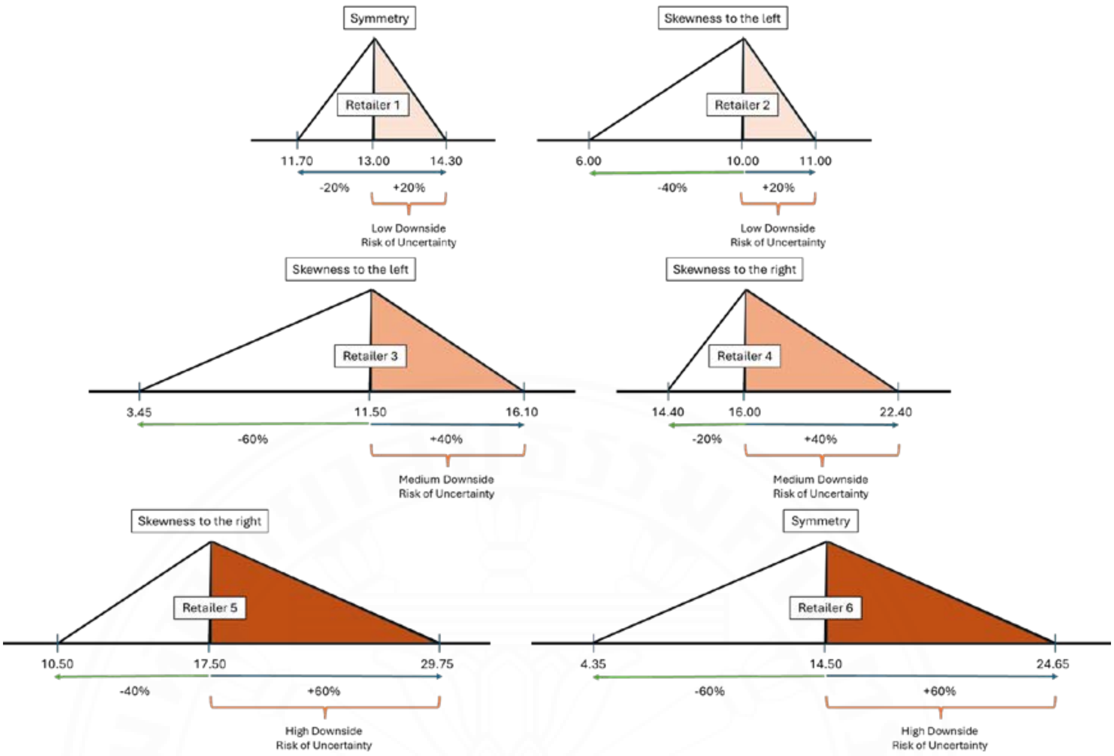

Figure 4.11 Different skewness patterns and uncertainty risks in transportation expenses.

Figure 4.11 illustrates the differences in transportation cost patterns and their corresponding downside risks across various retailers. Retailers 1 and 6 display symmetrical cost distributions; however, Retailer 6 encounters a significantly higher downside risk of uncertainty (+60% from the most likely value) compared to Retailer 1, which has a relatively modest downside risk of +20%. In contrast, Retailers 2 and 3 exhibit left-skewed distributions. Retailer 3 faces a moderate downside risk (+40%), whereas Retailer 2 is subjected to a lower level of risk (+20%). Retailers 4 and 5 both present right-skewed cost distributions, with Retailer 5 experiencing a higher downside risk than Retailer 4. These differences in transportation cost behavior and related risks highlight the varying logistical and financial complexities the supply chain must navigate in meeting retailer demand. By addressing these disparities, the study offers valuable guidance on optimizing transportation and inventory decisions while taking into account uncertainty and potential risk exposure.

During operations, several factors can significantly increase the likelihood of higher transportation costs for certain retailers. In this study, these factors include scenarios such as unusual traffic congestion in specific areas. High population density, especially in business districts during peak hours, can exacerbate traffic conditions. Additionally, events like protests, roadblocks near government offices, or mass gatherings (mobile vulgus) can severely disrupt traffic flow. Such situations not only lead to prolonged delays but can also heighten driver frustration, increasing the risk of road rage incidents and accidents, further compounding transportation delays. These circumstances collectively contribute to a higher probability of elevated transportation costs. A similar framework is applied to holding costs and penalty costs for lost sales, which can also escalate under adverse operational conditions. The interconnected nature of these factors highlights the importance of accounting for such risks in supply chain planning. The detailed impacts of these variables on transportation, holding, and penalty costs are summarized in Table 4.39.

The SCAPP model is developed based on the following assumptions:

- A predefined set of qualified suppliers is available, as outlined in Table 4.35.
- Retailer demand varies dynamically over the six-month planning horizon.
- Demand at each retailer may be completely fulfilled or partially unmet; any shortages result in penalty costs.
- All cost components within the supply chain are subject to uncertainty and display different forms of skewness, which are assumed to remain stable throughout the planning period.
- The use of subcontractors is excluded from this scenario.
- Delivery lead times are considered negligible.
- Initial inventory levels and available labor resources are known at the beginning of the planning period.

4.3.3 Results of Case 3

This analysis examines three separate strategies: focusing solely on cost minimization, solely on minimizing downside risk through the Mean-Conditional Value at Risk Gap (MCVaRG), and a combined approach that targets both objectives simultaneously. The resulting outcomes are carefully analyzed and compared to assess their performance, advantages, and trade-offs. Through this comparative evaluation, this study aims to offer meaningful insights into the effectiveness of each strategy, emphasizing the benefits of integrated optimization methods that align cost-efficiency with risk mitigation to support stronger and more resilient decision-making.

