# RISK-BASED OVERBOOKING MODEL

**Murati Somboon**

**A Dissertation Submitted in Partial**
**Fulfillment of the Requirements for the Degree of**
**Doctor of Philosophy (Statistics)**
**School of Applied Statistics**
**National Institute of Development Administration**
**2016**

**RISK-BASED OVERBOOKING MODEL**

**Murati Somboon**

**School of Applied Statistics**

---

Associate Professor ............................................................ Major Advisor

(Kannapha Amaruchkul, Ph.D.)

The Examining Committee Approved This Dissertation Submitted in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy (Statistics).

Professor ............................................................ Committee Chairperson

(Prachoom Suwattee, Ph.D.)

Associate Professor ............................................................ Committee

(Jirawan Jitthavech, Ph.D.)

Associate Professor ............................................................ Committee

(Vichit Lorchirachoonkul, Ph.D.)

Associate Professor ............................................................ Committee

(Kannapha Amaruchkul, Ph.D.)

Instructor ............................................................ Dean

(Siwiga Dusadenoad, Ph.D.)

October 2016

# ABSTRACT

| | |
|---|---|
| **Title of Dissertation** | Risk-Based Overbooking Model |
| **Author** | Miss Murati Somboon |
| **Degree** | Doctor of Philosophy (Statistics) |
| **Year** | 2016 |

---

In this study, a two-class revenue management (RM) model, which combines two of the most important RM strategies for a passenger airline: overbooking and seat inventory control is proposed. Using this model, it is possible to concurrently find both the optimal booking limit and the optimal overbooking limit. Consequently, on a closed-form expression for an optimal booking/overbooking limit, sensitivity analysis was analytically assessed by changing model parameters such as revenue, the penalty cost associated with unsatisfied demand, the show-up probability, refunds, denied boarding cost, and plane capacity, and a study of its properties and expected profit function carried out.

Numerical studies were carried out in two parts. The first part was performed to check the results of the sensitivity analysis using simulated data and the second to evaluate the performance of the proposed model using real-life data. Finally, three hypotheses were tested using real-life data. 5,184 sets of conditions were used to check the result of the sensitivity analysis, the properties of the optimal booking/overbooking limit, and the expected profit function. It was concluded that the booking limit is affected by the demand for class 1 seat allocations and all model parameters except for denied boarding cost and overbooking limit is affected by all of the model parameters for class 2 seat allocations, including denied boarding cost and capacity. Using real-life data, the performance of the policy from the proposed model was evaluated against the fixed-booking limit policy of an airline and was found to outperform it. Moreover, three hypotheses: the effect of varying the number of update booking limit points, the effect of an incorrect initial mean for demand, and the effect of a number of smoothing constants on an exponential smoothing method were tested using real-life data. At the 0.05 significance level, it was found that different numbers of update booking limit points affected profit, incorrect initial mean for demand did not affect profit when a high number of update booking limit points was set, and all of the smoothing constants in exponential smoothing method affected profit to some extent.

# ACKNOWLEDGEMENTS

I would like to express my deepest gratitude to everyone for their help in completing this dissertation. In particular, I wish to thank my advisor, Associate Professor Dr. Kannapha Amaruchkul, for always giving invaluable suggestions, motivation, and encouragement; she was always available for consultation whenever I needed her. I also gratefully acknowledge my committee members: Professor Prachoom Suwattee, Associate Professor Dr. Jirawan Jitthavech and Associate Professor Dr. Vichit Lorchirachoonkul, for their helpful comments and suggestions.

I would like to express my gratitude to Professor Dr. Prachoom Suwattee, Associate Professor Dr. Virool Boonyasombat, and Associate Professor Dr. Pachitjanut Siripanich for their invaluable lectures. I am thankful to my friends at the School of Applied Statistics, National Institute of Development Administration (NIDA), for their help, encouragement, and sharing of their knowledge and experience.

Finally, I gratefully acknowledge to the School of Applied Statistics, NIDA for providing the facilities, and NIDA for financial support.

Murati Somboon
October 2016

# TABLE OF CONTENTS

# LIST OF TABLES

**Tables** **Page**

# LIST OF FIGURES

**Figures** **Page**

# SYMBOLS

| | |
|---|---|
| $\mathbb{R}$ | A set of real numbers |
| $\mathbb{Z}$ | A set of integers |
| $\mathbb{Z}_+$ | A set of non-negative integers |
| $(y)^+$ | The maximum number of $y$ and 0 |
| $\inf A$ | The greatest lower bound of $A$ |

# CHAPTER 1

# INTRODUCTION

## 1.1 Background

In 2014, the global economic crisis damaged most economic sectors but commercial airlines were still able to grow by 2.6% and increase their worth to $704 billion (FlightGlobal, 2015). According to 2014 aerospace industry financial data, the commercial airline industry was likely to continue to grow over the next ten years (Thisdell, 2015). One of the keys to success is revenue management (RM) (Sabre, 2015). RM can be defined as “selling the right product to the right customer at the right time for the right price” (Cross, 1997: 52), and originated in the airline industry with American Airlines, who are pioneers of RM; the company estimates that RM increased its revenue by $1.4 billion between 1989 and 1991 (Smith, Leimkuhler and Darrow, 1992). To this day, RM has been effectively applied to other business areas, such as hotels, rental cars, restaurants, health care, cruise industries, railways, manufacturing, IT services, etc. (Chiang, Chen and Xu, 2006). Major research areas in RM can be categorized into 1) seat inventory control, 2) overbooking, 3) pricing, and 4) demand forecasting (McGill and Van Ryzin, 1999). In this dissertation, the focus is on two RM strategies: seat inventory control and overbooking practiced by passenger airlines.

The seat inventory control method is concerned with mixing passengers from different fare classes in the same aircraft compartment. An airline may offer discounted fares to stimulate demand, such as for early booking, hoping to fill seats that would otherwise be empty, and so the assumption is that demand for the lower-fare class will occur before the higher-fare class. The booking limit for each fare class is set before the start of reservations to protect seats for higher fare-paying customers. The objective of seat inventory control is to determine how to allocate capacity to the different classes so that expected revenue or profit is maximized. This technique also yields the number of seats that should be protected for late booking, full fare passengers.

Overbooking means that the airline intentionally sells more reservations for a flight than physical capacity on the aircraft to compensate for cancellations and no-show passengers. Overbooking is the oldest technique and generally accepted to be the most successful in the practice of RM. When a company emphasizes on cancellations and no-shows passenger, overbooking control is used. Without overbooking, about 15% of seats would be unused at the time of service (Smith et al., 1992). One of the objectives of overbooking is to find an optimal overbooking limit that maximizes the expected profit.

Overbooking and seat inventory control approaches have the same objective in terms of maximizing expected profit. The former manages cancellations and no-shows by accepting a certain number of reservations above capacity, whereas with the latter, discounted fares are used to stimulate demand to fill seats. When there are cancellations and no-shows, the airline is likely to have many empty seats at departure time if it does not overbook. Thus, combining overbooking and seat inventory control techniques may increase seat utilization and expected profit.

There are many multiple-class booking control models which allow overbooking (see e.g. Brumelle and McGill, 1989; Subramanian, Stidham and Lautenbacher, 1999; Gosavi, Bandla and Das, 2002; Lan, Ball and Karaesmen, 2011; Aydin, Birbil, Frenk and Noyan, 2012; and Lan, Ball, Karaesmen, Zhang and Liu, 2015). These models have been formulated as Markov decision processes and most of them do not possess the closed-form solutions, except for Aydin et al. (2012); their closed-form solution is achieved by assuming that booking requests follow a multinomial distribution. The assumption for the model in this study is that booking requests follow a general distribution. In practice most commercial RM systems are based upon a two-class model instead of a multi-class model. In this study, a closed-form solution for the proposed two-class model is presented.

Most airlines start accepting reservations about a year prior to flight departure. The airlines set an initial booking limit and/or overbooking limit based on the number of available seats, demand forecasting for each booking class, and show-up probability. Periodically, new booking limits or overbooking limits are recalculated based on a new demand forecast, remaining available seats, and show-up probability when demand and show- up probability are reforecast. Typically, airlines only update the booking/overbooking limit monthly for a flight at least six months before departure (Phillips, 2005: 135-136), and most airlines recalculate the booking/overbooking limit daily during the last week before departure. In addition, a reasonable amount of time to update the booking/overbooking limit is required.

An optimal booking/overbooking limit is determined such that expected profit is maximized. The expected profit is the result of revenue minus the summation of refunds and penalty costs (penalty costs when booking requests are rejected and when passengers are denied boarding). The booking/overbooking limit is set before any bookings are made and the company will accept bookings until the total number of reservations reaches the booking/overbooking limit.

Therefore, to find the optimal booking/overbooking limit in a realistic model over an appropriate amount of time it is necessary to the update booking/overbooking limit to maximize expected profit.

## 1.2 Objectives of the Study

This dissertation focuses on the static two-class model that applied overbooking. The objective of the study are:

1) To propose a static two-class overbooking model and to find the optimal booking/overbooking limit.

2) To obtain the properties of the optimal booking/overbooking limit and expected profit.

3) To evaluate the performance of the policy from the proposed model using real-life data.

## 1.3 Scope of the Study

The proposed static two-class model is analyzed under a single-leg problem with a single resource.

## 1.4 Usefulness of the Study

The proposed model could be applied to an airline operating with multiple fare classes. The closed-form solution received from the proposed model could be extended to a heuristic method in a multiple fare class model, which is better than using the Markov decision process. Moreover, the proposed model could be applied to other businesses utilizing perishable assets, such as hotels, car rental, restaurants, and the cruise ship industry in order to maximize expected profit.

## 1.5 Organization of this Dissertation

This dissertation is organized as follows. Chapter 2 provide an overview of RM and other details of seat inventory control, overbooking, updating booking limits, the unconstraining method, and simple exponential smoothing. Additionally, a literature review of static two-class models is presented.

In Chapter 3, a static two-class overbooking model with discrete demand is proposed. An analysis of the proposed model and sensitivity analysis with proof is also shown in this chapter. Sensitivity analysis is performed by changing model parameters such as revenue, penalty cost, refund cost, show-up probability, denied boarding cost, and capacity.

In Chapter 4, a simulation study carried out to check the result of sensitivity analysis is described along with a study with real-life data to evaluate the performance of the policy from the proposed model by comparing it with an airline's policy and to test three hypotheses which may affect profit.

Chapter 5 contains a summary of this study and recommendations for future research.

# CHAPTER 2

# LITERATURE REVIEW

In this chapter, a review of two important techniques for success in revenue management (RM): seat inventory control and overbooking are examined. Other tactics for success in RM are updating booking limits, unconstraining methods, and forecasting. Exponential smoothing is a forecasting technique that is used to forecast future booking requests. Available literature on static-two class models is presented in the last section.

## 2.1 An Overview of RM

RM or yield management originated from the airline industry is an appropriate tool that can deliver the right product to the right customer at the right time for the right price on behalf of a company (Cross, 1997: 52). Three major traditional applications of RM are in the booking systems for airlines, hotels, and the rental car industry. At this time, RM has also been effectively applied to other areas such as restaurants, the cruise ship industry, restaurants, railways, manufacturing, IT services, etc. (Chiang et al., 2006). The appropriate conditions for RM contain the following characteristics (Phillips, 2005: 120):

1) Capacity is perishable and limited.
2) Capacity is booked before departure.
3) The available perishable capacity can be sold at different prices through different booking classes.
4) The availability of fare classes can change over time.

RM is a process by which allocation of capacity to different fare classes is made over time to maximize profit. Passenger airlines face largely fixed costs and small variable costs per passenger. Profit increases when the airlines can accept a higher number of reservations, and so an optimal booking limit policy is necessary for this business. Talluri and Van Ryzin (2004), and Phillips (2005), describe two techniques for single-leg RM . First, capacity allocation or seat inventory control is the technique used to determine which fare classes should be open and which closed at any given time of a product consisting of a constrained resource. Second, overbooking is the technique to determine how many units should be reallocated to compensate for cases where customers may not show up or may cancel.

Under the single-leg approach, seat inventory control and overbooking are two techniques to handle the RM objectives. Seat inventory control is a method to determine how many seats of the low-fare class are allowed to be booked when there is the possibility of future high-fare demand. Overbooking is concerned with how many seats should be sold more than capacity to replace customers who may cancel or not show up. The next two sections address seat inventory control and overbooking in more detail.

## 2.2 Seat Inventory Control

Seat inventory control or capacity allocation is an approach to merge passengers from different fare classes in the same aircraft compartment. Many airlines segment marketing between early-booking, lower-fare passengers and later-booking, higher-fare passenger based on the RM strategy. Any business applying RM has the opportunity to restrict early lower-price booking in order to reserve capacity for later higher-price booking. The seat inventory control approach was developed from Littlewood's two-class model.

### 2.2.1 Littlewood's Two-Class Model

Littlewood's two-class model is a simple two-class allocation rule with fixed capacity $\kappa$ (Littlewood, 1972). The assumption for this model is that there are two classes of customers: class 2 customers who book early with a low fare $p_2 > 0$ and class 1 customers who book later with a high fare $p_1 > p_2 > 0$ . The airline earns revenue $p_i$ when a class $i$ customer is accepted. We assume all class 2 reservations arrive before class 1 and assume no cancellations or overbooking; all customers with reservations show-up at the time of departure.

Let $x_L$ be the booking limit for class 2 of Littlewood's two-class model, i.e. class 2 booking requests are accepted up to $x_L$. From here, we use the term booking limit instead of the booking limit for class 2. For $i = 1, 2$, let $D_i$ be the demand for class $i$, the number of class $i$ booking requests, which are assumed to be independent random variables. The problem is to decide how much class 2 demand to accept so that the expected profit is maximized. Therefore, we let $x_L^*$ be an optimal booking limit that maximizes the expected profit:

$$x_L^* = \left[\kappa - F_{D_1}^{-1}(1 - \frac{p_2}{p_1})\right]^+ ,$$

where $F_{D_1}^{-1}(a)$ is a quantile function of distribution function of random variable $D_1$ and define $F_{D_1}^{-1}(a) = \inf\{x : P(D_1 \le x) \ge a\}$.

## 2.3 Overbooking

Overbooking is the oldest technique that has been most successful in RM practices. This technique increases the capacity utilization in a reservation system by selling more reservations of a flight than physical capacity on an aircraft to compensate for cancellations and no-show passengers. Passengers cancelling means passengers who return or change their booking before departure time and no-show passengers means passengers who fail to show up at departure time.

An overbooking limit is set before any booking is accepted. An airline accepts booking requests until the total number of reservations reaches the overbooking limit. At departure time, the airline may not need to deny boarding to the excess reservations because of an adequate number of no-show passengers. Finally, an airline with overbooking earns more from reservations that show up than without overbooking (see Figure 2.1).

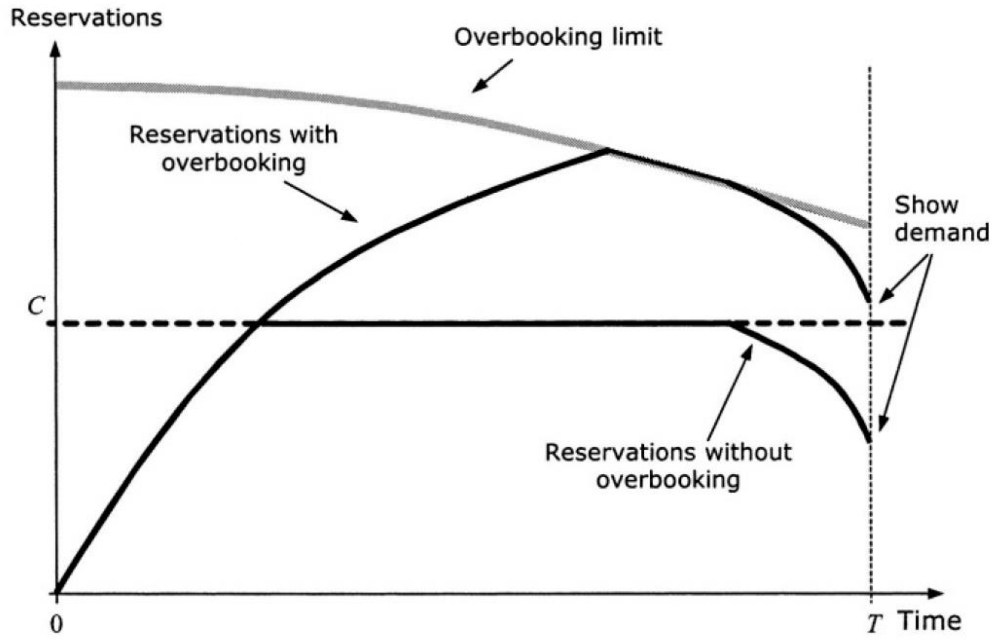



**Figure 2.1** Overbooking Limit and Reservations Oime.

**Source:** Talluri and Van Ryzin, 2004: 140.

### 2.3.1 Deterministic Heuristic

An overbooking limit $(\text{OBL})$ can be simply calculated when an airline has a show-up probability for a passenger, which the airline can calculate using historical data. OBL can be written as

$$\text{OBL} = \frac{\kappa}{\theta},$$

where $\kappa$ is capacity and $\theta$ is the show-up probability.

For example, an airplane has 162 available seats for passengers and the expected show-up probability is 0.9, the deterministic heuristic gives

$$\text{OBL} = \frac{162}{0.9} = 180 \text{ seats.}$$

### 2.3.2 A Risk-Based Policy

The purpose of a risk-based policy is to balance the expected cost of denying service with the expected revenue by accepting more bookings. The denied boarding cost consists of the compensation paid to each passenger denied boarding at the time of the flight, and is usually higher than the fare. The denied boarding cost includes one or more of four components (Phillips, 2005: 214):

1) A fare on the next flight and/or hotel accommodation
2) Providing the cost of a meal and/or accomodation
3) The cost of putting denied boarding passengers on another flight to their destination
4) Ill-will compensation

For calculating the overbooking limit, we assume $D$ is demand, the number of booking requests, assumed to be a random variable, and $B(x)$ is the number of reservations when given the overbooking limit $x$. Given the number of reservations $B(x) = y$ and the number of show-ups, denoted by $W(y)$, we define $F(y;x)$ as the cumulative distribution function of $W(y)$. Capacity $\kappa$ is fixed. The airline earns a revenue of $p$ when accepting a booking request and pays a denied boarding cost $h$ to each denied boarding passengers, $h > p$. The airline wants to choose an optimal overbooking limit $x^*$ that maximizes its expected profit:

$$\pi(x) = pE[B(x)] - hE[(W(B(x)) - \kappa)^+].$$

## 2.4 Updating Booking Limits

Most airlines have historical data of passenger bookings that shows capacity, the number of passengers in each fare class, the number of no-show passengers, the number of boarding passengers, etc. Based on the historical data, an airline is able to forecast the demand and show-up probability for each booking class to set the initial booking/overbooking limit in a reservation system, which accepts booking requests up to the initial booking/ overbooking limit set beforehand. Periodically, the booking/overbooking limit is recalculated based on a new forecast of demand, show-up probability, and/or remaining capacity. This is known as reoptimization or an update. Three different situations leading to updating the booking/ overbooking limit are as follows (Phillips, 2005: 135):

1) Periodic updates occur at scheduled intervals.