- **Result of Purely Minimizing the Total Supply Chain Operational Costs**

The Supply Chain Aggregate Production Planning (SCAPP) problem is addressed using Fuzzy Linear Programming (FLP), with an emphasis on minimizing total operational costs across the entire supply chain. The optimal outcomes, presented in Table 4.40, highlight key decision variables and cost-saving strategies. Polyethylene Terephthalate (PET) is primarily procured from Suppliers 1 and 2 due to their lower material costs. Production of plastic bottles initially takes place during regular working hours; however, after reaching 28,000 units, overtime is employed to produce an additional 7,000 units. Any demand beyond this capacity results in shortages, particularly evident during peak seasons, 3,025 units in period 1 and 4,001 units in period 6. Workforce levels are adjusted between 40 and 55 employees over the six-month planning horizon to align with production requirements. Retailer service levels are managed with a primary focus on minimizing costs. Retailer 5 achieves full demand fulfillment, while Retailers 4, 6, 1, 2, and 3 attain service levels of 99.00%, 96.00%, 93.39%, 91.71%, and 89.59%, respectively. This approach strictly targets cost efficiency without accounting for the downside risk linked to cost uncertainty. Although total operational expenses are reduced to $7,712,875, the corresponding downside risk escalates to $2,147,100, potentially surpassing acceptable risk thresholds. This outcome highlights the critical need to incorporate risk considerations alongside cost objectives in supply chain planning.

Table 4.40 Result obtained exclusively aiming to minimize total operational costs in the supply chain.

	Value						
Minimum Total Supply Chain Operation Costs	$7,712,875.00						
Downside Risk of Total Supply Chain Operation Costs	$2,147,100.00						
	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	
Production volume during regular working hours in period *d*	28,000 units	28,000 units	20,364 units	25,455 units	28,000 units	28,000 units	
Production volume during overtime working hours in period *d*	7,000 units	4,566 units	0 unit	0 unit	4,950 units	7,000 units	
Amount of unmet product demand at the plant in period *d* (units)	3,025 units	0 unit	0 unit	0 unit	0 unit	4,001 unit	
Amount of overall labors in period *d*	55 persons	55 persons	40 persons	50 persons	55 persons	55 persons	
	Total raw materials furnished by supplier s in period *d* (units)						
	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	
Supplier 1	**25,000**	**25,000**	**20,364**	**25,000**	**25,000**	**25,000**	
Supplier 2	**10,000**	**7,566**	**0**	**455**	**7,950**	**10,000**	
Supplier 3	0	0	0	0	0	0	
Supplier 4	0	0	0	0	0	0	
	Volume of products transported to retailer *r* in period *d* (units)						Average Service Level (%)
	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	
Retailer 1	4,685	4,747	3,080	3,935	4,780	4,879	93.39%
Retailer 2	3,711	3,772	1,526	2,770	4,194	4,021	91.71%
Retailer 3	2,805	2,281	838	1,841	2,504	2,755	89.59%
Retailer 4	**7,168**	**6,792**	**4,878**	**5,250**	**7,162**	**8,529**	**99.00%**
Retailer 5	**9,380**	**8,678**	**6,125**	**6,131**	**8,081**	**9,060**	**100.00%**
Retailer 6	**7,251**	**6,296**	**3,917**	**5,528**	**6,229**	**5,756**	**96.00%**

The highlighted cells indicate the suppliers selected to provide Polyethylene Terephthalate (PET) and the retailers who achieved the highest service level satisfaction percentages.

- **Result of Purely Minimizing Mean-Conditional Value at Risk Gap (MCVaRG) of Total Supply Chain Operation Costs**

Fuzzy Linear Programming (FLP) is utilized to tackle the SCAPP problem, focusing primarily on minimizing the Mean-Conditional Value at Risk Gap (MCVaRG) related to the total operational costs of the supply chain. The optimal results, shown in Table 4.41, emphasize values of decision variable aimed at lowering downside risk. Suppliers 3 and 4 are chosen to provide PET to the production plant because of their reduced downside risk, indicating a lower likelihood of cost increases. Despite this change in suppliers, production volumes, shortages, and labor levels remain similar to those in the cost-minimization approach, ensuring that retailer demand is still met. In this scenario, Retailer 1 attains a full-service level of 100%, followed by Retailer 4 at 99.00%, Retailer 2 at 97.61%, Retailer 5 at 94.80%, Retailer 3 at 94.19%, and Retailer 6 at 92.34% across the six-month planning horizon. This strategy prioritizes minimizing downside risk from cost uncertainties, with supplier and retailer choices guided by risk reduction rather than solely cost considerations. Consequently, the downside risk decreases substantially to \$1,731,676.04, though the total operational costs rise to \$8,429,300, which may be above the acceptable limits for decision-makers. This outcome highlights the essential trade-off between managing costs and mitigating risk in supply chain planning.

Table 4.41 Result obtained exclusively aiming to minimize Mean-Conditional Value at Risk Gap (MCVaRG) of total operation costs in the supply chain.

	Value						
Minimum Downside Risk of total supply chain operation costs	$1,731,676.00						
Total Supply Chain Operation Costs	$8,429,300.00						
	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	
Production volume during regular working hours in period *d*	28,000 units	28,000 units	20,364 units	25,455 units	28,000 units	28,000 units	
Production volume during overtime working hours in period *d*	7,000 units	4,566 units	0 unit	0 unit	4,950 units	7,000 units	
Amount of unmet product demand at the plant in period *d* (units)	3,025 units	0 unit	0 unit	0 unit	0 unit	4,001 units	
Amount of overall labors in period *d*	55 persons	55 persons	40 persons	50 persons	55 persons	55 persons	
	Total raw materials furnished by supplier s in period *d* (units)						
	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	
Supplier 1	0	0	0	0	0	0	
Supplier 2	0	0	0	0	0	0	
Supplier 3	**10,000**	**7,566**	**0**	**455**	**7,950**	**10,000**	
Supplier 4	**25,000**	**25,000**	**25,000**	**25,000**	**25,000**	**25,000**	
	Volume of products transported to retailer *r* in period *d* (units)						Average Service Level (%)
	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	
Retailer 1	**5,460**	**4,747**	**3,080**	**3,935**	**4,780**	**5,951**	**100.00%**
Retailer 2	**4,113**	**3,772**	**1,526**	**2,770**	**4,194**	**4,905**	**97.61%**
Retailer 3	3,475	2,281	838	1,841	2,504	2,755	94.19%
Retailer 4	**7,168**	**6,792**	**4,878**	**5,250**	**7,162**	**8,529**	**99.00%**
Retailer 5	8,704	8,678	6,125	6,131	8,081	7,266	94.80%
Retailer 6	6,080	6,296	3,917	5,528	6,229	5,594	92.34%

The highlighted cells indicate the suppliers selected to provide Polyethylene Terephthalate (PET) and the retailers who achieved the highest service level satisfaction percentages.

- **Result of Considering Both Cost and Downside Risk Minimization**

Fuzzy Linear Programming (FLP) is utilized to address the SCAPP problem with the dual goal of simultaneously minimizing total costs and downside risk (MCVaRG). The optimal results, detailed in Table 4.42, illustrate the effectiveness of a multi-objective fuzzy linear programming approach that achieves an overall satisfaction level of 94.93% by maximizing the minimum satisfaction level. The lowest total supply chain operational cost is $7,832,100 with a satisfaction level of 98.37%, while the minimum downside risk stands at $1,921,500 with a satisfaction level of 94.93%. Additionally, Table 4.42 presents the decision variable values corresponding to the joint minimization of costs and associated downside risks. In this scenario, Suppliers 2 and 3 are chosen to supply PET to the production plant due to their balanced profiles in terms of cost and moderate downside risk. The production plan, including both regular and overtime hours, remains consistent, aligning with retailer demand. Retailer priorities in terms of service levels over the six-month planning period place Retailer 4 at 100%, followed by Retailers 2, 6, 1, 5, and 3, with service levels of 98.29%, 96.67%, 95.33%, 93.71%, and 92.30% respectively. This method represents a trade-off approach that balances cost reduction with risk mitigation, selecting suppliers and retailers that provide the most favorable combination of these objectives. Among the retailers, 1, 3, and 5 are given lower priority for order fulfillment: Retailer 1 is chosen last due to its low cost but elevated risk; Retailer 3 is deprioritized for both higher costs and risks; and Retailer 5 is ranked last because of its low risk but comparatively higher costs.

Table 4.42 Result obtained by integrating both cost and downside risk minimization.

	Value						
Overall Satisfaction Level	94.93%						
Satisfaction Level of Minimizing Total Supply Chain Operation Costs	98.37%						
Satisfaction Level of Minimizing Downside Risk of Total Supply Chain Operation Costs	94.93%						
Minimum Total Supply Chain Operation Costs	$7,832,100.00						
Minimum Downside Risk of Total Supply Chain Operation Costs	$1,921,500.00						
	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	
Production volume during regular working hours in period *d*	28,000 units	28,000 units	20,364 units	25,455 units	28,000 units	28,000 units	
Production volume during overtime working hours in period *d*	7,000 units	4,566 units	0 unit	0 unit	4,950 units	7,000 units	
Amount of unmet product demand at the plant in period *d* (units)	3,025 units	0 unit	0 unit	0 unit	0 unit	4,001 units	
Amount of overall labors in period *d*	55 persons	55 persons	40 persons	50 persons	55 persons	55 persons	
	Total raw materials furnished by supplier s in period *d* (units)						
	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	
Supplier 1	0	0	0	0	0	0	
Supplier 2	**25,000**	**25,000**	**25,000**	**25,000**	**25,000**	**25,000**	
Supplier 3	**10,000**	**7,566**	**0**	**455**	**7,950**	**10,000**	
Supplier 4	0	0	0	0	0	0	
	Volume of products transported to retailer *r* in period *d* (units)						Average Service Level (%)
	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	
Retailer 1	5,050	5,086	3,300	4,216	5,121	5,776	95.33%
Retailer 2	**4,535**	**4,041**	**1,635**	**2,968**	**4,493**	**5,285**	**98.29%**
Retailer 3	3,090	2,444	898	1,972	2,682	3,289	92.30%
Retailer 4	**8,108**	**7,277**	**5,226**	**5,625**	**7,673**	**9,138**	**100.00%**
Retailer 5	8,250	9,298	6,562	6,569	8,658	8,307	93.71%
Retailer 6	**7,207**	**6,746**	**4,197**	**5,922**	**6,674**	**6,991**	**96.67%**

The highlighted cells indicate the suppliers selected to provide Polyethylene Terephthalate (PET) and the retailers who achieved the highest service level satisfaction percentages.