2) Event-driven updates are triggered by events such as a booking class closing, a change in aircraft, and an unanticipated spike in demand.

3) Requested updates may be applied at any time by a flight controller or revenue manager based on competitive actions, changes in fares, anticipated changes in future demand, or for any other reason.

## 2.5 Unconstraining of Airline Demand Data

One of tactics for success in RM is accurately forecasting demand. The reservation system accepts the booking requests up to a pre-determined booking limit. Hence, demand in that fare class for a given flight may exceed the booking limit, but historical data shows only the number of reservations. At the booking limit, the demand is called censored demand in the field of statistics or constrained demand for a passenger airline. The method to uncensor data is called unconstraining. In 2002, Weatherford and Pölt reviewed unconstraining methods; the simplest three are as follows:

1) Naïve 1 (N1) – replace all constrained observations with the mean of all observations.

2) Naïve 2 (N2) – replace all constrained observation with the mean of all unconstrained observations.

3) Naïve 3 (N3) – replace constrained observation less than the mean of all observations with the mean of all unconstrained observations.

## 2.6 Simple Exponential Smoothing

Effective seat inventory control needs demand forecasting for each booking class. After uncensoring the data, the airlines forecast demand for a given flight by using exponential smoothing techniques. Suppose that time series data are from a constant process represented as

$$y_t = \mu + \varepsilon_t \, ,$$

where $\mu$ represents the basic constant level of system response and $\varepsilon_t$ is the noise at time *t*. The $\varepsilon_t$ are often assumed to be uncorrelated with zero mean and constant variance $\sigma^2$. The least square estimator of $\mu$ is

$$\hat{\mu} = \bar{y} = \frac{1}{n}\sum_{t=1}^{n} y_t \; .$$

Simple exponential smoothing is used for forecasting a time series when there is no trend or seasonal pattern, but the mean of the time series $y_t$ is slowly changing over time. A simple exponential smoothing method will give larger weights to more recent observations, and the weights decrease exponentially as the observations become more distant. Let $l_0$ be the mean of the series at time $t = 0$. Procedures of simple exponential smoothing are as follows:

1) Compute the initial estimate of the mean of the series at time period $t = 0$:

$$l_0 = \bar{y} = \frac{1}{T}\sum_{t=1}^{T} y_t \; .$$

2) Compute the update estimate by using the smoothing equation

$$l_T = \rho y_T + (1-\rho) l_{T-1} ,$$

where $\rho$ is a smoothing constant between 0 and 1, and $l_{T-1}$ is the estimate of the mean of the time series during time period $T-1$.

Note that

$$\begin{aligned} l_T &= \rho y_T + (1-\rho) l_{T-1} \\ &= \rho y_T + (1-\rho)[\rho y_{T-1} + (1-\rho) l_{T-2}] \\ &= \rho y_T + (1-\rho)\rho y_{T-1} + (1-\rho)^2 l_{T-2} \\ &\vdots \\ &= \rho y_T + (1-\rho)\rho y_{T-1} + (1-\rho)^2 l_{T-2} + \ldots + (1-\rho)^{T-1} y_1 + (1-\rho)^T l_0. \end{aligned}$$

### 2.6.1 The value of smoothing constant $\rho$

Smoothing constant $\rho$ is set between 0 and 1, and so the two extremes are $\rho = 0$ and $\rho = 1$. As $\rho$ gets closer to 1, and more emphasis is put on the last observation, the smoothed values will approach the original observations. We can expect the variance of the simple exponential smoother to vary between 0 and the variance of the original time series based on the choice of $\rho$. The value of $\rho$ that performs well in practice and often recommended is usually between 0.1 and 0.4 (Montgomery, Jennings and Kulahci, 2008: 179), and can be found by using historical data. We define the sum of squared error $(SSE)$ as

$$SSE(\rho) = \sum_{t=1}^{T} e_{t-1}^2 .$$

We can calculate the $SSE$ for various values of $\rho$ and pick the value of $\rho$ that gives the smallest forecased $SSE$.

## 2.7 Review of Static Two-Class Models

The two-class model, which focuses on the booking control problem with fixed capacity, dates back to Littlewood (1972). This assumption for this model is that two product classes with associated prices arrive sequentially. The booking requests for the two classes are assumed to be discrete and continuous random variables, and overbooking is not allowed. All booking requests show up at the time of departure, i.e. there are no cancellations or no- shows. The booking limit is determined as the maximum number of low fare bookings allowed. This model forecasts booking control that protect seats for late, high fare booking. Shlifer and Vardi (1975) studied the two-class overbooking model but did not include the booking control problem. Demand in this model is assumed to follow a normal distribution along with a show-up pattern. The researchers determined the optimal overbooking limit under each of three criteria: 1) the probability that the rejection level does not exceed a given limit, 2) the expected rejection level does not exceed a given limit, and 3) the loss incurred by rejecting a passenger relative to the profit of carrying one.

Two-class models which include both overbooking and booking control methods can be found in Sawaki (1989), and Ringbom and Shy (2002). Sawaki (1989), and Ringbom and Shy (2002), extended Littlewood's (1972) method to allow for no-show passengers. Sawaki (1989) proposed two two-class models with an associated price. The first model only consists of a booking control method with an added penalty cost for high-fare passengers who reject booking requests. The second model is a two-class overbooking model that allows only high-fare class overbooking. The arrival pattern for this model is assumed to be the same in Littlewood's, i.e. low-fare passengers are booked before high-fare ones.

In Sawaki (1989), booking requests for the two classes are assumed to be continuous random variable whereas Ringbom and Shy (2002) follow a bivariate normal distribution. The show-up passengers in both methods follow a binomial distribution. In this study, it is not necessary for booking requests to follow a normal distribution; they can be any non-negative integer-valued random variable with a general distribution.

In Ringbom and Shy (2002), the refund cost is added to the two-class overbooking model; the full refund cost is given to denied high-fare passengers (i.e. the refund is exactly equal to the fare) and low-fare passengers receive no refund, whereas in this study, refunds are given to both classes, and the refund need not be fully given (i.e. the refund can be expressed as a percentage of the fare). Similar to other overbooking models, booking requests are accepted up to the overbooking limit after which additional requests are rejected. In Sawaki (1989), the airline incurs a penalty (loss-of-goodwill) cost for only class 1 rejected booking requests, whereas in this study, the penalty cost is given to each rejected booking request. The refund and penalty scheme of this study are more general and fit more cases in practice. A summary of two-class models is shown in Table 2.1 below.

**Table 2.1** Comparison of Static Two-Class Models

| | **Littlewood (1972)** | **Shlifer and Vardi (1975)** | **Sawaki (1989)** | | **Ringbom and Shy (2002)** | **Study Model** |
|---|---|---|---|---|---|---|
| | | | **1st model** | **2nd model** | | |
| Capacity | fixed | fixed | fixed | fixed | fixed | fixed |
| Arrival Pattern | yes | - | yes | yes | yes | yes |
| Overbooking | no | yes | no | only class 1 | only class 2 | only class 2 |
| Demand Random Variable | Discrete and Continuous | Continuous (Assume Normal) | Continuous | Continuous | Continuous (Assume Normal) | Discrete |
| Show-up Pattern | - | Normal | - | Binomial | Binomial | Binomial |
| Price | $p_1 > p_2$ | - | $p_1 > p_2$ | $p_1 > p_2$ | $p_1 > p_2$ | $p_1 > p_2$ |
| Refund Cost | - | - | - | - | fully refundable only class 1 | no-show (percentage of price) |
| Denied Boarding Cost | - | - | - | only class 1 | only class 2 | only class 2 |
| Penalty Cost | - | - | only class 1 | only class 1 | - | all rejected requests |

# CHAPTER 3

## THE PROPOSED MODEL: THE STATIC TWO-CLASS OVERBOOKING MODEL

In this chapter, the proposed two-class overbooking model that combines the booking control and overbooking strategies from RM is presented. Details of the proposed model start with its formulation and assumptions of this model. After this, an analysis of the proposed model is performed to study the properties of the optimal booking/overbooking limit and expected profit.

### 3.1 Formulation

Let $\mathbb{R}$ be a set of real numbers and $\mathbb{Z}_+$ be a set of non-negative integers, and let $(y)^+ = \max(0, y)$, for $y \in \mathbb{R}$. The quantile function of the distribution function of random variable $D$ is denoted as $F_D^{-1}(a) = \inf\{x : P(D \le x) \ge a\}$.

Consider an airline with fixed capacity $\kappa$ and two customer classes with fares $p_1 > p_2 > 0$. We assume that all class 2 reservations have been made before class 1 reservations start. For each $i =$ 1, 2, the airline earns revenue $p_i$ when a class $i$ customer is accepted; on the other hand, if rejected the airline incurs a penalty cost $g_i$, where $g_1 > g_2 > 0$. The penalty cost when a customer is rejected includes, e.g. the loss-of-goodwill cost, which measures customer satisfaction, and the opportunity cost, which measures future revenue loss. The loss-of-goodwill cost may be intangible and can be difficult to estimate in practice. The opportunity cost depends on what happens after the lost sales occur. If a customer is likely to return to make a booking request, then the opportunity cost is the expected revenue loss from this event; however, if a customer never returns to make another booking with the airline, then the opportunity cost includes all future revenues the customer might have brought to the airline.

Let $x \in \mathbb{R}_+$ be the class 2 booking limit; class 2 booking requests are accepted up to $x$. We allow overbooking; i.e. $x$ can be greater than capacity $\kappa$. Let $D_i$ be the demand for class $i$, the number of class $i$ booking requests. Assume that $D_1$ and $D_2$ are two independent $\mathbb{R}_+$-valued random variables. The number of class 2 reservations is $\min(x, D_2)$, and the number of class 2 booking requests rejected is $(D_2 - x)^+$.

After all class 2 reservations have been made, class 1 customers can start to book. The remaining capacity after class 2 bookings have been completed is $(\kappa - \min(x, D_2))^+$. We do not overbook class 1 because class 1 passengers are high priority, hence there is an extremely high penalty cost for overbooking. Class 1 customers are accepted up to the remaining capacity. For $i =$ 1, 2, let $B_i(x)$ be the number of class $i$ reservations:

$$B_2(x) = \min(x, D_2), \quad B_1(x) = \min((\kappa - B_2(x))^+, D_1).$$

Some reservations may be cancelled prior to or do not show up at the time of departure. In this model, we assume that cancellations and no-show passengers are the same. Given that the number of class $i$ reservations is $B_i(x) = y_i$, the number of class $i$ show-ups, denoted by $W_i(y_i)$, is assumed to follow a binomial distribution with parameters $y_i$ and $\theta_i$, where $\theta_i \in (0,1]$ is the show-up probability for class $i$. Note that when the show-up probability for class $i$ is equal to 1 $(\theta_i = 1)$, this means that all class $i$ passengers show up at the time of departure.

That the binomial distribution is an adequate model for the show- ups distribution has been demonstrated in Tasman Empire Airways (Thompson, 1961). Each class $i$ reservation that does not show up receives a refund $r_i$, which is a proportion $\gamma_i$ of cost of revenue where $\gamma_i \in (0,1)$; $r_i = \gamma_i p_i$, for $i =$ 1, 2.

At the time of departure, the number of show-up passengers may be over the capacity of the aircraft. Recall that we overbook only class 2 passenger, so all denied boarding passengers are class 2. The airline pays a compensation $h$ to each denied boarding passenger where $h > p_2$. This compensation may include the fare for a higher booking class on the next flight, vouchers for cash or tickets for future travel, and/or hotel accommodation. The airline wants to choose an optimal booking limit $x^*$ that maximizes its expected profit:

$$\pi(x) = E\left[\sum_{i=1}^{2}\left[p_i B_i(x) - r_i\left(B_i(x) - W_i\left(B_i(x)\right)\right)\right]\right] - E\left[h\left(W_2\left(B_2(x)\right) - \kappa\right)^+\right]$$
$$- E\left[g_2(D_2 - x)^+ + g_1\left(D_1 - \left(\kappa - B_2(x)\right)^+\right)^+\right]. \qquad (3.1)$$

The first term in $(3.1)$ is the expected revenue $p_i B_i(x)$ minus the expected refund cost for no-show reservations $r_i(B_i(x) - W_i(B_i(x)))$. The second term is the expected denied boarding cost the company pays out to the denied boarding passengers when the number of show-ups is more than capacity. Recall that we do not overbook class 1 customer, so all denied boarding passengers are class 2. The last term is the expected penalty cost: the expected revenue lost when we reject class 1 or class 2 booking requests. Note that, in $(3.1)$, there are two sources of uncertainty, namely demand uncertainty and the number of show-ups. In practice, an optimal booking limit is re–solved periodically to account for changes in show-up probability and the proportion of refund cost over time, resulting in booking limits that vary over time. The airlines accept reservations at any time up to the current booking limits. Typically, airlines may only update the optimal booking limit for a flight monthly at least six months before departure (Phillips, 2005: 135-136). Most airlines recalculate the optimal booking limit every day during the last week before departure.

## 3.2 Analyses

In this section, increasing (the opposite being decreasing) means non-decreasing (the opposite being non-increasing). For $i = 1, 2$, let $\alpha_i = p_i + g_i - r_i + r_i\theta_i$ and $\tau = \alpha_2 / \alpha_1$. Let $\bar{F}(t; x, \theta_2) = 1 - \sum_{j=0}^{t}\binom{x}{j}\theta_2^j(1-\theta_2)^{x-j}$ be the tail-sum probability (complementary cumulative distribution function) of a binomial distribution with parameters $x$ and $\theta_2$.

**Lemma 1**. The expected profit $\pi(x)$ can be written as follows:

For $x=0,$

$$\pi(x)=\alpha_1 E[B_1(x)]-\sum_{i=1}^{2} g_i E[D_i]. \tag{3.2}$$

For $x=1,2,\ldots,\kappa-1,$

$$\pi(x)=\sum_{i=1}^{2}[\alpha_i E[B_i(x)]-g_i E[D_i]]. \tag{3.3}$$

For $x=\kappa,\kappa+1,\ldots,$

$$\pi(x)=(\alpha_2-h\theta_2)E[B_2(x)]+\alpha_1 E[B_1(x)]-\sum_{i=1}^{2} g_i E[D_i]+h\sum_{t=0}^{\kappa-1} P(W_2(B_2(x))>t), \tag{3.4}$$

where the expected number of class 2 reservations is

$$E[B_2(x)]=\begin{cases} 0 & ; x=0 \\ \sum_{t=0}^{x-1} P(D_2>t) & ; x=1,2,\ldots \end{cases}$$

and the expected number of class 1 reservations is

$$E[B_1(x)]=\begin{cases} \sum_{t=0}^{\kappa-1} P(D_1>t) & ; x=0 \\ \sum_{t=0}^{\kappa-x-1} P(D_1>t)+\sum_{t=\kappa-x}^{\kappa-1} P(D_2<\kappa-t)P(D_1>t) & ; x=1,2,\ldots,\kappa-1 \\ \sum_{t=0}^{\kappa-1} P(D_2<\kappa-t)P(D_1>t) & ; x=\kappa,\kappa+1,\ldots \end{cases}$$

**Proof** From (3.1), we obtain

$$\begin{aligned}\pi(x)=&\sum_{i=1}^{2}\left[(p_i-r_i)E[B_i(x)]+r_i E[W_i(B_i(x))]\right]-hE[(W_2(B_2(x))-\kappa)^+] \\ &-g_2 E[(D_2-x)^+]-g_1 E[(D_1-(\kappa-B_2(x))^+)^+].\end{aligned} \tag{3.5}$$

If $x>0,$ we use a tail-sum formula for expectation to find the expected number of class 2 reservations:

$$E[B_2(x)] = \sum_{t=0}^{\infty} P(\min(x, D_2) > t) = \sum_{t=0}^{x-1} P(D_2 > t).$$

Similarly,

$$\begin{aligned} E[B_1(x)] &= \sum_{t=0}^{\infty} P(\min((\kappa - B_2(x))^+, D_1) > t) \\ &= \sum_{t=0}^{\infty} P((\kappa - B_2(x))^+ > t) P(D_1 > t) \\ &= \sum_{t=0}^{\kappa} [1 - P((\kappa - B_2(x))^+ \le t)] P(D_1 > t). \end{aligned} \tag{3.6}$$

Clearly, if $x = 0,$ then $E[B_2(x)] = 0.$

If $x = 0,$ then $E[B_1(x)] = \sum_{t=0}^{\kappa-1} P(D_1 > t).$

If $x = 1, 2, ..., \kappa - 1,$ the probability mass function of $\kappa - B_2(x)$ is given as

$$P(B_2(x) = \kappa - k) = \begin{cases} P(D_2 \ge x) & ; k = \kappa - x \\ P(D_2 = \kappa - k) & ; k = \kappa - x + 1, ..., \kappa \\ 0 & ; \text{otherwise} \end{cases} \tag{3.7}$$

If $x = \kappa, \kappa + 1, ...,$ the probability mass function of $(\kappa - B_2(x))^+$ is given as

$$P((\kappa - B_2(x))^+ = k) = \begin{cases} P(D_2 \ge x) & ; k = 0 \\ P(D_2 = \kappa - k) & ; k = 1, 2, ..., \kappa \\ 0 & ; \text{otherwise} \end{cases} \tag{3.8}$$

By substituting (3.7) and (3.8) into (3.6), we obtain

$$E[B_1(x)] = \begin{cases} \sum_{t=0}^{\kappa-1} P(D_1 > t) & ; x = 0 \\ \sum_{t=0}^{\kappa-x-1} P(D_1 > t) + \sum_{t=\kappa-x}^{\kappa-1} P(D_2 < \kappa - t) P(D_1 > t) & ; x = 1, 2, ..., \kappa - 1 \\ \sum_{t=0}^{\kappa-1} P(D_2 < \kappa - t) P(D_1 > t) & ; x = \kappa, \kappa + 1, ... \end{cases}$$

The number of class *i* show-ups, $W_i(y_i)$, has a binomial distribution with parameters $y_i$ and $\theta_i \in (0,1]$, where $y_i$ is the number of class *i* reservations and $\theta_i$ is the show-up probability for class *i*.