4.3.3.1 Case 3's Comparison of the Results

Table 4.43 and Figure 4.12 present a comparative analysis of the results from three fuzzy linear programming models, focusing on supplier selection, retailer prioritization, and objective function values. In the model aimed at minimizing total supply chain operational costs, the emphasis is placed entirely on cost-efficiency, with no consideration for downside risk. Consequently, suppliers offering the most economical Polyethylene Terephthalate (PET) are chosen, and retailers with the lowest associated servicing costs are prioritized. This outcome is consistent with conventional specific fuzzy programming approaches, where uncertainties are modeled using symmetrical fuzzy numbers and resolved through standard defuzzification methods. The decision rule in this context favors options with the lowest cost. Conversely, in the model that targets the minimization of downside risk while disregarding cost, the focus shifts entirely to risk mitigation. Suppliers with the least downside risk in PET pricing are selected, and the production facility gives preference to fulfilling demands from retailers that present the lowest exposure to risk. This approach clearly emphasizes reducing uncertainty and potential losses rather than achieving cost savings.

The third scenario addresses the simultaneous minimization of both total supply chain operational costs and downside risk. This approach achieves a balanced trade-off between reducing expenses and mitigating downside risk, an aspect that conventional specific fuzzy programming techniques often overlook due to their inability to effectively handle the asymmetrical nature of risk, particularly downside risk. In this model, suppliers are selected based on a moderate combination of cost and risk, while the production plant prioritizes fulfilling demands from retailers that contribute to minimizing both factors. The combined total operational costs and associated risks are minimized, as illustrated in Figure 4.12. This is accomplished by maximizing the minimum satisfaction level between cost reduction and risk mitigation, ensuring a balanced performance across both objectives. However, such a compromise is feasible only when downside risk has a substantial impact on costs, warranting the integration of both considerations into the decision-making framework.

Table 4.43 Result comparison of three fuzzy linear programming models' outcomes.

	FLP Model (Minimizing total supply chain operation costs)	FLP Model (Minimizing downside risk of total supply chain operation costs)	MOFLP Model (Minimizing both total supply chain operation costs and downside risk of total supply chain operation costs)
Supplier Selection	Supplier 1 and Supplier 2	Supplier 3 and Supplier 4	Supplier 2 and Supplier 3
Retailer Selection: (Average Service Level)	Retailer 1: 93.39% Retailer 2: 91.71% Retailer 3: 89.59% ***Retailer 4: 99.00%*** ***Retailer 5: 100.00%*** ***Retailer 6: 96.00%***	***Retailer 1: 100.00%*** ***Retailer 2: 97.61%*** Retailer 3: 94.19% ***Retailer 4: 99.00%*** Retailer 5: 94.80% Retailer 6: 92.34%	Retailer 1: 95.33% ***Retailer 2: 98.29%*** Retailer 3: 92.30% ***Retailer 4: 100.00%*** Retailer 5: 93.71% ***Retailer 6: 96.67%***
Minimizing Total Supply Chain Operation Costs	$7,712,875.00	$8,429,300.00	$7,832,100.00
Minimizing Downside Risk (MCVaRG)	$2,147,100.00	$1,731,676.00	$1,921,500.00
Possible Range of Total Supply Chain Operation Costs	From $7,712,875.00 to $9,859,975.00	From $8,429,300.00 to $10,160,976.00	From $7,832,100.00 to $9,753,600.00

For each model, the bold and italicized retailers denote those with the highest three service level satisfaction rates.

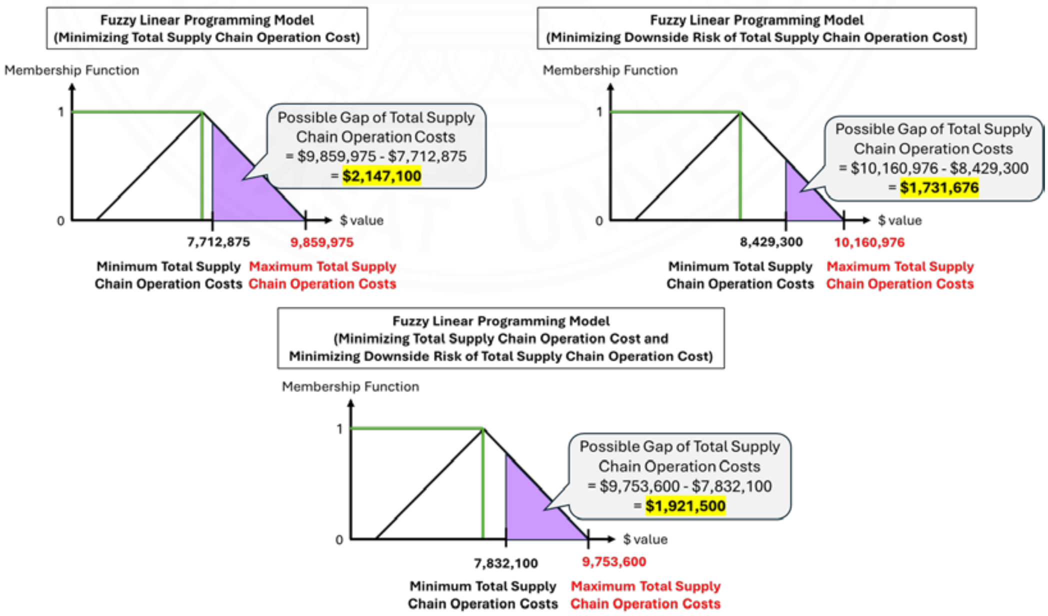

Figure 4.12 Demonstration of how to determine maximum overall supply chain operation expenses in a pessimistic case.

4.3.3.2 Case 3's Sensitivity Analysis

The developed Multi-Objective Fuzzy Linear Programming (MOFLP) model incorporates a credibility level parameter (γ), which reflects the decision maker's judgment regarding downside risk, measured by the Mean-Conditional Value at Risk Gap (MCVaRG). This parameter can impact on the resulting plans and their comparative performance. Therefore, a sensitivity analysis will be performed to evaluate the effect of varying this parameter, as described in the following section.

- **Sensitivity Analysis of the Percentage of Credibility**

As outlined earlier, the credibility parameter represents the level of confidence or belief in the model's outcomes, where a higher value of γ indicates stronger trust in the results. In this study, downside risk is influenced by changes in the credibility level, which is examined within the range of 0.5 to 1. The sensitivity analysis evaluates how varying γ affects the outcomes, as presented in Tables 4.44 and 4.45. When γ is set to 0.5, corresponding to the most probable scenario, the model produces the lowest total supply chain operational costs and downside risk. Conversely, at $\gamma = 1$, representing the most conservative or pessimistic scenario, both the operational costs and downside risk reach their highest levels.

Table 4.44 Result of sensitivity analysis of the percentage of credibility.

γ	Minimizing Total Supply Chain Operation Costs	Minimizing Downside Risk (MCVaRG)
50%	$6,665,479.13	$888,131.70
60%	$7,097,234.97	$1,151,982.54
70%	$7,455,783.73	$1,530,145.78
80%	$7,832,100.00	$1,921,500.00
90%	$8,271,553.91	$2,365,962.17
100%	$8,649,609.54	$2,740,306.33

The highlighted cell displays the results of applying γ at 80%, which served as the initial benchmark in this case study.

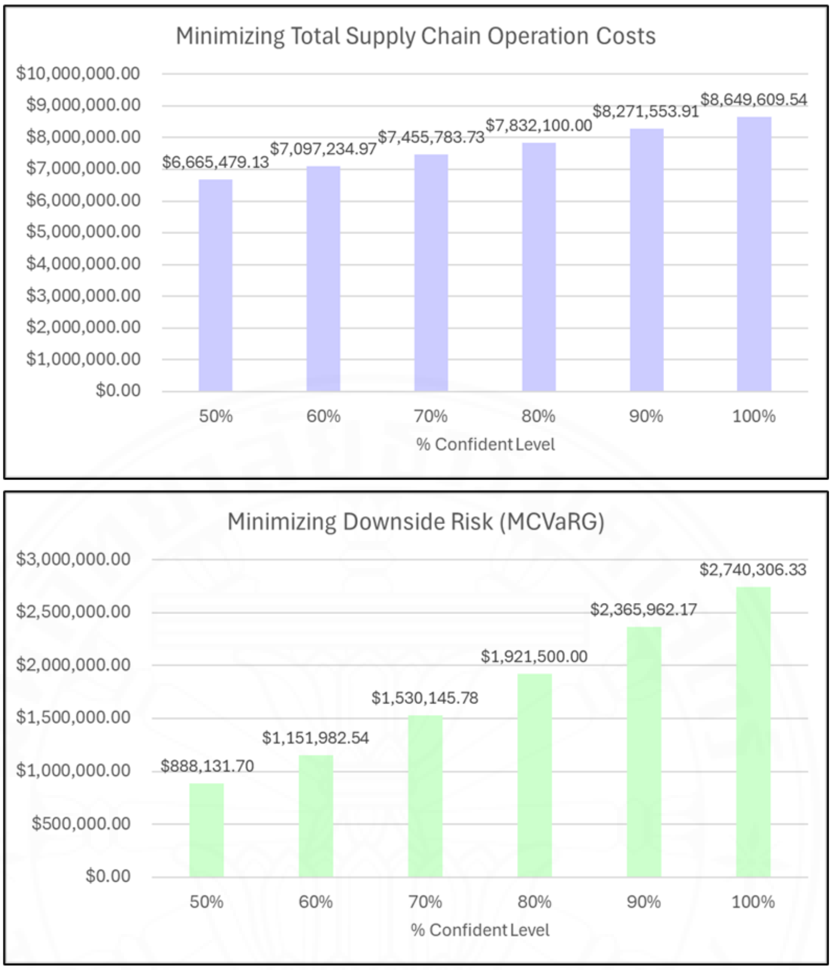

Figure 4.13 Objective value results correspond to different percentages of credibility.

Table 4.44 and Figure 4.13 illustrate that as the credibility level (γ) increases from 50% to 100%, total supply chain operational costs rise from $6,665,479.13 to $8,649,609.54. Likewise, the minimum downside risk (MCVaRG) escalates from $888,131.70 to $2,740,306.33. This trend offers valuable insights for decision-makers by highlighting the potential variability in outcomes and supporting more informed, proactive planning. A higher credibility level represents a more conservative outlook, where both operational costs and downside risk are elevated. Thus, increasing γ reflects a stronger level of confidence in the reliability of the results produced by the model.

Table 4.45 Result reflecting how average service levels change with varying percentage of credibility of each retailer.