Following this, $E[W_i(B_i(x)) \mid B_i(x) = y_i] = \theta_i y_i$, and so

$$E[W_i(B_i(x))] = \theta_i E[B_i(x)] \qquad ; i = 1, 2. \tag{3.9}$$

We know that $(a-b)^+ = a - \min(a,b)$, thus

$$E[(D_2 - x)^+] = E[D_2 - \min(x, D_2)] = E[D_2] - E[B_2(x)]. \tag{3.10}$$

Similarly,

$$E[(D_1 - (\kappa - B_2(x))^+)^+] = E[D_1] - E[B_1(x)]. \tag{3.11}$$

The expected number of class 2 passengers who are denied boarding is

$$\begin{aligned} E[(W_2(B_2(x)) - \kappa)^+] &= E[W_2(B_2(x))] - E[\min(W_2(B_2(x)), \kappa)] \\ &= \theta_2 E[B_2(x)] - \sum_{t=0}^{\kappa-1} P(W_2(B_2(x)) > t). \end{aligned} \tag{3.12}$$

By substituting (3.9), (3.10), (3.11), and (3.12) into (3.5), we obtain

$$\pi(x) = \sum_{i=1}^{2} \left[\alpha_i E[B_i(x)] - g_i E[D_i]\right] - h\left(\theta_2 E[B_2(x)] - \sum_{t=0}^{\kappa-1} P(W_2(B_2(x)) > t)\right). \tag{3.13}$$

After substituting $E[B_2(x)]$ and $E[B_1(x)]$ into (3.13), the expected profit becomes (3.2) - (3.4). □

**Theorem 1.** The expected profit function $\pi(x)$ is piecewise on $x = 0, 1, ..., \kappa - 2$ and $x = \kappa, \kappa + 1, ...$, and is unimodal in each piece.

1. For $x = 0, 1, ..., \kappa - 2$, the expected profit $\pi(x)$ has a local maximum point $x'$ given by

$$x' = \begin{cases} 0 & ; 0 \le \tau < P(D_1 > \kappa - 1) \\ \kappa - F_{D_1}^{-1}(1-\tau) & ; P(D_1 > \kappa - 1) \le \tau \le P(D_1 > 0) \\ \kappa - 2 & ; P(D_1 > 0) < \tau \le 1 \end{cases}. \tag{3.14}$$

2. For $x=\kappa,\kappa+1,\ldots$, if $0<\alpha_2/(h\theta_2)<\bar{F}(\kappa-1;\kappa,\theta_2)$, then the expected profit $\pi(x)$ has a local maximum point $x''$ given by

$$x''=\arg\min\{x\in\{\kappa,\kappa+1,\ldots\}:\bar{F}(\kappa-1;x,\theta_2)>\frac{\alpha_2}{h\theta_2}\}. \tag{3.15}$$

Otherwise, the expected profit function is increasing.

**Proof** For $x=0,1,2,\ldots,$ let $\delta(x)=\pi(x+1)-\pi(x)$ be the forward difference of the expected profit. Denote $G_i(x)=B_i(x+1)-B_i(x);\ i=1,2.$ Clearly,

$$E[G_2(x)]=P(D_2>x) \qquad \forall x\in\mathbb{R}_+$$

and

$$E[G_1(x)]=\begin{cases}-P(D_2>x)P(D_1>\kappa-x-1) & ;x=0,1,\ldots,\kappa-1\\ 0 & ;x=\kappa,\kappa+1,\ldots\end{cases}.$$

After some lengthy algebra, we obtain an expression for the difference as follows:

For $x=0,1,\ldots,\kappa-2$,

$$\delta(x)=P(D_2>x)[\alpha_2-\alpha_1P(D_1>\kappa-x-1)].$$

For $x=\kappa-1$,

$$\delta(x)=P(D_2>x)[\alpha_2-\alpha_1P(D_1>\kappa-x-1)]-hE[(W_2(B_2(x))-\kappa)^+]$$

For $x=\kappa,\kappa+1,\ldots$,

$$\delta(x)=P(D_2>x)[\alpha_2-h\theta_2\bar{F}(\kappa-1;x,\theta_2)],$$

where $\bar{F}(t;x,\theta_2)=1-\sum_{j=0}^{t}\binom{x}{j}\theta_2^j(1-\theta_2)^{x-j}$ is the tail-sum probability of a binomial distribution with parameters $x$ and $\theta_2$.

Here we consider two pieces: $x=0,1,\ldots,\kappa-2$ and $x=\kappa,\kappa+1,\ldots$.

Define $\eta(x)=\alpha_2-\alpha_1P(D_1>\kappa-x-1)$. Consider the first piece,

$$\delta(x)=P(D_2>x)[\alpha_2-\alpha_1P(D_1>\kappa-x-1)]=P(D_2>x)\eta(x).$$

We can see that $\delta(x)$ has the same sign as term $\eta(x)$, and so:

1) If $\delta(x)\geq 0$ for $x=0,1,\ldots,\kappa-2$, then the expected profit $\pi(x)$ is increasing and a local maximum point is $\kappa-2$.

2) If $\delta(x) \le 0$ for $x = 0,1,...,\kappa-2$, then the expected profit $\pi(x)$ is decreasing and a local maximum point is 0.

3) We can show that $\eta(x)$ is decreasing in $x$. If $\delta(x) > 0, \forall x < x'$ and $\delta(x) \le 0,\ \forall x \ge x'$, then there exists a local maximum point at $x'$ such that $\pi(x)$ is increasing for $x < x'$ and decreasing for $x \ge x'$.

If $P(D_1 > 0) < \alpha_2 / \alpha_1 < 1$, then $\eta(x) \ge 0$ for all $x = 0,1,...,\kappa-2$, so $\delta(x) \ge 0$ for $x = 0,1,...,\kappa-2$. The expected profit function is increasing in $x$, and the local maximum point is $\kappa - 2$.

If $0 < \alpha_2 / \alpha_1 < P(D_1 > \kappa-1)$, then $\eta(x) \le 0$ for all $x = 0,1,...,\kappa-2$, so $\delta(x) \le 0$ for $x = 0,1,...,\kappa-2$. The expected profit function is decreasing in $x$, and the local maximum point is 0.

Recall that $P(D_1 > \kappa - x - 1)$ is increasing in $x$, so $\eta(x)$ is decreasing in $x$. If $P(D_1 > \kappa-1) < \alpha_2 / \alpha_1 < P(D_1 > 1)$, then $\eta(0) > 0$ and $\eta(\kappa-2) < 0$, i.e. there exists a local maximum point $x'$ such that $\eta(x) > 0, \forall x < x'$ and $\eta(x) < 0, \forall x \ge x'$. Therefore, $\delta(x) > 0, \forall x < x'$ and $\delta(x) < 0, \forall x \ge x'$, i.e. the local maximum point $x'$ given by

$$x' = \arg\min\{x \in \{0,1,...,\kappa-2\} : P(D_1 > \kappa - x - 1) > \tau\}. \tag{3.16}$$

Let $y = \kappa - x - 1$, then, for $x = 0,1,...,\kappa-2$, we obtain $y = 1,2,...,\kappa-1$. Additionally, $y' = \kappa - x' - 1$, and the local maximum point condition (3.16) becomes

$$y' = \arg\max\{y \in \{1,2,...,\kappa-1\} : P(D_1 \le y) < 1-\tau\} = F_{D_1}^{-1}(1-\tau) - 1,$$

so

$$x' = \kappa - F_{D_1}^{-1}(1-\tau).$$

Next, consider the second piece, $x = \kappa, \kappa+1, \ldots$.

Let $\zeta(x) = \alpha_2 - h\theta_2 \bar{F}(\kappa-1; x, \theta_2)$, and $\delta(x) = P(D_2 > x)[\alpha_2 - h\theta_2 \bar{F}(\kappa-1; x, \theta_2)]$ $= P(D_2 > x)\zeta(x).$

We find that $\delta(x)$ has the same sign as the term $\zeta(x)$, and so:

1) If $\delta(x) \ge 0$, for $x = \kappa, \kappa+1, \ldots$, then the expected profit $\pi(x)$ is increasing and the local maximum point is set as large as possible.

We can show that $\zeta(x)$ is decreasing in $x$. If $\delta(x) > 0, \forall x < x''$ and $\delta(x) \le 0$, $\forall x \ge x''$, then there exists a local maximum point at $x''$ such that $\pi(x)$ is increasing for $x < x''$ and decreasing for $x \ge x''$.

If $\alpha_2 / (h\theta_2) > 1$, then $\zeta(x) \ge 0$ for all $x = \kappa, \kappa+1, \ldots$, and so $\delta(x) \ge 0$ for all $x = \kappa, \kappa+1, \ldots$. The expected profit function is increasing in $x$, and the local maximum point is set as large as possible.

Recall that $\bar{F}(\kappa-1; x, \theta_2)$ is increasing in $x$, so $\zeta(x)$ is decreasing in $x$. If $\bar{F}(\kappa-1; \kappa, \theta_2) < \alpha_2 / (h\theta_2) < 1$, then $\zeta(0) > 0$ and $\lim_{x\to\infty} \zeta(x) < 0$, i.e. there exists a local maximum point $x''$ such that $\zeta(x) > 0, \forall x < x''$ and $\zeta(x) < 0, \forall x \ge x''$. Therefore, $\delta(x) > 0, \forall x < x''$ and $\delta(x) < 0, \forall x \ge x''$, i.e. the local maximum point $x''$ is given by

$$x'' = \arg\min\{x \in \{\kappa, \kappa+1, \ldots\} : \bar{F}(\kappa-1; x, \theta_2) > \frac{\alpha_2}{h\theta_2}\}. \tag{3.17}$$

□

From Theorem 1, we can find the optimal booking limit $x^*$ from three points: $x', \kappa-1$, and $x''$. Suppose that $0 < \alpha_2 / (h\theta_2) < \bar{F}(\kappa-1; \kappa, \theta_2)$, then

$$x^* = \arg\max\{\pi(x'), \pi(\kappa-1), \pi(x'')\}.$$

Assume $\alpha_2 / (h\theta_2) \ge \bar{F}(\kappa-1; \kappa, \theta_2)$. If $\lim_{x\to\infty} \pi(x) < \max\{\pi(x'), \pi(\kappa-1)\}$, then $x^* = \arg\max\{\pi(x'), \pi(\kappa-1)\}$. Otherwise, $\pi(x)$ is increasing, and the airline should set the optimal booking limit to be as large as possible.

The optimal booking limit in Theorem 1 has a closed-form that is easy to calculate, and this solution can be extended to the heuristic method in a multiple fare class model, which is better than using the Markov decision process.

## 3.3 Sensitivity Analysis

**Corollary 1.** For $x = 0, 1, \ldots, \kappa-2$, suppose that the ratio $\alpha_2 / \alpha_1$ or capacity $\kappa$ increases or $D_1$ decreases with respect to the usual stochastic order. Then, the local maximum point $x'$ increases.

**Proof** Note that a function $P(D_1 > \kappa - x - 1)$ is increasing in $x$, and the directional change of $x'$ with respect to $\tau = \alpha_2 / \alpha_1$ is obvious in equation (3.16). Let $\tau$ and $\hat{\tau}$ be functions that has the local maximum points $x'$ and $\hat{x}'$, respectively. Since $\tau < \hat{\tau}$, then $x' \le \hat{x}'$ .

Note that $P(D_1 \le \kappa - x - 1)$ is increasing in $\kappa$, and consider the directional change of $x'$ with respect to $\kappa$. Let $\kappa$ and $\hat{\kappa}$ be of a capacity such that $\kappa < \hat{\kappa}$, $y = \kappa - x - 1$, and $\hat{y} = \hat{\kappa} - x - 1$:

$$P(D_1 \le y) \le P(D_1 \le \hat{y}),$$

$$\begin{aligned} \kappa - F_{D_1}^{-1}(1-\tau;\kappa) &\le \kappa - F_{D_1}^{-1}(1-\tau;\hat{\kappa}) \\ &\le \hat{\kappa} - F_{D_1}^{-1}(1-\tau;\hat{\kappa}). \end{aligned}$$

Thus, $x' \le \hat{x}'$.

Consider the directional change of $x'$ with respect to $D_1$, and assume that $D_1 \le_{st} \hat{D}_1$, i.e. $D_1$ is smaller than $\hat{D}_1$ with respect to the usual stochastic order (Müller and Stoyan, 2002).

Let $F$ and $G$ be distribution functions of $D_1$ and $\hat{D}_1$, and $\bar{F}(t) = P(D_1 > t)$ and $\bar{G}(t) = P(\hat{D}_1 > t)$. Therefore, $\bar{F}(\kappa - x - 1) \le \bar{G}(\kappa - x - 1)$ for all $x \in \{0,1,...,\kappa - 2\}$. Let $x'$ and $\hat{x}'$ be the local maximum points of $D_1$ and $\hat{D}_1$, respectively. Since $\bar{F}$ and $\bar{G}$ are increasing in $x$, $x' \ge \hat{x}'$. □

Suppose that we do not overbook; e.g. (i) mean demand for class 1 is larger than capacity, or (ii) mean demand for class 2 is much lower than capacity. There are many cases that increase the optimal booking limit corresponding to $\alpha_2 / \alpha_1$ increases, e.g. 1) the cost of revenue for class 2 increases (or the cost of revenue for class 1 decreases), 2) the penalty cost for class 2 increases (or the penalty cost for class 1 decreases), 3) the show-up probability for class 2 increases (or the show-up probability for class 1 decreases), or 4) the refund cost for class 2 decreases (or the refund cost for class 1 increases).

In Corollary 2, we indicate how the local maximum point $x''$ changes when the model parameter of class 2 and capacity are varied.

**Corollary 2.** For $x = \kappa, \kappa+1, \ldots,$ suppose that the ratio $\alpha_2 / (h\theta_2)$ or capacity $\kappa$ increases. Then, the local maximum point $x''$ increases.

**Proof** Note that function $\bar{F}(\kappa-1; x, \theta_2)$ is increasing in $x$. The directional change of $x''$ with respect to $\alpha_2 / (h\theta_2)$ is obvious in (3.17). Let $\xi = \alpha_2 / (h\theta_2)$ and let $x''$ be the local maximum point associated with $\xi$. Similarly, let $\hat{\xi} = \hat{\alpha}_2 / (\hat{h}\hat{\theta}_2)$ and let $\hat{x}''$ be the local maximum point associated with $\hat{\xi}$. Since, $\xi < \hat{\xi}$, then $x'' \le \hat{x}''$.

Recall that $\bar{F}(\kappa-1; x, \theta_2)$ is decreasing in $\kappa$. Consider the directional change of $x''$ with respect to $\kappa$. Let $\kappa$ and $\hat{\kappa}$ be capacity such that $\kappa < \hat{\kappa}$, then

$$\bar{F}(\kappa-1; x, \theta_2) \ge \bar{F}(\hat{\kappa}-1; x, \theta_2).$$

Thus, $x'' \le \hat{x}''$. □

Suppose that we overbook; e.g. (i) class 2 revenue is close to class 1 revenue, or (ii) the mean demand for class 2 is much higher than capacity. There are many cases that increase the optimal booking limit with corresponding $\alpha_2 / (h\theta_2)$ increases, e.g. 1) the cost of revenue for class 2 increases, 2) the penalty cost for class 2 increases, 3) the show-up probability for class 2 decreases, 4) the refund cost for class 2 decreases, or 5) the denied boarding cost decreases.

Corollary 2 implies that an airline may need to update the optimal booking limit when there is an unusual situation, such as a disaster, terrorist attack or demonstration, which affects some of the parameters in the model. For instance, an incident such as the bombing at the Erawan shrine in Bangkok on 17 August 2015 may decrease the show-up probability of tourists who plan to travel to Bangkok. When the show-up probability decreases, the airline may need to set a higher booking limit. On the other hand, during a long holiday such as Christmas, the New Year festival, or the Songkran festival, the show-up probability may be higher; consequently, the airline may decrease the booking limit.

# CHAPTER 4

# NUMERICAL EXPERIMENTS

This chapter contains the details of numerical experiments using simulated as well as real-life data. Numerical experiments using simulated data were performed to check the results of sensitivity analysis and numerical experiments using real-life data were performed to evaluate performance of the policy from the proposed model. Moreover, numerical experiments using real-life data were carried out to test the effects of various numbers of update booking limit points, an incorrect initial mean, and smoothing constants in an exponential smoothing method. Details of these experiments are described below.

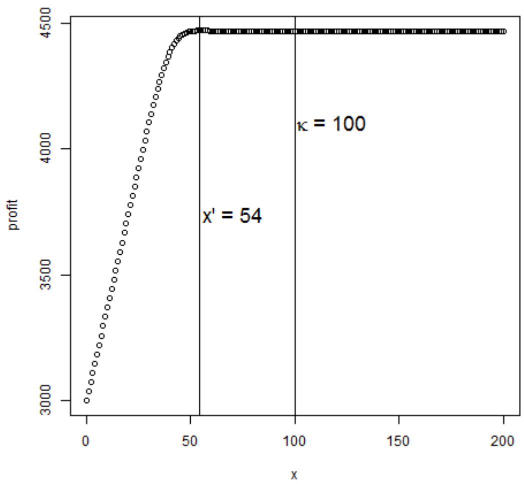
## 4.1 Numerical Experiments with Simulated Data

To check the results of the sensitivity analysis, the effects of changing the model parameters were investigated by conducting numerical experiments. Airlines are able to change the following seven parameters in the model: 1) the cost of revenue for class $i$ $(p_i)$, 2) the penalty cost for class $i$ $(g_i)$, 3) the refund cost for class $i$ $(r_i)$, for each $i = 1, 2$, and 4) the denied boarding cost $(h)$. The rest, namely the demand distribution of class $i$ $(D_i)$ and show-up probability for class $i$ $(\theta_i)$; $i = 1, 2$ are exogenously given and cannot be directly controlled by the airline in a real-life situation.