	Retailer 1	Retailer 2	Retailer 3	Retailer 4	Retailer 5	Retailer 6
γ = 50%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
γ = 60%	100.00%	100.00%	94.69%	100.00%	98.27%	100.00%
γ = 70%	97.42%	100.00%	93.49%	100.00%	94.52%	100.00%
γ = 80%	95.33%	98.29%	92.30%	100.00%	93.71%	96.67%
γ = 90%	94.56%	97.64%	91.85%	100.00%	92.82%	95.76%
γ = 100%	93.03%	96.04%	88.99%	100.00%	90.85%	94.56%

Table 4.45 shows that as the credibility level (γ) rises from 50% to 100%, the service levels for each retailer vary accordingly. This variation occurs because higher γ values correspond to increased retailer demands, which are modeled as fuzzy and defuzzified using chance-constrained programming. At the most probable scenario (γ = 0.5), all retailers' demands are fully met, achieving a 100% service level. However, as γ increases, service levels for certain retailers gradually decline due to production capacity constraints. The order of decreasing service levels is Retailer 3, Retailer 5, Retailer 1, Retailer 6, and then Retailer 2. Importantly, Retailer 4 consistently maintains a 100% service level, even at the highest credibility level (γ = 1), because it incurs the highest penalty cost for lost sales and is subject to the greatest uncertainty risk. Allowing shortages for Retailer 4 would therefore lead to substantial costs and risks.

4.3.4 Summary

This research offers important managerial insights and practical implications for those involved in SCAPP. Conventional SCAPP approaches typically concentrate on reducing overall operational costs, often overlooking the risks tied to uncertainty. The methodology presented in this study introduces a novel framework that accounts for asymmetrical skewness, thereby enhancing decision-making under uncertain conditions. By incorporating different forms of skewness into the model, it enables a more thorough evaluation of risk, leading to improved allocation of resources and greater operational effectiveness. Consequently, organizations are better equipped to formulate customized strategies that optimize performance in evolving and complex supply chain environments.

While the proposed model effectively targets the downside risk associated with uncertainty, it does not account for the potential benefits of favorable cost outcomes. By utilizing the Mean-Conditional Value at Risk Gap (MCVaRG) to measure and reduce downside risk, the model emphasizes limiting the negative impacts of worst-case scenarios. This risk-averse approach is particularly relevant for decision-makers focused on avoiding adverse outcomes. However, the model's focus on minimizing losses means it may not fully encompass the broader spectrum of uncertainty affecting supply chain operational costs, which are subject to a variety of unpredictable influences. Therefore, the study highlights the need for comprehensive risk management strategies that address the full range of uncertainties in supply chain planning.

Integrating multiple objectives into SCAPP, particularly under uncertain conditions, yields significant benefits. Relying on a single-objective model often falls short when dealing with volatile markets and unexpected disruptions. A multi-objective approach enables decision-makers to craft strategies that are both flexible and resilient, allowing them to address weaknesses throughout the supply chain. This broader perspective helps organizations respond more effectively to uncertainty, promoting greater long-term stability and sustainability. The framework introduced in this study supports decision-makers by offering tools to evaluate a wide range of cost scenarios and manage risks proactively, ultimately fostering more robust and adaptable supply chain systems.

Finally, this study presents an innovative multi-objective fuzzy linear programming model designed to optimize SCAPP by simultaneously balancing cost reduction and downside risk management. The proposed framework is both flexible and robust, improving decision-making in complex and uncertain settings. This study also identifies opportunities for future work, including investigating alternative fuzzy distribution models, accounting for the dynamic behavior of parameters, and examining the influence of external factors on outcomes. Additionally, incorporating advanced optimization methods such as meta-heuristic algorithms and machine learning could further strengthen the model's performance, offering more adaptable and sophisticated solutions. Future studies should also explore a variety of risk metrics and optimization approaches to better reflect diverse risk preferences and priorities, thereby advancing the development of more effective supply chain optimization models under uncertainty.

CHAPTER 5
DISCUSSION AND CONCLUSIONS

This chapter presents a comprehensive analysis of the research findings, highlighting the theoretical contributions and practical implications of the developed fuzzy optimization models for SCAPP. It explores how the proposed framework addresses the inherent uncertainties and conflicting objectives in modern supply chains, offering adaptive and resilient solutions for cost minimization, resource optimization, and risk mitigation. This chapter also reflects on the broader impact of these findings on industry practices and academic discourse, providing actionable insights for decision-makers and laying the groundwork for future research to advance the field further.

5.1 Discussion and Conclusions

- Case 1 (A Five-Phase Hybrid Fuzzy Optimization Approach for Supply Chain Aggregate Production Planning)

This study addresses the dual challenges of data uncertainty and conflicting objectives in SCAPP, highlighting the inseparable nature of supply chain operations and production planning in real-world scenarios. It proposes a five-phase hybrid fuzzy optimization approach that integrates procurement, production, and distribution planning while accommodating imprecise, incomplete, and noisy data. Utilizing advanced fuzzy optimization techniques, including Triangular Intuitionistic Fuzzy Numbers, (α, β)-cut, Realistic Robust Programming, Chance-Constrained Programming, Intuitionistic Fuzzy Linear Programming, and the AUGMECON method, the model generates a diverse set of Pareto optimal solutions tailored to decision-makers' varying risk preferences. This adaptability enables strategic planning across optimistic to pessimistic scenarios, providing decision-makers with a flexible range of solutions that can be customized to specific operational needs. The managerial implications are significant, offering a practical framework for addressing real-world complexities by demonstrating the necessity of integrating supply chain operations with production planning. This study encourages managers to adopt robust decision-making tools that can accommodate imprecise data, noisy inputs, and hesitation in human judgment. By generating Pareto optimal solutions, the proposed approach empowers managers to conduct trade-off analyses among competing objectives, ensuring that supply chain goals align with organizational priorities and enhancing the reliability of decisions in uncertain environments.