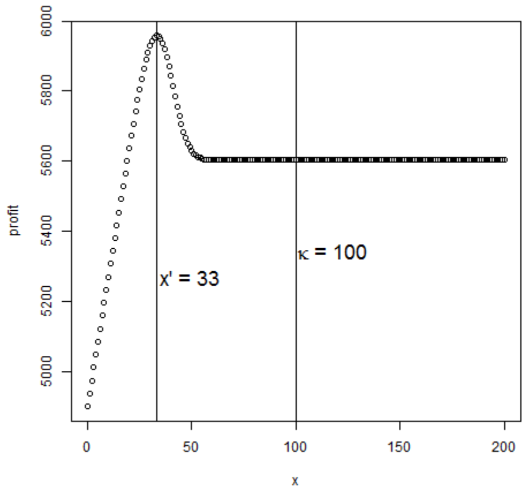
For the purposes of the experiment, the capacity $\kappa = 100$ and $p_1 = g_1 = 100$ were fixed, $g_2 = p_2$ was set, and $p_2$ set at three different levels: 20, 50, and 80. The refund cost for class *i* was a proportion $(\gamma_i)$ of the cost of revenue where $\gamma_i \in \{0.5, 0.8\}$. The refund cost $r_i = \gamma_i p_i$, and so the set of all refund costs is $r_1 \times r_2$, where $r_i \in \{0.5p_i, 0.8p_i\}$. The denied boarding cost $(h)$ was set at three different levels: 100, 300, and 500. The demand for class *i* was assumed to be a Poisson distribution with mean $\lambda_i$, for $i = 1, 2$, and was set at six different levels: 40, 60, 80, 100, 120, and 140. The set of all mean demands was $\lambda_1 \times \lambda_2$, where $\lambda_i = \{40, 60, 80, 100, 120, 140\}$. From Section 3.1, the number of class $i$ show-ups follows a binomial distribution with parameters $y_i$ and $\theta_i$, where $y_i$ is the number of class $i$ reservations and $\theta_i$ is the show-up probability for class $i$. $\theta_i \in \{0.7, 0.9\}$, $i = 1, 2$, was set with the set of all show-up probabilities being $\theta_1 \times \theta_2$. 5,184 sets of conditions were run with booking limit $x$ from 0 to $x_{ub}$, where $x_{ub} = F^{-1}_{D_1+D_2}(\vartheta)$, $\vartheta \in (0,1)$ in each set of conditions. In each case, the quantile function of the summation of two-class demand at $\vartheta = 0.999999$ was used as an upper bound of the booking limit $x_{ub}$ because the optimal booking limit must be less than the summation of demand for the two classes. The response variable observed is the expected profit assuming that $x$ is given. The expectation is to observe different shapes of the expected profit distribution under various circumstances. For each case, the location of $x^*$ was recorded.

Different shapes of the expected profit were derived for two cases; no overbooking and overbooking. Figure 4.1 shows the results for no overbooking; i.e. $x^* < \kappa$. The maximum expected profit in the first piece (in Theorem 1) is greater than that in the second piece; i.e. $\pi(x') > \pi(x'')$. In Figure 4.1(d), the optimal booking limit is $x^* = 0$, which corresponds to the first case in (3.14); this means that class 2 reservations are not accepted when the mean demand for class 1 is large. In Figure 4.1(a-c), the optimal booking limit $x^*$ is given by the second case in (3.14). Note that as the mean demand of class 1 $\lambda_1$ decreases, the optimal booking limit increases.

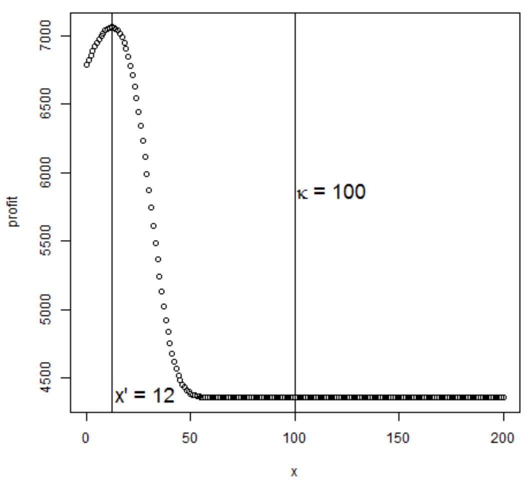
Figure 4.2 indicates the results of overbooking; i.e. $x^* > \kappa$. The maximum expected profit in the second piece (in Theorem 1) is greater than that in the first piece; i.e. $\pi(x'') > \pi(x')$. Note that as the denied boarding cost $h$ decreases, the optimal booking limit increases.



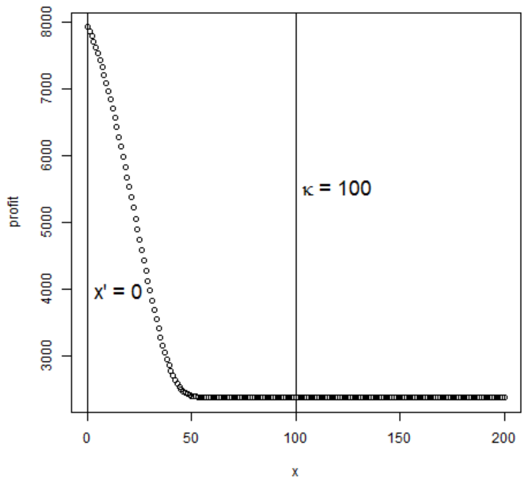

(a) $\lambda_1 = 40$




(b) $\lambda_1 = 60$

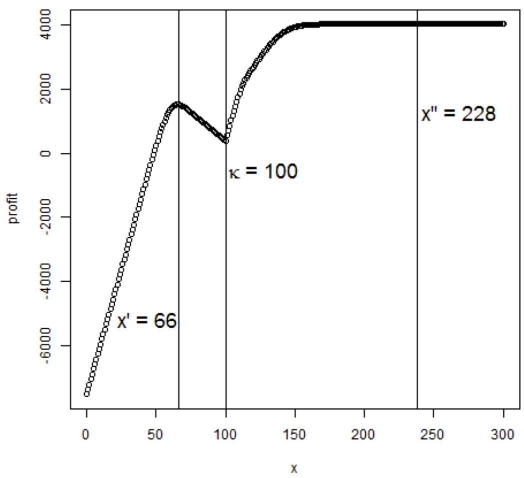



(c) $\lambda_1 = 80$

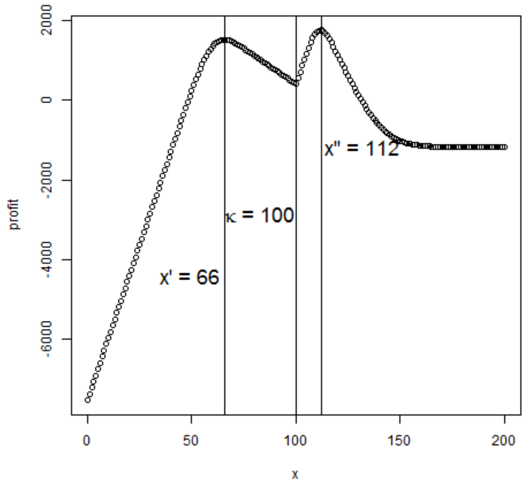



(d) $\lambda_1 = 100$

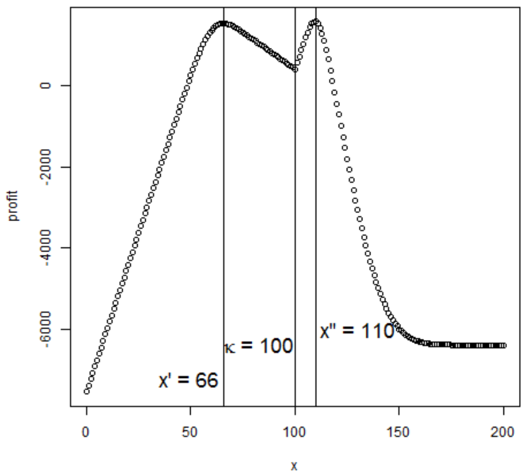
**Figure 4.1** The Expected Profit when $\lambda_1 = 40, 60, 80,$ and 100. Other Parameters are $p_1 = g_1 = 100, p_2 = g_2 = 20, r_1 = 50, r_2 = 10, \theta_1 = 0.9, \theta_2 = 0.7$ and $h = 300$.




(a) $h = 100$




(b) $h = 300$




(c) $h = 500$

**Figure 4.2** The Expected Profit when $h = 100, 300$, and 500. Other Parameters are $p_1 = g_1 = 100, p_2 = g_2 = 80, r_1 = 80, r_2 = 40, \theta_1 = 0.9, \theta_2 = 0.7$ and $\lambda_1 = 40$.

The properties of the local maximum point $x'$ were studied by varying the parameters in the proposed model. The first step was to study the effect of $\lambda_1$ on the local maximum point $x'$ by setting $p_1 = g_1 = 100, p_2 = g_2 = 20, r_1 = 0.8p_1, r_2 = 0.5p_2$, $\theta_1 = 0.9, \theta_2 = 0.9, h = 300$, and $\lambda_2 = 80$. The results are shown in Figure 4.3.

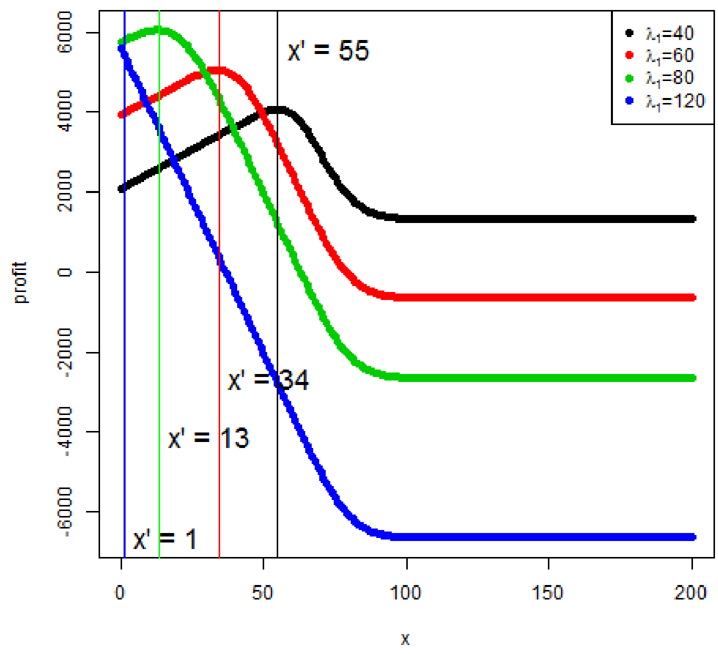



**Figure 4.3** The Expected Profit when $\lambda_1 = 40, 60, 80$, and 120.

From Figure 4.3, it can be seen that the local maximum point $x'$ increases when the mean demand for class 1 $\lambda_1$ decreases.

The next step was to study the effect of $p_2$ on the local maximum point $x'$ by setting $p_1 = g_1 = 100, p_2 = g_2$, $r_1 = 0.8p_1$, $r_2 = 0.5p_2$, $\theta_1 = 0.9$, $\theta_2 = 0.9$, $h = 300$, $\lambda_1 = 40$, and $\lambda_2 = 80$. The results depicted in Figure 4.4 show that the local maximum point $x'$ increases when the revenue from class 2 $p_2$ increases.

The effect of $\theta_2$ on the local maximum point $x'$ was studied by setting $p_1 = g_1 = 100$, $p_2 = g_2 = 20$, $r_1 = 0.8p_1$, $\theta_1 = 0.7$, $h = 100$, $\lambda_1 = 60$, and $\lambda_2 = 60$. The results are shown in Figure 4.5, in which we notice that the local maximum point $x'$ increases when the show-up probability for class 2 $\theta_2$ increases.

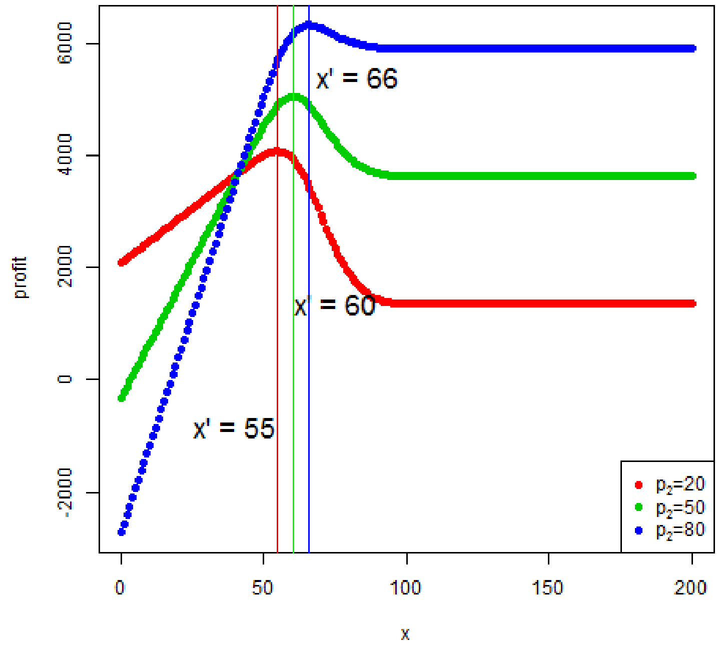



**Figure 4.4** The Expected Profit when $p_2 = 20, 50$, and 80.

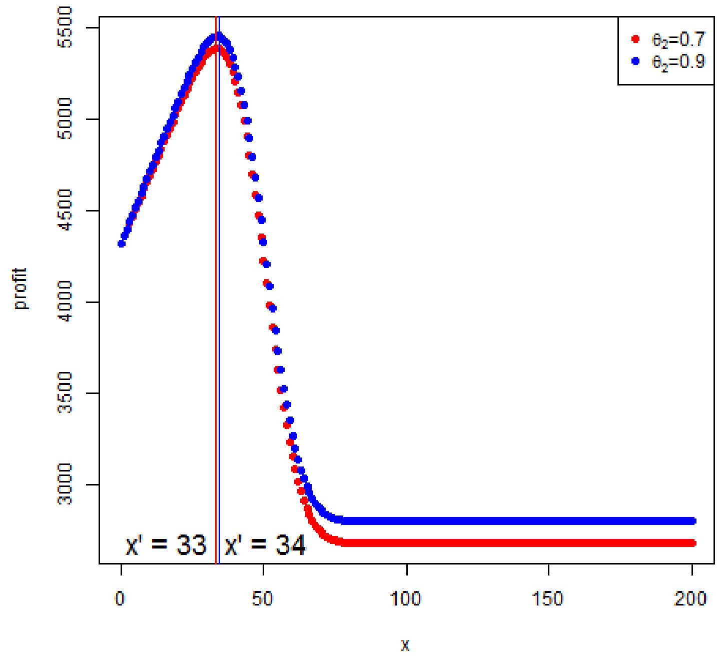



**Figure 4.5** The Expected Profit when $\theta_2 = 0.7$ and 0.9.

The last parameter in the first piece, the refund cost $r_2$ was tested by setting $p_1 = g_1 = 100,\ p_2 = g_2 = 50,\ r_1 = 0.8p_1,\ \theta_1 = 0.9,\ \theta_2 = 0.7,\ h = 300,$ and $\lambda_2 = 120$ to study its effect on the local maximum point $x'$. The results in Figure 4.6 show that the local maximum point $x'$ increases when the refund cost for class 2 $r_2$ decreases.

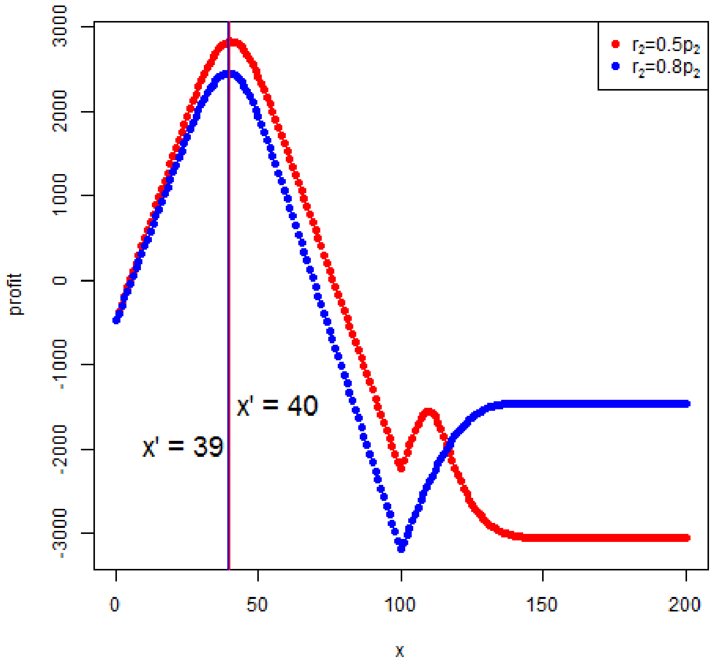



**Figure 4.6** The Expected Profit when $r_2 = 0.5p_2$ and $r_2 = 0.8p_2$.

Figures 4.4 – 4.6 show that when the revenue $p_2$ for class 2 increases, show-up probability $\theta_2$ for class 2 increases, or refund cost for class 2 decreases, the local maximum point $x'$ increases; these results agree with Corollary 1. Moreover, we can see that the expected profit at the local maximum point $x'$ also increases when the revenue $p_2$ for class 2 increases, show-up probability $\theta_2$ for class 2 increases, or refund cost for class 2 decreases. We can extend Corollary 1 to confirm the relationship between the model parameters and the expected profit at the local maximum point $x'$ in the following manner.

**Corollary 3.** For $x = 1, ..., \kappa - 2$, and $P(D_1 > \kappa - 1) \le \alpha_2 / \alpha_1 \le P(D_1 > 0)$ :

1) Suppose that $\alpha_1$ decreases when the other parameters are fixed, then the local maximum point $x'$ increases and the expected profit at the local maximum point decreases.

2) Suppose that $\alpha_2$ increases when the other parameters are fixed, then the local maximum point $x'$ increases and the expected profit at the local maximum point increases.

**Proof** From Lemma 1, the expected profit when $x = 1, ..., \kappa - 2$ is

$$\pi(x) = \alpha_1 E[B_1(x)] + \alpha_2 E[B_2(x)] - \sum_{i=1}^{2} g_i E[D_i].$$

Let $\alpha_1 > \hat{\alpha}_1$ and the other parameters be fixed, then the expected profit with respect to $\hat{\alpha}_1$ is

$$\hat{\pi}(x) = \hat{\alpha}_1 E[B_1(x)] + \alpha_2 E[B_2(x)] - \sum_{i=1}^{2} g_i E[D_i].$$

Obviously, the expected profit function is increasing in $\alpha_1$, i.e.

$$\pi(x) > \hat{\pi}(x) \qquad \forall x \in \{1, 2, ..., \kappa - 2\}. \tag{4.1}$$

Suppose that $P(D_1 > \kappa - 1) \le \alpha_2 / \alpha_1 \le P(D_1 > 0)$ and the local maximum point $x'$ exists (by Theorem 1), i.e.

$$\pi(x') > \pi(x), \quad \forall x \in \{0, 1, ..., \kappa - 2\}, \tag{4.2}$$

and let $\hat{x}'$ be the local maximum point with respect to $\hat{\alpha}_1$.

From Corollary 1, if $\alpha_1$ decreases, then the local maximum point $x'$ increases, and so $x' < \hat{x}'$.

From (4.2), we obtain

$$\pi(x') > \pi(\hat{x}'). \tag{4.3}$$

From (4.1), we obtain

$$\pi(\hat{x}') > \hat{\pi}(\hat{x}'). \tag{4.4}$$

From (4.3) and (4.4), we can conclude that $\pi(x') > \hat{\pi}(\hat{x}')$.

Let $\alpha_2 < \tilde{\alpha}_2$ and the other parameters be fixed, then the expected profit with respect to $\tilde{\alpha}_2$ is

$$\tilde{\pi}(x) = \alpha_1 E[B_1(x)] + \tilde{\alpha}_2 E[B_2(x)] - \sum_{i=1}^{2} g_i E[D_i].$$

Obviously, the expected profit function is increasing in $\alpha_2$, i.e.

$$\pi(x) < \tilde{\pi}(x) \qquad \forall x \in \{1, 2, ..., \kappa - 2\}. \tag{4.5}$$

Let $\tilde{x}'$ be a local maximum point with respect to $\tilde{\alpha}_2$.

From Corollary 1, if $\alpha_2$ increases, then the local maximum point $x'$ increases, and so $x' < \tilde{x}'$.

From (4.5), we obtain

$$\pi(x') < \tilde{\pi}(x'). \tag{4.6}$$

From (4.2), we obtain

$$\tilde{\pi}(x') < \tilde{\pi}(\tilde{x}'). \tag{4.7}$$

From (4.6) and (4.7), we can conclude that $\pi(x') < \tilde{\pi}(\tilde{x}')$. □

If overbooking is not applied, then there are many cases that increase the expected profit with corresponding $\alpha_i$ increases, e.g. 1) the cost of revenue for class *i* increases, 2) the penalty cost for class *i* increases, 3) the show-up probability for class *i* increases, or 4) the refund cost for class *i* decreases. All cases resulting in an increase in the optimal booking. Note that an increasing in the optimal booking limit does not imply the expected profit at this optimal booking limit increasing.

Properties of the local maximum point $x''$ were studied by varying the parameters for class 2 and denied boarding cost in the proposed model. The first step was to study the effect of $p_2$ on the local maximum point $x''$ by setting $p_1 = g_1 = 100$, $g_2 = p_2$, $r_1 = 0.8p_1$, $r_2 = 0.5p_2$, $\theta_1 = 0.9$, $\theta_2 = 0.7$, $h = 500$, $\lambda_1 = 40$, and $\lambda_2 = 140$;

the results are shown in Figure 4.7, in which we can see that the local maximum point $x''$ increases when the cost of revenue $p_2$ for class 2 increases.

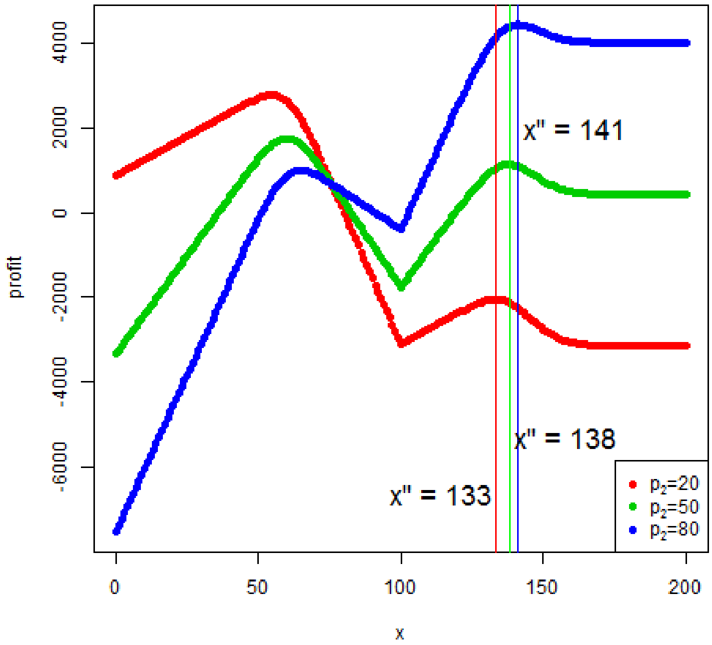



**Figure 4.7** The Expected Profit when $p_2 = 20, 50$, and 80.

The next step was to study the effect of the show-up probability $\theta_2$ on the local maximum point $x''$ by setting $p_1 = g_1 = 100$, $p_2 = g_2 = 80$, $r_1 = 0.8p_1$, $r2 = 0.5p_2$, $\theta_1 = 0.9$, $h = 500$, $\lambda_1 = 40$, and $\lambda_2 = 140$. The results in Figure 4.8 show that the local maximum point $x''$ increases when the show-up probability $\theta_2$ for class 2 decreases.

The effect of the refund cost $r_2$ on the local maximum point $x''$ was tested by setting $p_1 = g_1 = 100, p_2 = g_1 = 80, r_1 = 0.8p_1, \theta_1 = 0.9, \theta_2 = 0.7, h = 300, \lambda_1 = 40,$ and $\lambda_2 = 140$. The results are shown in Figure 4.9 and indicate that the local maximum point $x''$ increases when the refund cost $r_2$ for class 2 decreases.

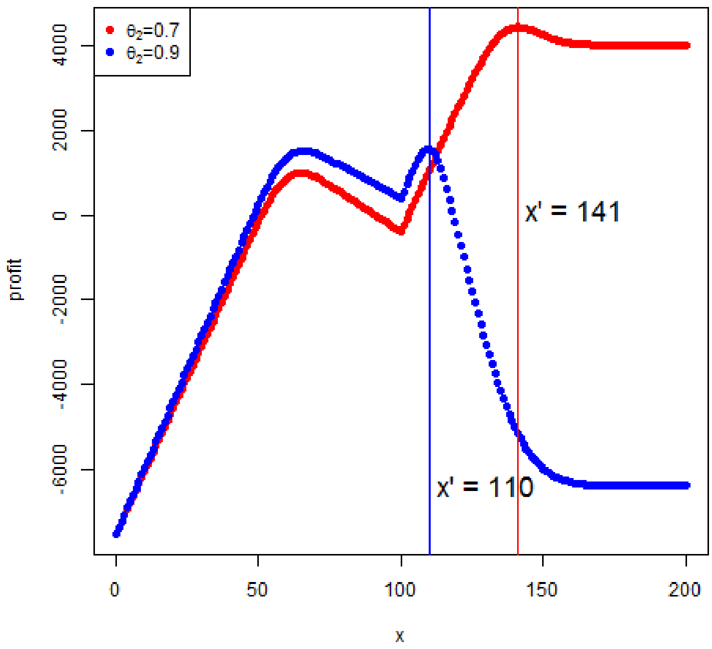



**Figure 4.8** The Expected Profit when $\theta_2 = 0.7$ and 0.9.

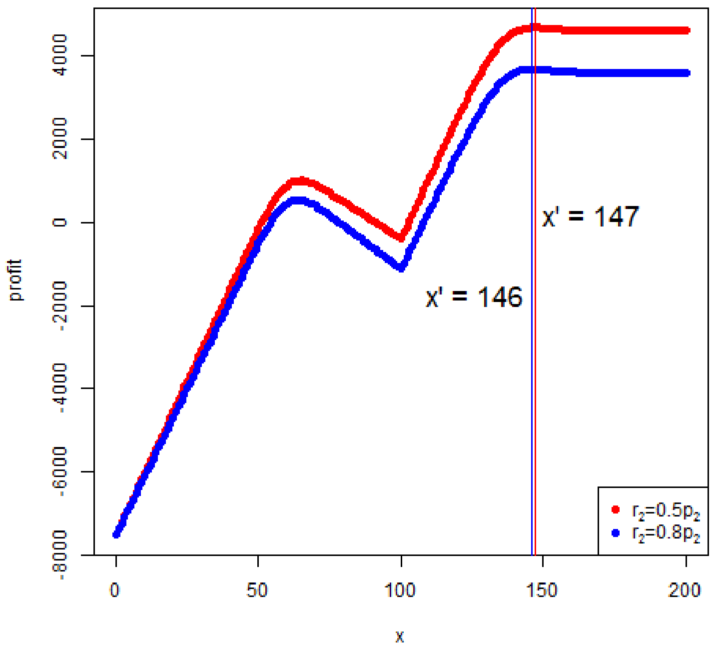



**Figure 4.9** The Expected Profit when $r_2 = 0.5p_2$ and $r_2 = 0.8p_2$.

Lastly, the effect of the denied boarding cost $h$ on the local maximum point $x''$ was examined by setting $p_1 = g_1 = 100$, $p_2 = g_2 = 80$, $r_1 = 0.8p_1$, $r_2 = 0.5p_2$, $\theta_1 = 0.9$, $\theta_2 = 0.7$, $\lambda_1 = 40$, and $\lambda_2 = 140$. From the results shown in Figure 4.10, we can see that the local maximum point $x''$ increases when the denied boarding cost $h$ decreases.

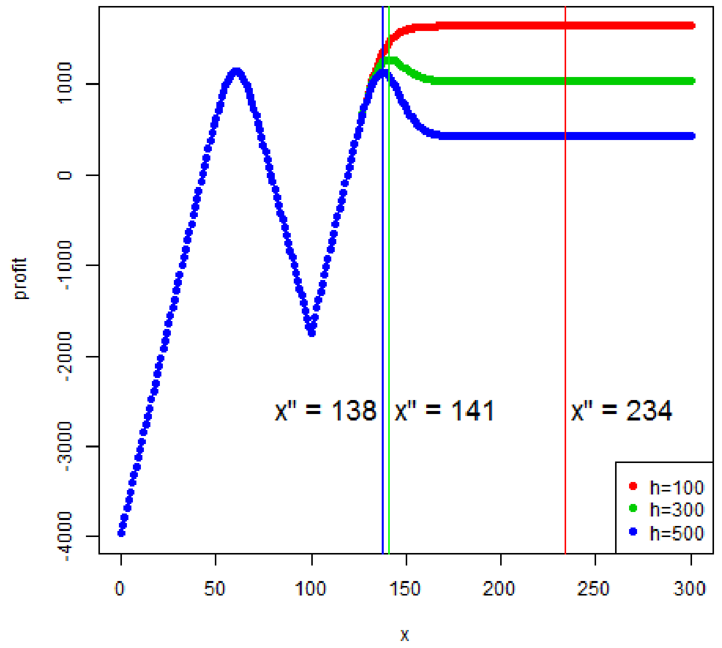



**Figure 4.10** The Expected Profit when $h = 100, 200$, and 300.

Figures 4.7 – 4.10 show that when the revenue $p_2$ for class 2 increases, the show-up probability $\theta_2$ for class 2 decreases, the refund cost for class 2 decreases, or the denied boarding cost $h$ decreases, the local maximum point $x''$ increases; these results are in accordance with Corollary 2. Moreover, we can see that the expected profit at the local maximum point $x''$ also increases when the revenue $p_2$ for class 2 increases, the show-up probability $\theta_2$ for class 2 decreases, the refund cost for class 2 $r_2$ decreases, or the denied boarding cost $h$ decrease. We can extend Corollary 2 to confirm the relationship between the model parameters and the expected profit at the local maximum point $x''$ in the following manner.

**Corollary 4.** For $x=\kappa,\kappa+1,\ldots$ and $0<\alpha_2/(h\theta_2)\le\bar{F}(\kappa-1;x,\theta_2)$, suppose that $\alpha_2$ increases with fixed $\theta_2$ and other parameters. Then, the local maximum point $x''$ increases and the expected profit at the local maximum point increases.

**Proof** From Lemma 1, the expected profit when $x=\kappa,\kappa+1,\ldots$ is

$$\pi(x)=(\alpha_2-h\theta_2)E[B_2(x)]+\alpha_1E[B_1(x)]-\sum_{i=1}^{2}g_iE[D_i]+h\sum_{t=0}^{\kappa-1}P(W_2(B_2(x))>t).$$

Let $\alpha_2<\tilde{\alpha}_2$, $\theta_2$ and the other parameters be fixed, then the expected profit with respect to $\tilde{\alpha}_2$ is

$$\tilde{\pi}(x)=(\tilde{\alpha}_2-h\theta_2)E[B_2(x)]+\alpha_1E[B_1(x)]-\sum_{i=1}^{2}g_iE[D_i]+h\sum_{t=0}^{\kappa-1}P(W_2(B_2(x))>t).$$

Obviously, the expected profit function is increasing in $\alpha_2$, i.e.

$$\pi(x)<\tilde{\pi}(x)\qquad\forall x\in\{\kappa,\kappa+1,\ldots\}\tag{4.8}$$

Suppose that $0<\alpha_2/(h\theta_2)\le\bar{F}(\kappa-1;x,\theta_2)$ and the local maximum point $x''$ exists (by Theorem 1), i.e.

$$\pi(x'')>\pi(x),\quad\forall x\in\{\kappa,\kappa+1,\ldots\},\tag{4.9}$$

and let $\tilde{x}'$ be the local maximum point with respect to $\tilde{\alpha}_2$.

From Corollary 2, if $\alpha_2$ increases, then the local maximum point $x''$ decreases, and so $x''>\tilde{x}''$.

From (4.8), we obtain

$$\pi(x'')<\tilde{\pi}(x'').\tag{4.10}$$

From (4.9), we obtain

$$\tilde{\pi}(x'')<\tilde{\pi}(\tilde{x}'').\tag{4.11}$$

From (4.10) and (4.11), we can conclude that $\pi(x'')<\tilde{\pi}(\tilde{x}'')$. □

If overbooking is allowed, there are many cases that increase the expected profit with a corresponding $\alpha_2$ increase, e.g. 1) the cost of revenue for class 2 increases, 2) the penalty cost for class 2 increases, or 3) the refund cost for class 2 decreases.

## 4.2 Discussions of the Simulation Study

Some managerial insights that can be drawn from the above numerical experiments are as follows. From Figure 4.1, the airline should not overbook when mean demand for class 1 is higher than capacity $(\lambda_1 \geq \kappa)$, or the mean demand for class 2 is much lower than capacity $(\lambda_2 << \kappa)$. The optimal booking limit is zero when the mean demand of class 1 is high. When the mean demand for class 1 exceeds or equals capacity, the airline should reject all class 2 passengers and accept only class 1. The optimal booking limit is the local maximum point $x'$ when the sum of mean demand of the two classes is approximately equal to the capacity $(\lambda_1 + \lambda_2 \approx \kappa)$; The airline has the remaining capacity $\kappa - x'$ to accept class 1 customers. From Figure 4.2, overbooking occurs when class 2 revenue is close to class 1 revenue and the mean demand for class 2 is much higher than capacity $(\lambda_2 >> \kappa)$, and the airline should accept all class 2 passengers up to the local maximum point $x''$; in most cases, all passengers in these flights are class 2.

## 4.3 Details of the Real-life Data

Information on passengers from two flights between Bangkok and Phuket on every Sunday in 2014 was received from an airline. The first flight, flight A, is scheduled for Sunday mornings and the second flight, flight B, is scheduled for Sunday evenings. These data contain the number of reservations and the number of show-up passengers at flight departure time. The airline uses an airplane that has 162 available seats. The revenue for this airline is classified into eleven classes. The percentages of passengers classified by revenue are shown in Table 4.1.

**Table 4.1** Percentage of Passengers from 2014 Reclassified by Revenue.

| Class | Revenue | Percent |
|---|---|---|
| Y | 4675 | 10.74 |
| M | 4350 | 4.97 |
| K | 3850 | 13.82 |
| N | 3500 | 3.72 |
| T | 3050 | 20.47 |
| L | 2800 | 5.27 |
| H | 2550 | 8.02 |
| Q | 2100 | 4.47 |
| V | 1900 | 9.23 |
| G | 1690 | 12.58 |
| B | 945 | 6.71 |

**Note:** The percentages of passengers in Table 4.1 was received from the airline company in Thailand. The name of the company has not been disclosed to protect its and interviewees' privacy.

In this experiment, the eleven classes were reclassified into two classes according to the proposed model and the revenue for the two classes calculated using a weighted average mean. All classes except class B were grouped into class 1 and class B assigned to class 2. Using a weighted average mean, the revenue for class 1, $p_1$, was 3043 baht and that for class 2, $p_2$, 945 baht. The number of reservations from real-life data were assumed to be the demand for the flight. This demand was partitioned into two classes by using the demand function for airline industry in Thailand proposed by Komsan Suriya (2009).

The demand function for the airline industry in Thailand given that $q_i$ is the demand for class *i*, for $i = 1, 2$.

Class 1 demand function

$$p_1 = 3588 - 188.605 q_1. \tag{4.12}$$

Class 2 demand function

$$p_2 = 1763 - 188.605 q_2. \tag{4.13}$$

After substituting $p_1 = 3043$ in (4.12) and $p_2 = 945$ in (4.13), we obtain the demand for class 1, $q_1$, as 4.6 million persons and the demand for class 2, $q_2$, as 6.9 million persons. The proportion of class 1 ($\upsilon_1$) and class 2 ($\upsilon_2$) passengers is

$$\upsilon_1 = 0.4 \text{ and } \upsilon_2 = 0.6. \tag{4.14}$$

We used the above proportions to partition the number of reservations into two classes.