- Case 2 (A Unified Fairness and Robustness Fuzzy Optimization Approach for Supply Chain Aggregate Production Planning)

This study provides valuable insights into SCAPP under uncertainty, emphasizing the advantages of a multi-objective approach over conventional single-objective models. By enhancing flexibility and resilience, the multi-objective strategy enables decision-makers to effectively manage supply network dynamics and fluctuations in demand, contributing to long-term supply chain sustainability. The integration of Chance-Constrained Programming (CCP) introduces probabilistic elements that improve decision-making and risk management by accounting for the likelihood of various outcomes. Furthermore, the incorporation of fairness within the optimization framework strengthens stakeholder relationships, fostering trust and collaboration critical to ensuring a stable supply chain environment. This study also highlights the importance of robustness in maintaining operational stability and efficiency in dynamic, uncertain circumstances. The proposed unified fuzzy optimization approach, which integrates fairness and robustness, outperforms conventional methods by simultaneously optimizing conflicting objectives such as cost reduction, workforce stabilization, and purchasing maximization. The use of triangular fuzzy numbers and Realistic Robust Programming (RRP) enhances the optimization process, making it more adaptable and effective in addressing the complex challenges faced by modern supply chains.

- Case 3 (A Downside Risk Mitigation Approach for Supply Chain Aggregate Production Planning)

This study offers valuable insights for decision-makers in SCAPP, emphasizing the importance of incorporating multiple objectives into supply chain strategies. Traditional SCAPP models typically focus on minimizing operational costs, often neglecting the associated risks. The proposed methodology enhances decision-making under uncertainty by integrating various types of skewness, providing a more comprehensive risk assessment and facilitating improved resource allocation. By addressing downside risks through the Mean-Conditional Value at Risk Gap (MCVaRG), the model helps mitigate the adverse effects of worst-case scenarios, improving risk management and operational efficiency. This approach enables organizations to develop more tailored strategies, optimizing performance in dynamic and complex environments. However, while the model focuses on downside risk, it may not fully capture all uncertainties impacting supply chain operation costs, which are influenced by unpredictable factors. The managerial implications are significant, as the model offers a strategic tool for managers to adopt a more holistic approach that moves beyond cost minimization to incorporate risk management and resource optimization. By proactively addressing potential losses through skewness and downside risk assessment, managers can enhance decision robustness and reliability, fostering resilient, adaptive supply chain strategies that align with organizational risk profiles. Additionally, the model’s focus on fair resource allocation and efficient planning contributes to improved stakeholder trust, better alignment of supply chain functions, and long-term sustainability.

- Overall Discussion and Managerial Implications

This study represents a significant advancement in Supply Chain Aggregate Production Planning (SCAPP) by integrating advanced fuzzy optimization models to address the inherent uncertainties within modern supply chains. Unlike conventional models that rely on static assumptions, the proposed framework leverages fuzzy logic to manage the unpredictable nature of supply chain systems, enabling decision-makers to balance cost minimization, resource allocation, and risk mitigation effectively. The model's systematic approach to quantifying and managing uncertainties ensures resilience against both external shocks and internal variability, making it particularly relevant for industries affected by market volatility, global uncertainties, and rapid technological changes. Empirical results demonstrate that this framework enhances operational efficiency, mitigates cost-related risks, and optimizes resource utilization, positioning it as a critical tool for businesses striving to maintain stability and competitiveness in uncertain environments.

Beyond its theoretical significance, this study offers important managerial insights, including the following:

- **Enhanced Decision-Making Under Uncertainty**: Managers can benefit from a multi-objective approach that accommodates uncertainty and hesitation in human judgment, enabling more informed and balanced decisions in fluctuating supply chain environments.
- **Strategic Trade-off Analysis**: The availability of multiple optimal solutions enables managers to conduct trade-off analyses among conflicting objectives, supporting more balanced and aligned organizational strategies.
- **Flexible Strategic Planning**: By offering Pareto optimal solutions across a range of optimistic to pessimistic scenarios, the model allows managers to select strategies that best align with their risk tolerance and business conditions.
- **Strengthening Stakeholder Relationships**: By incorporating fairness into the optimization framework, the proposed approach promotes equitable decision-making, enhancing trust, cooperation, and long-term partnerships within the supply chain.

- **Operational Stability in Uncertain Environments**: The use of robustness in the optimization process enhances operational consistency, reducing the impact of variability and disruptions.
- **Risk-Awareness Planning**: By integrating Chance-Constrained Programming (CCP), managers can incorporate probabilistic reasoning into their planning processes, enhancing risk management by assessing the likelihood of various outcomes. Additionally, the inclusion of downside risk analysis through the Mean-Conditional Value at Risk Gap (MCVaRG) empowers managers to proactively address worst-case scenarios, reducing potential losses and strengthening the overall resilience of the SC.
- **Strategic Use of Skewness in Planning**: Leveraging skewness as part of risk assessment empowers managers to better evaluate asymmetric uncertainties and plan accordingly.
- **Future-Ready Planning Framework**: By integrating advanced methodologies, the study provides a forward-thinking framework that prepares managers to navigate increasingly uncertain and complex supply chain environments.