Properties of the Expected Profit Function for Real-life Data

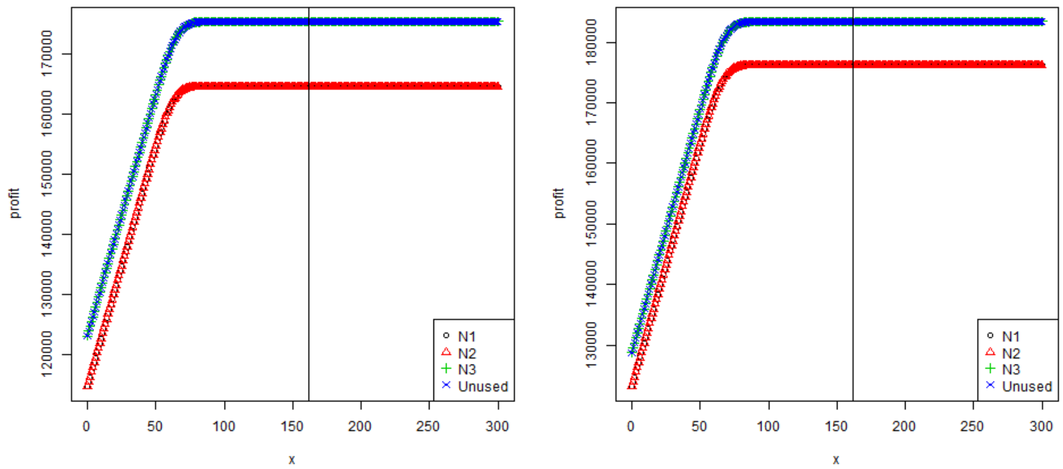
The properties of the expected profit function were studied by setting a numerical experiment with the real-life data detailed above. This experiment was set to find the shape of the expected profit function and the position of the optimal booking limit. To increase accuracy, the data received from the airline was uncensored using an unconstraining method: N1, N2, or N3. After that, the data was partitioned into two classes using the proportions in $(4.14)$; the estimated mean demand of the two classes are shown in Table 4.2.

**Table 4.2** Estimated Mean Demand of the Two Classes after using Unconstraining Methods

| **Unconstraining Method** | **Flight A** | | **Flight B** | |
|---|---|---|---|---|
| | $\hat{\lambda}_1$ | $\hat{\lambda}_2$ | $\hat{\lambda}_1$ | $\hat{\lambda}_2$ |
| N1 | 41 | 62 | 44 | 66 |
| N2 | 41 | 62 | 44 | 66 |
| N3 | 44 | 65 | 46 | 68 |
| Unused | 44 | 65 | 46 | 68 |

In the numerical experiment, the show-up probability for class $i$ $\theta_i$ was set at four different levels; 0.7, 0.8, 0.9, and 0.95. The set of all show-up probabilities was $\theta_1 \times \theta_2$, and the set of all refunded costs was $r_1 \times r_2$, where $r_i \in \{0.5p_i, 0.8p_i\}$. The denied boarding cost $h$ was set at three different levels: 1500, 2000, and 2500 baht, and the penalty cost $(g_i)$ for the two classes was assumed to be zero because the number of rejected customers was not available. The capacity $\kappa$ of these flights was 162 and the revenue for class 1 and class 2 after reclassification was $p_1 = 3043$ and $p = 945$. A plot of the expected profit when given $x$ from 0 to 300 was carried out. 192 sets of conditions were applied.

Similar shapes of the expected profit function when varying the parameters are shown in Figure 4.11. The two graphs only show two lines because the expected profit using N1 and N2 are equal, and the expected profit using N3 and an unused unconstraining method are equal. The expected profit function of real-life data has the same shape as Figure 4.1 (a) with simulated data, i.e. the global maximum point occurs at the local maximum point $x'$. This means that the airline should not overbook for the two flights.




(a) Flight A (b) Flight B

**Figure 4.11** The Expected Profit Function using Real-life Data for Parameters $p_1 = 3043, p_2 = 945, g_1 = g_2 = 0, r_1 = 2434.4, r_2 = 472.5, \theta_1 = 0.9, \theta_2 = 0.7,$ and $h = 1,500.$

## 4.4 Numerical Experiments with Real-life Data

Numerical experiments with real-life data were partitioned into two parts. The first part was carried out to evaluate the performance of the policy from the proposed model. The second part was carried out to test three hypotheses: 1) a different number of update booking limit points affects profit, 2) an incorrect initial mean demand affects profit, and 3) a different smoothing constant $(\rho)$ used in exponential smoothing affects profit.

### 4.4.1 Comparing the Performance of This Study's Policy to the Airline's Policy

The airline's policy has a fixed booking limit $(x_A)$ for all periods, and so it was assumed that the airline set booking limits $x_A$ are 9, 17, 41, 81, 122, and 171, which are 5%, 10%, 25%, 50%, 75%, and 105% of capacity, respectively. The performance of this study's policy using an optimal booking limit $x^*$ was compared to the airline's policy of a fixed booking limit $x_A$.

The capacity $\kappa$ from real-life data was 162 and revenues for class 1 and class 2 after reclassification were $p_1 = 3043$ and $p = 945$ baht, respectively, were used in this experiment. Since, the number of rejected customers was not available, the penalty cost $(g_i)$ of the two classes was assumed to be zero. The refund cost for class $i$ $(r_i)$ is a proportion $(\gamma_i)$ of the cost of revenue, which is the same assumption as with the numerical experiments using simulated data. The set of all refund costs is $r_1 \times r_2$, where $r_i \in \{0.5p_i, 0.8p_i\}$. The denied boarding cost was set at three different levels: 1500, 2000, and 2500 baht, corresponding to the class 2 revenue for this airline. The show-up probability for class $i$ $(\theta_i)$ was set at four different levels: 0.7, 0.8, 0.9, and 0.95, and the set of all show-up probabilities is given by $\theta_1 \times \theta_2$. 192 sets of conditions were run to compare this study's policy to the airline's policy. This study's profit is calculated using

$$\hat{\pi}(x^*) = \sum_{i=1}^{2}\left[ p_i B_i(x^*) - r_i(B_i(x^*) - W_i(B_i(x^*)))\right] - h\left(W_2(B_2(x^*)) - \kappa\right)^+ . \quad (4.15)$$

The airline's profit is calculated using

$$\hat{\pi}(x_A) = \sum_{i=1}^{2}\left[ p_i B_i(x_A) - r_i(B_i(x_A) - W_i(B_i(x_A)))\right] - h\left(W_2(B_2(x_A)) - \kappa\right)^+ . \quad (4.16)$$

The steps to compare the performance between the two policies were as follows:

1) Randomly partition the real-life data into training and testing sets. A training set comprises 75% of the data and the testing set 25%.

2) For the training set:

(1) Uncensor data using one of the N1, N2, or N3 methods.

(2) Partition data into two classes using the proportions of class 1 and class 2 passengers in (4.14).

(3) Estimate the mean demand of the two classes.

(4) Calculate the optimal booking limit $x^*$ of the proposed model using \ Theorem 1.

3) For the testing set:

(1) Partition data into two classes using the proportions of class 1 and class 2 passengers to be the same as the training set.

(2) Use the optimal booking limit $x^*$ from the training set to calculate this study's profit from (4.15) and use fixed booking limit $x_A$ to calculate the airline's profit from (4.16).

(3) Average the profit for both policies separately.

4) Repeat all steps for 1,000 iterations

5) Average the profit from 1,000 iterations.

6) Compare the average profit between this study's policy and the airline's policy.

Figure 4. 12 shows the steps to compare the performance of this study's policy to the airline's policy.

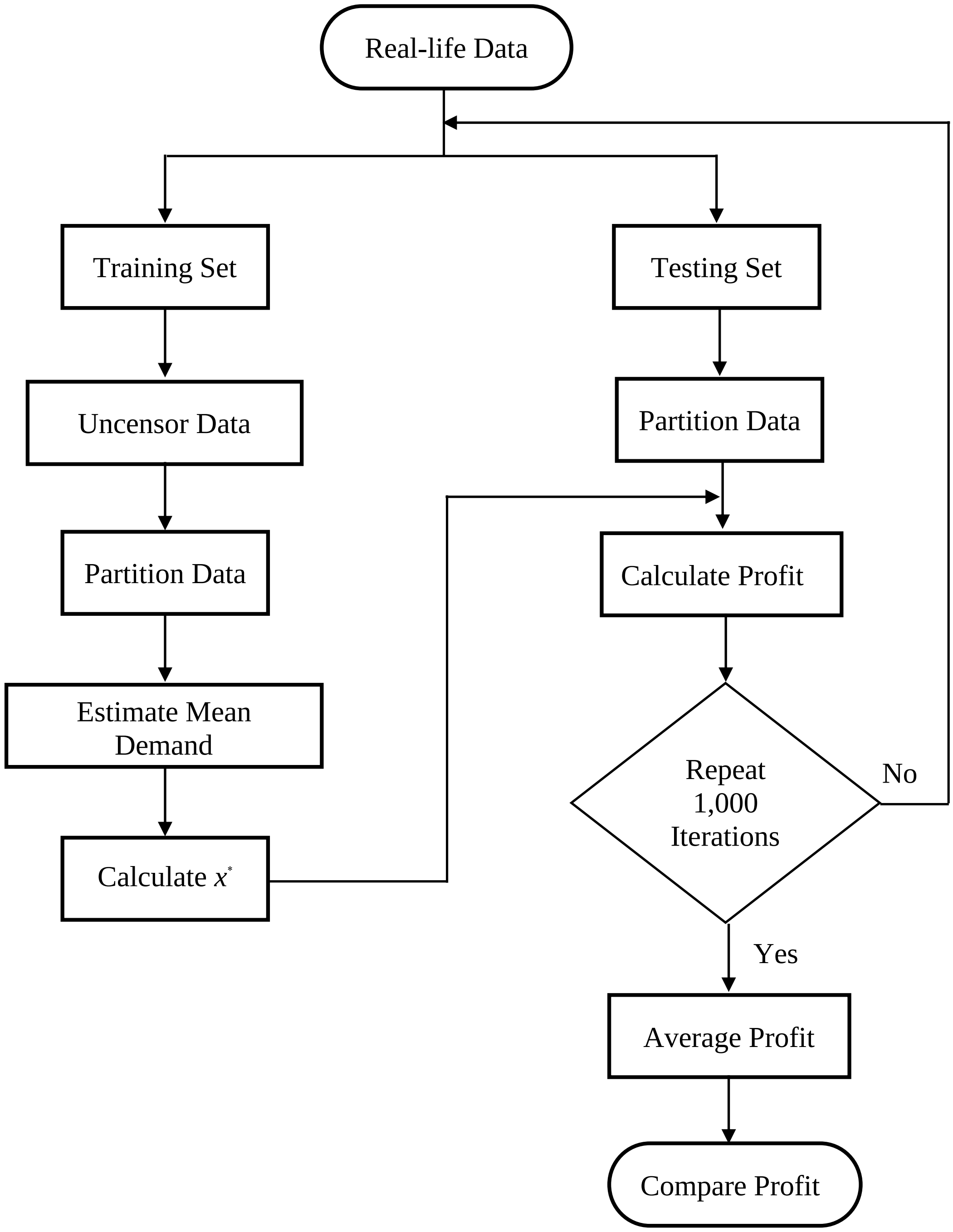



**Figure 4.12** Steps to Compare the Performance of This Study's Policy to the Airline's Policy.

### 4.4.2 Results of the Performance Comparison between the Policy from the Proposed Model and the Airline's Policy

In this experiment, the optimal booking limit $x^*$ of two flight occurs at the local maximum point $x'$, which means that the airline should not overbook these two flights. The optimal booking limit agrees with the property of real-life data in Section 4.3. The optimal booking limit under different parameter settings are only slightly different. The average of optimal booking limit in all cases of parameter setting are shown in Table 4.3.

**Table 4.3** The Average Optimal Booking Limit $x^*$

| **Unconstraining Method** | **Flight A** | **Flight B** |
|---|---|---|
| N1 | 117.61 | 114.46 |
| N2 | 117.88 | 114.51 |
| N3 | 115.40 | 113.24 |
| Unused | 115.40 | 113.24 |

To improve performance, three simple unconstraining methods were used to uncensor data before demand forecasting. Tables A.1 and A.2 in Appendix A display the profit using the different unconstraining methods and the different fixed booking limits of the airline when $p_1 = 3043$, $p_2 = 945$, $g_1 = g_2 = 0$, $r_1 = 1521.5$, $r_2 = 472.5$, $\theta_1 = 0.9$, $\theta_2 = 0.7$, and $h = 1500$. The profit shown with different unconstraining method only differs by a small amount. The N2 method provided the highest profit while the N1 method provided the lowest. Other cases of parameter settings gave similar results.

The performance between the policy from the proposed model and the airline's policy was compared in terms of profit. This study's policy uses $x^*$ as the booking limit whereas the other booking limits, $x_A = 9, 17, 41, 81, 122, 171$, were assumed to be the airline's policy. The minimum optimal booking limits $x^*$ for N1, N2, and N3 were

116.615, 116.885, and 113, respectively, and the maximum optimal booking limits $x^*$ for N1, N2, and N3 were 119.175, 119.546, and 116.528, respectively. If $x^* > x_A$, then the profit of this study's policy is higher than that of the airline' s policy, which means that the airline loses some profit per flight when they use a low booking limit, and loses even more profit per flight when $x^* >> x_A$. If $x^* < x_A$, then the profit of this study's policy and airline's policy are equal (see Tables A.1 and A.2 in Appendix A).

The difference in profit per flight between this study's policy and the airline's policy that are equal in all combination of denied boarding cost ($h$) and show-up probability ($\theta_i$) are shown in Tables A.3 and A.4 in Appendix A. The loss of profit per flight per year when the airline does not use an optimal booking limit are shown in Tables A.5 and A.6 in Appendix A. The results show that if the airline uses a booking limit that is too low, then it will lose more profit per year.

Nevertheless, the airline does not lose profit when they set a high booking limit (see Tables A.3-A.6), although this does not necessarily mean that the airline should set too high a booking limit to avoid losing profit. It is worth noting that the loss of profit in Tables A.3-A.6 has only been shown to occur with this real-life data; this may not be the case with other real-life data examples.

The effect of varying model parameters was also examined in this experiment. The denied boarding cost did not affect the profit for these real-life data, as shown in Tables A.7 and A.8 in Appendix A. Since there was no overbooking, the passengers who showed up were always less than capacity, i.e. no denied boarding passengers.

The profit when varying show-up probability $\theta_1$ of class 1 is shown in Tables A.9 and A.10 in Appendix A. When the show-up probability $\theta_1$ of class 1 decreases, the profit decreases for all cases of parameter settings. Since the show-up probability $\theta_1$ of class 1 decreases, $\alpha_1$ also decreases, which implies that profit decreases; these findings correspond to Corollary 3. In the same manner, as the show-up probability $\theta_2$ of class 2 increases, profit increases in all cases of parameter settings (see Tables A.11 and A.12), which is also in accordance with Corollary 3.

The last parameter with varied values in this experiment was refund cost. If refund cost for class $i$ $(r_i)$ increases, then profit decreases in all cases of parameters, which also corresponds with Corollary 3. The profit when varying refund cost for class 1 and class 2 is shown in Tables A.13 and A.14, respectively, in Appendix A.

## 4.5 Testing Hypotheses

It is expected that the number of update booking limit points, incorrect initial mean demand, and different smoothing constants affect profit. Hypotheses for testing these scenarios are as follows:

Hypothesis 1: The number of update booking limit points affects profit.

Hypothesis 2: Incorrect initial mean demand affects profit.

Hypothesis 3: The choice of smoothing constant in exponential smoothing affects profit.

The previous section shows that varying the denied boarding cost $h$ does not affect profit. $h = 2000$ was used to test all hypotheses. The penalty cost for the two classes was set to zero, as in the previous experiment. The capacity was 162 and the revenues for classes 1 and 2 were 3043 and 945 baht, respectively. In a real-life situation, the refund cost for the two classes is fixed and so, in this study, the refund cost for class 1, $r_1$, was fixed at 2434.4 baht and that for class 2, $r_2$, at 472.5 baht. The show-up probability for class $i$ was set at four different levels: 0.7, 0.8, 0.9, and 0.95 because show-up probability for class $i$ cannot be controlled by the airline. The set of all show-up probability $\theta_1 \times \theta_2$ was set in a randomized complete block design (RCBD). Since 1000 iterations were required to generate demand for the two classes and to calculate profit, which requires a great deal of time, it was thought that RCBD could reduce the time needed to test these hypotheses. Variability occurring from the show-up probability is also reduced by RCBD. Assumptions of the analysis of variance (ANOVA) are independence of observations, normality, and equality of variance. All of these assumptions were checked before all cases of hypotheses testing. To test all hypotheses, the significance level for selection was $\alpha = 0.05$.

### 4.5.1 Description of Hypothesis 1

In practice, most airlines update the booking limit before departure on a monthly basis for at least six months and update it every day in the last week before departure (Phillips, 2005: 135-136). For this hypothesis, we set four different cases of update booking limit points. In the first case, the booking limit was updated every day in the last week before departure; this case has seven points to update the booking limit and is referred to as 7 points. In the second case, the booking limit was the same as is used in practice, i.e. every month for six months before departure and every day in the last week before departure; this case has thirteen points to update the booking limit and is referred to as 13 points. In the third case, the booking limit was updated every three months before departure more than six months in advance and then the same as the second case; this case has fourteen points to update the booking limit and is referred to as 14 points. In the last case, the booking limit was updated every month before departure and every day in the last week before departure; this case has eighteen points to update the booking limit and is referred to as 18 points.

For $i = 1, 2$, let $b_i$ be the number of class i reservations at the departure time and $W_i(b_i)$ be the number of class i show-ups. The profit is calculated as

$$\hat{\pi} = \sum_{i=1}^{2} \left[ p_i b_i - r_i (b_i - W(b_i)) \right] - h\left(W_2(b_2) - \kappa\right)^+ . \qquad (4.17)$$

Let $t$ be the number of days before departure and $x_t^*$ be the optimal booking limit at the update booking limit point $t$ days before departure. All points that update the booking limit for the four cases are shown in Table 4.4.

**Table 4.4** The Update Booking Limit Points for the Four Cases

| Case | $t$ |
|---|---|
| 7 points | 6, 5, 4, 3, 2, 1, 0 |
| 13 points | 180, 150, 120, 90, 60, 30, 6, 5, 4, 3, 2, 1, 0 |
| 14 points | 270, 180, 150, 120, 90, 60, 30, 6, 5, 4, 3, 2, 1, 0 |
| 18 points | 330, 300, 270, 180, 150, 120, 90, 60, 30, 6, 5, 4, 3, 2, 1, 0 |

Furthermore, the performance of the study's policy was compared to first come first serve (FCFS) and no update booking limit policies. In the FCFS case, booking requests are accepted for each customer as they arrive in order until the flight is full. In the no update booking limit case, the optimal booking limit is used in the initial step ( $x^*_{360}$ ) for all periods to accept booking requests by customers. This means that class 2 customer are accepted up to $x^*_{360}$ and booking requests from class 1 customers are accepted up to the remaining capacity $\kappa - x^*_{360}$.

The notations of hypothesis testing for flight A and flight B are set as follows:

Hypothesis testing for flight A

$$H_0 : \mu_{A(FCFS)} = \mu_{A(no\ update)} = \mu_{A(7)} = \mu_{A(13)} = \mu_{A(14)} = \mu_{A(18)}$$

$$H_1 : \mu_{A(i)} \neq \mu_{A(j)} \text{ at least for } i \neq j$$

Hypothesis testing for flight B

$$H_0 : \mu_{B(FCFS)} = \mu_{B(no\ update)} = \mu_{B(7)} = \mu_{B(13)} = \mu_{B(14)} = \mu_{B(18)}$$

$$H_1 : \mu_{B(i)} \neq \mu_{B(j)} \text{ at least for } i \neq j$$

where

$\mu_{k(FCFS)}$ is the mean profit of flight *k* when the airline uses the FCFS policy,

$\mu_{k(no\ update)}$ is the mean profit of flight *k* when the airline does not use an update booking limit policy, and

$\mu_{k(i)}$ is the mean profit of flight *k* when the airline updates the booking limit by *i* points,

for $k = A, B$ and $i = 7, 13, 14, 18$.

The steps to generate data to test this hypothesis are as follows:

1) Use the real-life data to forecast the demand for the next flight $(\hat{\lambda})$ by using an exponential smoothing method with a smoothing constant determined by minimizing the sum of squared errors.

2) Partition the forecasted demand into two classes using the proportion of class 1 and class 2 passengers in (4.14). The mean demand of class 1 and class 2 are $\hat{\lambda}_{1,360}$ and $\hat{\lambda}_{2,360}$.

3) Generate the demand for class 1 and class 2 over 360 days, which is assumed to be a Poisson distribution with means $\hat{\lambda}_{1,360}$ and $\hat{\lambda}_{2,360}$, respectively.

4) Calculate the optimal initial booking limit $x^*_{360}$ using Theorem 1.

5) Calculate the number of class 1 and class 2 reservations every day using $x^*_{360}$.

6) Recalculate the optimal booking limit $x^*_t$ at update booking limit point *t* and forecast the demand for the two classes $(\hat{\lambda}_{1,t}, \hat{\lambda}_{2,t})$ over the remaining days before departure.

7) Use the forecasted demand for the two classes to calculate a new optimal booking limit $x^*_t$.

8) At the time of departure, generate the number of show-up passengers following binomial distribution with a mean equal to the number of class *i* reservations and the show-up probability is $\theta_i; i = 1, 2$, and calculate the profit using equation (4.17).

9) Repeat all steps for 1,000 iterations.

10) Average the profit from 1,000 iterations.

11) Test this hypothesis.

Figure 4.13 shows the steps to test hypothesis 1.

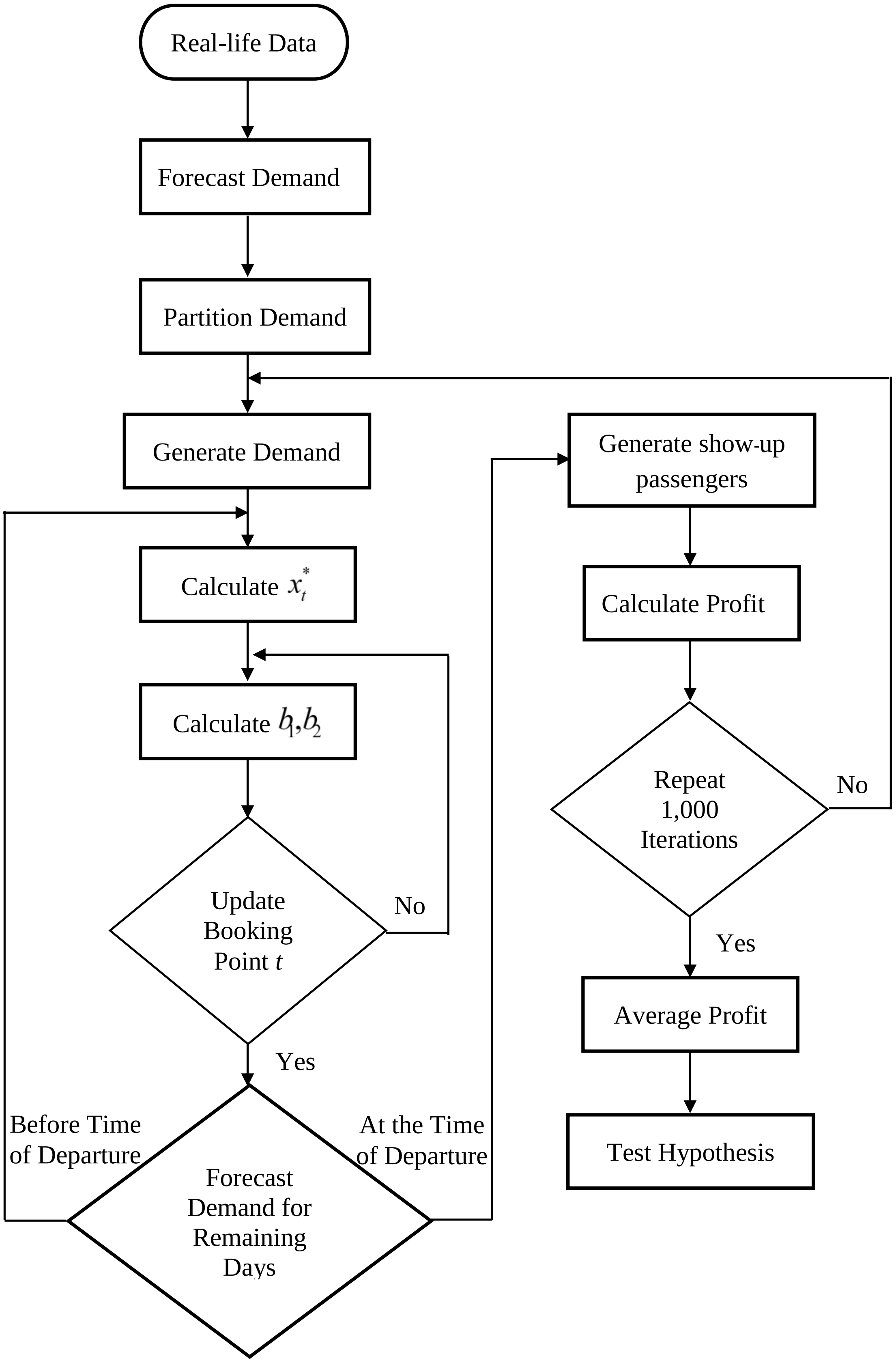



**Figure 4.13** Steps to Test Hypothesis 1.

### 4.5.2 Results for Hypothesis 1

The data used to test this hypothesis for flight A and flight B are shown in Tables A.15 and A.16 in Appendix A, respectively. The hypotheses for testing for flight A and flight B are shown below.

Hypothesis testing for flight A

$$H_0 : \mu_{A(FCFS)} = \mu_{A(Not-update)} = \mu_{A(7)} = \mu_{A(13)} = \mu_{A(14)} = \mu_{A(18)}$$

$H_1 : \mu_{A(i)} \neq \mu_{A(j)}$ at least for $i \neq j$

**Table 4.5** ANOVA of Profit on Block of Flight A

| SOV | DF | SSE | MSE | F | Sig. |
|---|---|---|---|---|---|
| Treatment | 5 | 3.65e+07 | 7.30e+06 | 270.53 | <2e-16 |
| Block | 15 | 2.25e+10 | 1.50e+09 | 55579.91 | <2e-16 |
| Error | 75 | 2.02e+06 | 2.70e+04 | | |

From Table 4.5, there is sufficient evidence at the $\alpha = 0.05$ significance level to reject the claim that varying the number of updates to the booking limit does not affect mean profit. After a multiple comparison test, it can be concluded that the mean profit of FCFS and no update to the booking limit cases are no different and the mean profit of 14 points and 18 points are also no different (see Table A.17). The highest profit occurs in cases with an update booking limit of 13 points.

Hypothesis testing for flight B

$$H_0 : \mu_{B(FCFS)} = \mu_{B(Not-update)} = \mu_{B(7)} = \mu_{B(13)} = \mu_{B(14)} = \mu_{B(18)}$$

$H_1 : \mu_{B(i)} \neq \mu_{B(j)}$ at least for $i \neq j$

**Table 4.6** ANOVA of Profit on Block of Flight B

| SOV | DF | SSE | MSE | F | Sig. |
|---|---|---|---|---|---|
| Treatment | 5 | 3.21e+07 | 6.43e+06 | 286.28 | <2e-16 |
| Block | 15 | 2.24e+10 | 1.49e+09 | 66550.11 | <2e-16 |
| Error | 75 | 1.68e+06 | 2.25e+04 | | |

From Table 4.6, there is sufficient evidence at the $\alpha = 0.05$ significance level to reject the claim that varying the number of updates to the booking limit does not affect mean profit. After a multiple comparison test, we can conclude that mean profit of the FCFS and no update to the booking limit cases are no different and the mean profit from 14 points and 18 points are also no different (see Table A.18). The highest profit occurs in the case with an update booking limit of 13 points.

The results of flight A and B with different numbers of update booking limit points are also similar.

For next hypothesis, only data from flight A is used for testing.

#### 4.5.3 Description of Hypothesis 2

The initial mean demand of class 1 and class 2 were set to underestimate the initial true demand by $50\%$ and $75\%$, and to overestimate the initial true demand by $125\%$ and $150\%$.

Let $\mu_{(i),\text{int}\, j}$ be the mean profit when update booking limit $i$ points are applied with initial mean demand $j$, where $i$ = 7, 13, 14, and 18, and $j$ = 0.5, 0.75, 1.25, and 1.5. The hypothesis for testing is set out below.

Hypothesis testing of update booking limit $i$ points for $i = 7, 13, 14, 18$

$$H_0 : \mu_{(i),\text{int}0.5} = \mu_{(i),\text{int}0.75} = \mu_{(i)} = \mu_{(i),\text{int}1.25} = \mu_{(i),\text{int}1.5}$$

$$H_1 : \mu_{(i),j} \neq \mu_{(i),k} \text{ at least for } i \neq j$$

The steps to generate data to test this hypothesis are laid out as follows:

1) Use the real-life data to forecast the demand for the next flight using an exponential smoothing method with a smoothing constant determined by minimizing the sum of squared errors.

2) Partition the forecasted demand into two classes using the proportions of class 1 and class 2 passengers in (4.14). Mean demand of class 1 and class 2 are $\hat{\lambda}_{1,360}$ and $\hat{\lambda}_{2,360}$.

3) Generate demand of class 1 and class 2 over 360 days, which is assumed to be a Poisson distribution with means $\hat{\lambda}_{1,360}$ and $\hat{\lambda}_{2,360}$, respectively.

4) Calculate the optimal initial booking limit $(x^*_{360})$ using incorrect initial means as define above.

5) Calculate the number of class 1 and class 2 reservations every day using $x^*_{360}$.

6) Recalculate the optimal booking limit $x^*_t$ at update booking limit point *t* and forecast demand for the two classes $(\hat{\lambda}_{1,t}, \hat{\lambda}_{2,t})$ over the remaining days before departure.

7) Use the forecasted demand for the two classes to calculate a new optimal booking limit $(x^*_t)$.

8) At the time of departure, generate the number of show-up passengers following a binomial distribution with mean equal to the number of class *i* reservations and show-up probability $\theta_i; i = 1, 2$, and calculate the profit using equation (4.17).

9) Repeat all steps over 1,000 iterations.

10) Average the profit from 1,000 iterations.

11) Test this hypothesis.

Figure 4.14 show the steps to test hypothesis 2.

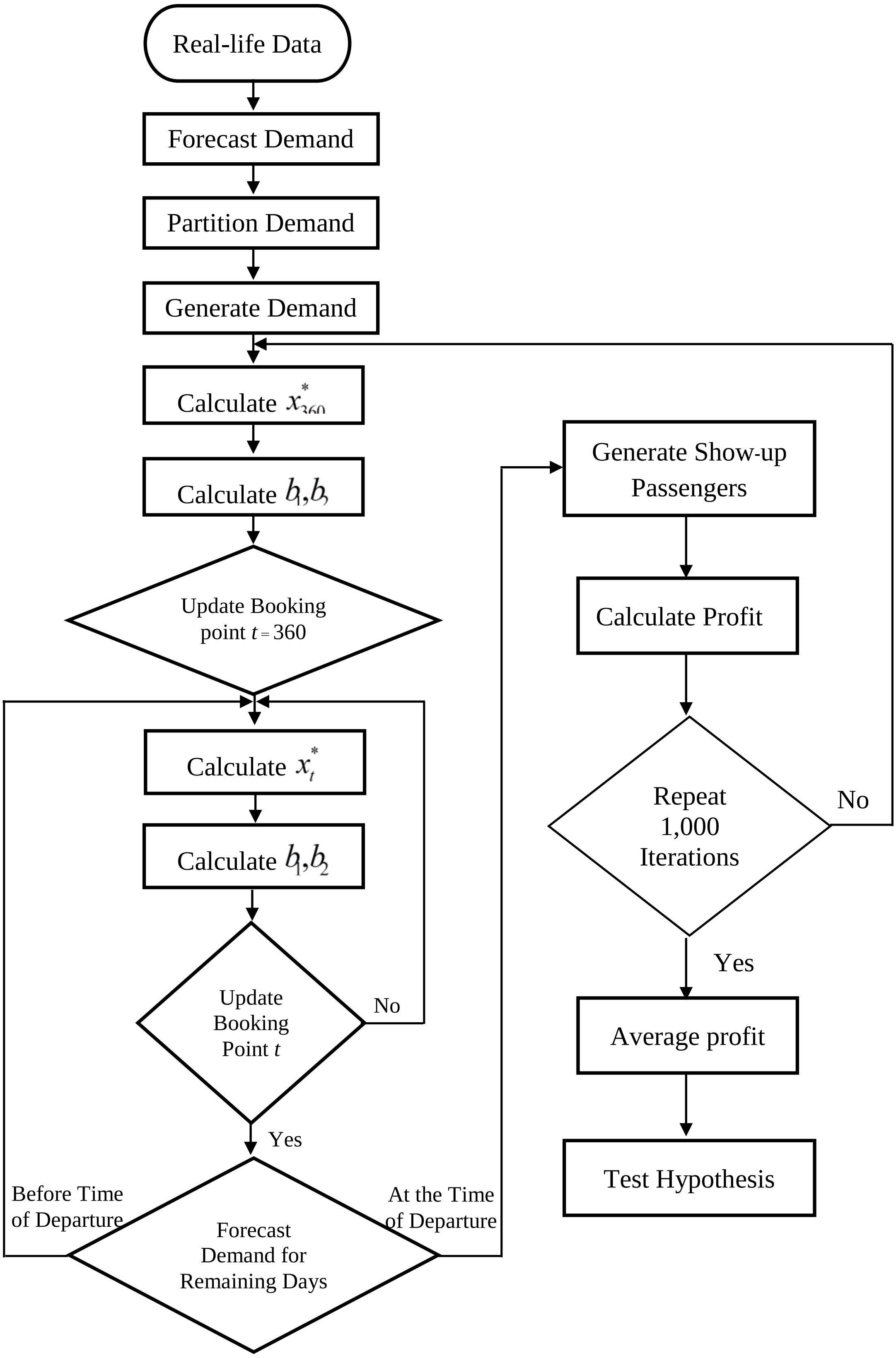



**Figure 4.14** Steps to Test Hypothesis 2.

### 4.5.4 Result for Hypothesis 2

Data used to test this hypothesis are show in Tables A.19-A.22 in Appendix A. Hypotheses for testing of 7 points, 13 points, 14 points, and 18 points with different initial means are shown below.

Hypothesis testing of update booking limit 7 points

$$H_0 : \mu_{(7),\text{int}0.5} = \mu_{(7),\text{int}0.75} = \mu_{(7)} = \mu_{(7),\text{int}1.25} = \mu_{(7),\text{int}1.5}$$

$H_1 : \mu_{(7),i} \neq \mu_{(7),j}$ at least for $i \neq j$

**Table 4.7** ANOVA of Profit on Block with Incorrect Initial Mean Demand for Update Booking Limit 7 Points

| SOV | DF | SSE | MSE | F | Sig. |
|---|---|---|---|---|---|
| Treatment | 4 | 6.71e+09 | 1.68e+09 | 6034.17 | <2e-16 |
| Block | 15 | 1.80e+10 | 1.20e+09 | 4320.40 | <2e-16 |
| Error | 60 | 1.67e+07 | 2.78e+05 | | |

From Table 4.7, there is sufficient evidence at the $\alpha = 0.05$ significance level to reject the claim that an incorrect initial mean does not affect the mean profit. After a multiple comparison test, it can be concluded that the mean profit of the two cases with an underestimated mean demand is no different and higher than true mean demand case and the mean profit of the two cases with an overestimated mean demand are different and lower than true mean demand case (see Table A.23).

In the 13 points, the results for different incorrect initial means for demad are exactly equal except the last case, overestimated 150%, as shown in Table A.20 in Appendix A. It was sufficient to test only true initial mean demand case and overestimated 150%.

Hypothesis testing of update booking limit 13 points

$$H_0 : \mu_{(13)} = \mu_{(13),\text{int}1.5}$$

$$H_1 : \mu_{(13)} = \mu_{(13),\text{int}1.5}$$

**Table 4.8** ANOVA of Profit on Block with Incorrect Initial Mean Demand for Update Booking Limit 13 Points

| SOV | DF | SSE | MSE | F | Sig. |
|---|---|---|---|---|---|
| Treatment | 1 | 2644.66 | 2644.66 | 307.22 | 2.12e-11 |
| Block | 15 | 7.60e+09 | 5.07e+08 | 5.89e+07 | < 2e-16 |
| Error | 60 | 129.13 | 8.61 | | |

From Table 4.8, there is sufficient evidence at the $\alpha = 0.05$ significance level to reject the claim that overestimate initial mean demand 150% does not affect the mean profit. Note that only effect of overestimate initial mean demand 150% can be test by using the independent sample *t*-test.