5.2 Limitations and Further Study

While this study offers meaningful insights into supply chain production planning, it also has certain limitations that future research should consider. The main constraints identified across the three cases are outlined below:

- **Data Availability and Quality**: The model relies heavily on imprecise, incomplete, and noisy data, which may impact its accuracy and effectiveness in real-world applications, especially when data quality is suboptimal.
- **Human Judgment Bias**: Although the model accommodates hesitation and human judgment, subjective biases in decision-making could still affect the quality of the results, particularly in the absence of sufficient decision-maker expertise or experience.

- **Computational Complexity**: The proposed fuzzy optimization approaches, while comprehensive, can become computationally intensive, especially when applied to large-scale supply chain scenarios, potentially limiting its practical application in time-sensitive environments.
- **Limited Consideration of External Factors**: Although the study addresses data uncertainty and conflicting objectives, it may not fully account for all external factors or unforeseen events (e.g., political, economic, or environmental disruptions) that could influence supply chain operations.
- **Scalability Issues**: The model may face challenges in scaling to large-scale supply chains with many variables, decision-makers, and operational complexities, potentially reducing its applicability in global or highly intricate systems.
- **Assumption of Linear Relationships**: The integration of various fuzzy optimization techniques may oversimplify some non-linear relationships and interactions in supply chain operations, which could affect the accuracy of the solutions in certain cases.
- **Assumptions of Static Risk Preferences**: The study assumed fixed risk preferences of decision-makers across scenarios, which may not reflect the dynamic nature of risk tolerance, particularly in fast-changing or uncertain environments.
- **Narrow Focus on Downside Risk**: The model primarily focuses on mitigating downside risks and worst-case scenarios, but it may not fully capture all types of uncertainties affecting supply chain operation costs, particularly those arising from unpredictable factors such as sudden market shifts or geopolitical events. Although the model integrates various types of skewness for a more comprehensive risk assessment, it may still overlook certain non-quantifiable risks or those driven by human factors, which can significantly influence supply chain performance.

- **Over-Simplification of Fairness**: While fairness is integrated into the optimization framework, its implementation might not fully capture the complex, diverse interests of all stakeholders, potentially simplifying the nuances of real-world stakeholder dynamics.
- **Simplification of Real-World Dynamics**: The study assumes that decision-makers can effectively adapt to a range of solutions, but real-world decision-making often involves more complex interactions and constraints that may not be fully captured in the proposed model.
- **Generalizability to All Industries**: The framework, though promising, may not be universally applicable across all industries, especially those with highly specific constraints or operational characteristics that differ significantly from the study's focus.

According to all limitations of this study, future research can be explored as follows:

- **Enhanced Data Quality Management**: Future studies could explore methods for improving data collection, processing, and validation, especially in the context of noisy or incomplete data, to enhance the accuracy and reliability of the model's predictions.
- **Human and Behavioral Factors**: Further studies could also incorporate insights from behavioral economics and decision-making theories to better account for human judgment and biases in the decision-making process within the supply chain context.
- **Human-Computer Decision-Making Integration**: Future studies could investigate ways to better integrate human decision-makers with the fuzzy optimization model, perhaps by using machine learning techniques to improve the model's adaptability to human intuition and reduce decision bias.
- **Computational Efficiency**: Future studies could focus on reducing the computational complexity of the model, making it more efficient for larger datasets and more scalable for dynamic, real-time supply chain decision-making.

- **Integration of External Factors**: Future studies could also explore the inclusion of additional external uncertainties (e.g., market fluctuations, geopolitical risks) that might influence supply chain operations and extend the model's applicability in dynamic environments.
- **Scalability and Efficiency Enhancements**: Future studies could explore ways to improve the scalability and efficiency of the multi-objective optimization approach, potentially using advanced algorithms or heuristic methods that reduce computational overhead for larger supply chains.
- **Stochastic and Non-linear Modeling**: Future studies could investigate the potential of incorporating stochastic or non-linear programming approaches into the model to better capture complex relationships and improve the robustness of solutions in more varied scenarios.
- **Dynamic Risk Assessment Models**: Further studies could also focus on developing dynamic models that evolve with changing market conditions, enabling ongoing adjustments to risk assessments and optimizing supply chain strategies over time.
- **Incorporating Broader Risk Categories**: Future research could expand the model to include other types of risks beyond downside risk, such as operational, financial, and reputational risks, to provide a more comprehensive risk assessment framework.
- **Exploration of Non-Quantifiable Risks**: Future studies could investigate the inclusion of qualitative or non-quantifiable risks, such as those related to human behavior, organizational culture, or customer sentiment, to improve the comprehensiveness of the risk assessment.
- **Fairness in Resource Allocation**: Further research could investigate alternative methods for quantifying and implementing fairness in resource allocation, especially in multi-stakeholder environments, to refine the model's practical applicability.
- **Real-Time Application and Testing**: The model could be tested in real-world supply chain scenarios to assess its practicality and effectiveness in real-time

decision-making, and adjustments could be made based on feedback from actual operations.

- **Cross-Industry Application**: Applying the model to different industries with unique supply chain characteristics (e.g., healthcare, perishable goods, or technology) could provide insights into its generalizability and identify potential modifications for specific sectors.
- **Comparative Analysis with Other Models**: Future studies could compare the proposed approaches with other supply chain optimization models to evaluate its relative advantages and limitations, potentially identifying areas for improvement or adaptation for different industry sectors.

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