In the 14 points and 18 points cases, the results for different incorrect initial means for demand are exactly equal, as shown in Tables A.21 and A.22 in Appendix A. It was not necessary to test this hypothesis for these two cases since an incorrect initial mean demand does not affect profit when higher numbers of update booking limit points are applied.

### 4.5.5 Description of Hypothesis 3

For this hypothesis, only the effect of a smoothing constant on the 13 points case was tested because it was the best case of the number of update booking limit points.

Smoothing constants $(\rho)$ were set at 0.1, 0.2, …, 0.9 and a smoothing constant that minimized the sum of squared errors $(\rho_{\min SSE} = 0.78338)$ was calculated using the R program.

Let $\mu_{\rho=i}$ be the mean profit with smoothing constant *i*, where $i = 0.1, 0.2, \ldots,$ 0.9 and 0.78338.

Hypothesis testing with a smoothing constant

$$H_0 : \mu_{\rho=0.1} = \mu_{\rho=0.2} = \cdots = \mu_{\rho=0.7} = \mu_{\rho_{\min SSE}} = \mu_{\rho=0.8} = \mu_{\rho=0.9}$$

$$H_1 : \mu_{\rho=i} \neq \mu_{\rho=j} \text{ at least for } i \neq j$$

The following steps were used to generate data to test this hypothesis:

1) Use real-life data to forecast demand for the next flight by using an exponential smoothing with a smoothing constant as defined above.

2) Partition the forecasted demand into two classes using the proportions of class 1 and class 2 passengers in (4.14). The mean demands for class 1 and class 2 are $\hat{\lambda}_{1,360}$ and $\hat{\lambda}_{2,360}$.

3) Generate demand for class 1 and class 2 over 360 days, which is assumed to be a Poisson distribution with means $\hat{\lambda}_{1,360}$ and $\hat{\lambda}_{2,360}$, respectively.

4) Calculate the optimal initial booking limit $(x^*_{360})$.

5) Calculate the number of class 1 and class 2 reservations every day using $x^*_{360}$.

6) Recalculate the optimal booking limit $x^*_t$ at update booking limit point $t$ and forecast demand for the two classes $(\hat{\lambda}_{1,t}, \hat{\lambda}_{2,t})$ over the remaining days before departure.

7) Use the forecasted demand for the two classes to calculate the new optimal booking limit $(x^*_t)$.

8) At the time of departure, generate the number of show-up passengers following a binomial distribution with a mean equal to the number of class $i$ reservations and the show-up probability $\theta_i; i = 1, 2$, and calculate profit using equation (4.17).

9) Repeat all steps over 1,000 iterations.

10) Average the profit from 1,000 iterations.

11) Test this hypothesis.

Figure 4.15 shows the steps to test hypothesis 3.

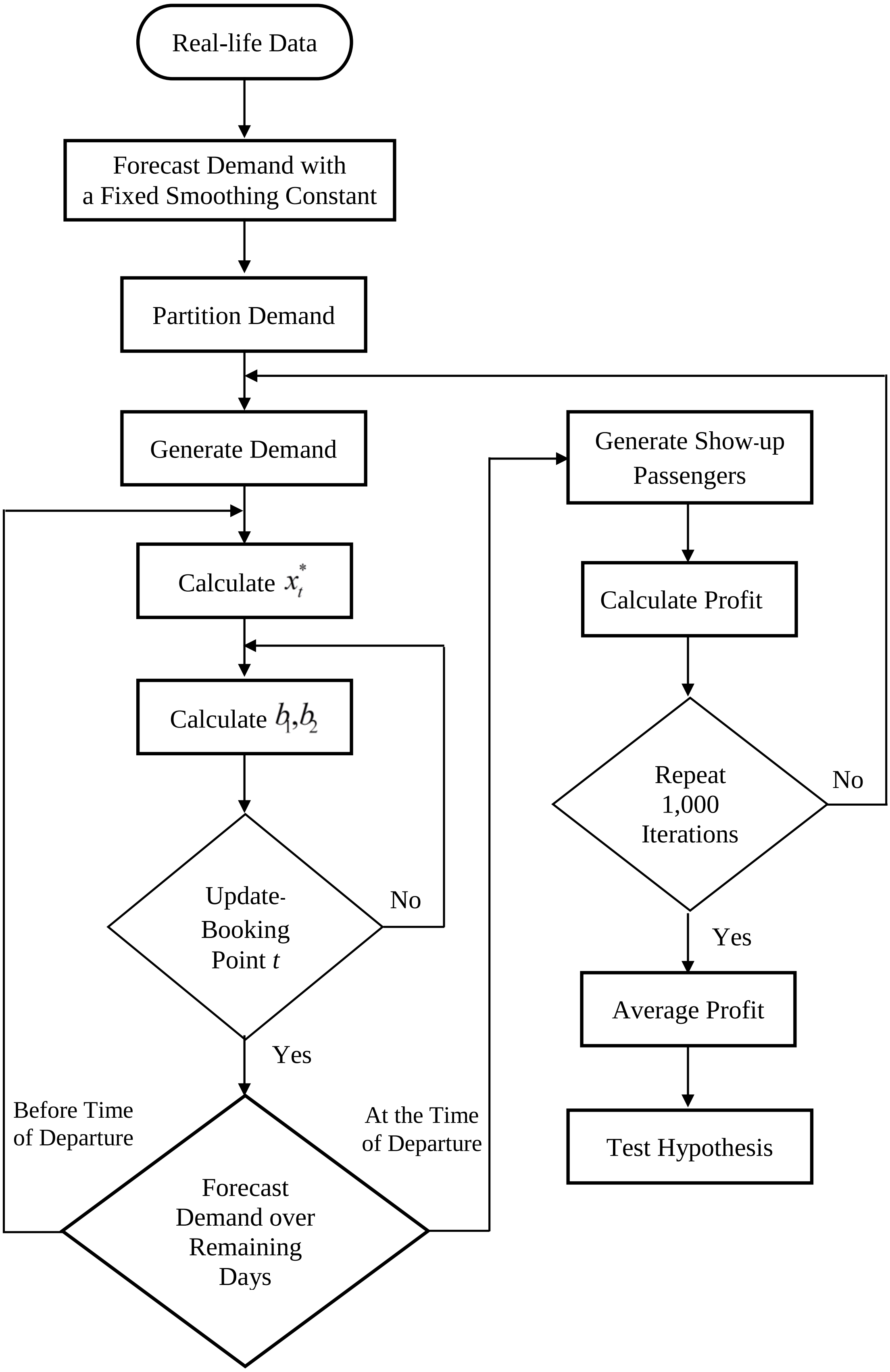



**Figure 4.15** Steps to Test Hypothesis 3.

### 4.5.6 Results for Hypothesis 3

The data used to test this hypothesis are shown in Table A.24 in Appendix A. Hypothesis testing for different smoothing constants is shown below.

Hypothesis for testing the smoothing constant:

$$H_0 : \mu_{\rho=0.1} = \mu_{\rho=0.2} = \cdots = \mu_{\rho=0.7} = \mu_{\rho_{\min SSE}} = \mu_{\rho=0.8} = \mu_{\rho=0.9}$$

$$H_1 : \mu_{\rho=i} \neq \mu_{\rho=j} \text{ at least for } i \neq j$$

**Table 4.9** ANOVA of Profit on Block with Different Smoothing Constants

| SOV | DF | SSE | MSE | F | Sig. |
|---|---|---|---|---|---|
| Treatment | 9 | 8.61e+10 | 9.57e+09 | 3528.84 | <2e-16 |
| Block | 15 | 3.11e+10 | 2.07e+09 | 765.05 | <2e-16 |
| Error | 135 | 3.66e+08 | 2.71e+06 | | |

From Table 4.9, there is sufficient evidence at the $\alpha = 0.05$ significance level to reject the claim that the different smoothing constants applied do not affect the mean profit. After a multiple comparison test, all cases of smoothing constant are different except for $\rho = 0.9$ and $\rho_{\min SSE}$ are no different. The smoothing constant $\rho_{\min SSE}$ provided the highest mean profit (see Table A.25). Moreover, the mean profit increases when the smoothing constant increases.

# CHAPTER 5

# SUMMARY AND CONCLUSION

## 5.1 Summary and Conclusion

A static two-class overbooking model with fixed capacity is proposed in this dissertation. This model combines two essential strategies in RM, namely overbooking and seat inventory control. Assume the arrival pattern in this model is low-before high fare, demand for the two classes are independent random variables, and the number of show-ups follows a binomial distribution. This model includes a penalty cost when rejecting booking requests in two classes, a refund cost for the two classes when passengers do not show up (or cancel) at the time of departure, and a denied boarding cost for class 2 passengers when denied boarding. The parameters in the model are cost of revenue $(p_i)$, penalty cost $(g_i)$, refund cost $(r_i)$, show-up probability $(\theta_i)$, denied boarding cost $(h)$, and capacity $(\kappa)$, for $i=1,2.$ The booking limit $(x)$ is the number of class 2 customers accepted. The optimal booking limit $(x^*)$ obtained from the proposed model is the same as the optimal booking limit $(x')$ for seat inventory control and the optimal overbooking limit $(x'')$ when overbooking is applied.

The optimal booking limit has a closed-form to calculate what will occur at three points: $x', \kappa-1, x''.$ For $i=1,2,$ let $\alpha_i = p_i + g_i - r_i + r_i\theta_i$ . Suppose that $0<\alpha_2/(h\theta_2)<\bar{F}(\kappa-1;\kappa,\theta_2)$ , then

$$x^* = \arg\max\{\pi(x'), \pi(\kappa-1), \pi(x'')\}\ .$$

Suppose that $\alpha_2/(h\theta_2) \ge \bar{F}(\kappa-1;\kappa,\theta_2)$ . If $\lim_{x\to\infty}\pi(x) < \max\{\pi(x'), \pi(\kappa-1)\}$ , then

$$x^* = \arg\max\{\pi(x'), \pi(\kappa-1)\}\ ,$$

where $\bar{F}(t;x,\theta_2) = 1 - \sum_{j=0}^{t} \binom{x}{j} \theta_2^j (1-\theta_2)^{x-j}$ is the tail-sum probability of a binomial distribution with parameters $x$ and $\theta_2$.

Otherwise, if $\pi(x)$ is increasing, the airline should set the optimal booking limit to be as large as possible.

By means of a sensitivity analysis, it was proved that $x'$ increases when the ratio $\alpha_2/\alpha_1$ or capacity $\kappa$ increases, or $D_1$ decreases with respect to the usual stochastic order, and $x''$ increases when the ratio $\alpha_2/(h\theta_2)$ or capacity $\kappa$ increases.

Using simulated data, some of the relationships between the model parameters and the expected profit were studied at local maximum points $x'$ and $x''$. By mathematical proof, the profit at the local maximum point $x'$ increases when $\alpha_i$ increases; the profit at the local maximum point $x'$ increases in many cases, e.g. 1) the cost of revenue for class *i* increases, 2) the penalty cost for class *i* increases, 3) the show-up probability for class *i* increases, and 4) the refund cost for class *i* decreases with other parameters fixed and the profit at the local maximum point $x''$ increases when $\alpha_2$ increases; the profit at the local maximum point $x''$ increase in many cases, e.g. 1) the cost of revenue for class 2 increases, 2) the penalty cost for class 2 increases, or 3) the refund cost for class 2 decreases with the show-up probability for class 2 and other parameters fixed.

Real-life data was used to compare performance of the policy from the proposed model to the airline's policy of a fixed booking limit over all time periods. The results show that the profit from this study's policy is higher than that of the airline's policy when overbooking is not applied, and the profit from this study's policy and the airline's policy are the same when overbooking is applied. Varying the number of update booking limit points affects profit, with the best number of update booking limit points being 13 points. This case is typically used in the current practice of updating the booking limit every month at least six months before departure and every day in the last week before departure. In a further real-life data numerical experiment for hypothesis testing, it was found that an incorrect initial mean does not affect profit when a high number of update booking limit points is set. The best case of update booking limit (13 points) was used to test the effect of a smoothing constant when using exponential smoothing. Different smoothing constants affect profit and maximum profit occurs when using a smoothing constant that minimized the sum of squared errors.

## 5.2 Future Research

It is possible to extend the study by estimating the parameters in the model with a statistical unconstraining method, e.g projection detruncation, expectation maximization, or forecast demand. Non-stationary demand is generated in numerical experiments which simulate data from real-life situations, which may improve the accuracy of the optimal booking limit and profit. In addition, we may consider this study's two-class model where capacity is random and try to find an optimal booking limit for this study's model and other two-class models. Moreover, a model in which demand is dependent could be studied. Finally, overbooking for class 1 could be allowed, leading to a model with two overbooking limits. It is hoped that pursuit of some of these related problems may be possible in the future.

# BIBLIOGRAPHY


Aydin, N.; Birbil, S. I.; Frenk, J. B. G. and Noyan, N. 2012. Single-leg Airline Revenue Management with Overbooking. **Transportation Science.** 47(4): 560–583.

Brumelle, S. L. and McGill, J. I. 1989. **A General Model for Airline Overbooking and Two-class Revenue Management with Dependent Demands.** Working Paper, University of British Columbia, Vancouver, BC.

Chiang, W.-C.; Chen, J. C. H. and Xu, X. 2006. An Overview of Research on Revenue Management: Current Issues and Future Research. **International Journal of Revenue Management.** 1(1): 97–128.

Cross, R. G. 1997. **Revenue Management.** New York: Crown Business.

FlightGlobal. 2015. **Top Ten: How the World's Biggest Airlines ranked in 2014.** Retrieved September 12, 2015 from https://www.flightglobal.com/news/ articles/ top-ten-how-the-worlds-biggest-airlines-ranked-in-415274/.

Gosavi, A., Bandla, N. and Das, T. K. 2002. A Reinforcement Learning Approach to a Single Leg Airline Revenue Management Problem with Multiple Fare Classes and Overbooking. **IIE Transactions.** 34(9): 729–742.

Komsan Suriya. 2009. The Impact of Low Cost Airlines to Airline Industry: An Experience of Thailand. **Jurnal Ekonomi Malaysia.** 43: 3–25.

Lan, Y.; Ball, M. O. and Karaesmen, I. Z. 2011. Regret in Overbooking and Fare-class Allocation for Single Leg. **Manufacturing & Service Operations Management.** 13(2): 194–208.

Lan, Y.; Ball, M. O.; Karaesmen, I. Z.; Zhang, J. X. and Liu, G. X. 2015. Analysis of Seat Allocation and Overbooking Decisions with Hybrid Information. **European Journal of Operational Research.** 240(2): 493–504.

Littlewood, K. 1972. Special Issue Papers: Forecasting and Control of Passenger Bookings. **Journal of Revenue and Pricing Management.** 4(2): 111–123.

McGill, J. I. and Van Ryzin, G. J. 1999. Revenue Management: Research Overview and Prospects. **Transportation Science.** 33(2): 233–256.

Montgomery, D. C.; Jennings, C. L. and Kulahci, M. 2008. **Introduction to Time Series Analysis and Forecasting.** Hoboken, NJ: John Wiley & Sons.

Müller, A. and Stoyan, D. 2002. **Comparison Methods for Stochastic Models and Risks.** Hoboken, NJ: John Wiley & Sons.

Phillips, R. L. 2005. **Pricing and Revenue Optimization.** Palo Alto, CA: Stanford University Press.

Ringbom, S. and Shy, O. 2002. The "Adjustable-Curtain" Strategy: Overbooking of Multiclass Service. **Journal of Economics.** 77(1): 73–90.

Sabre. 2015. **Maximize your Revenue on Each and Every Flight.** Retrieved Setember 12, 2015 from http://www.sabreairlinesolutions.com/home/industry_challenges/revenue_growth/.

Sawaki, K. 1989. An Analysis of Airline Seat Allocation. **Journal of the Operations Research Society of Japan.** 32(4): 411–419.

Shlifer, E. and Vardi, Y. 1975. An Airline Overbooking Policy. **Transportation Science.** 9(2): 101–114.

Smith, B. C.; Leimkuhler, J. F. and Darrow, R. M. 1992. Yield Management at American Airlines. **Interfaces.** 22(1): 8–31.

Subramanian, J.; Stidham, S. and Lautenbacher, C. J. 1999. Airline Yield Management with Overbooking, Cancellations, and No-Shows. **Transportation Science.** 33(2): 147–167.

Talluri, K. T. and Van Ryzin, G. J. 2004. **The Theory and Practice of Revenue Management.** Berlin: Springer Science & Business Media.

Thisdell, D. 2015. **Size Matters: Ten Biggest Players in Aerospace.** Retrieved September 12, 2015 from https://www.flightglobal.com/news/articles/size-matters-ten-biggest-players-in-aerospace-416563/.

Thompson, H. R. 1961. Statistical Problems in Airline Reservation Control. **Operation Reaseach Quarterly.** 12(3): 167–185.

Weatherford, L. R. and Pölt, S. 2002. Better Unconstraining of Airline Demand Data in Revenue Management Systems for Improved Forecast Accuracy and Greater Revenues. **Journal of Revenue and Pricing Management.** 1(3): 234–254.

# APPENDICES

# Appendix A

# TABLE

**Table A.1** Profit of This Study's Policy and the Airline Policy using Different Unconstraining Methods on Flight A.

| $r_1$ | $r_2$ | Booking Limit | Profit | | |
|---|---|---|---|---|---|
| | | | N1 | N2 | N3 |
| 1521.5 | 472.5 | $x^*$ | 177,072.79 | 178,328.65 | 177,668.75 |
| | | 9 | 150,845.89 | 151,889.04 | 151,339.13 |
| | | 17 | 154,625.89 | 155,669.04 | 155,119.13 |
| | | 41 | 165,676.29 | 166,720.13 | 166,167.88 |
| | | 81 | 176,159.90 | 177,355.27 | 176,735.64 |
| | | 122 | 177,072.79 | 178,328.65 | 177,668.75 |
| | | 171 | 177,072.79 | 178,328.65 | 177,668.75 |
| 1521.5 | 756 | $x^*$ | 171,550.60 | 172,768.78 | 172,127.85 |
| | | 9 | 161,160.82 | 162,296.94 | 161,698.46 |
| | | 17 | 162,672.82 | 163,808.94 | 163,210.46 |
| | | 41 | 167,092.98 | 168,229.37 | 167,629.96 |
| | | 81 | 171,286.42 | 172,483.43 | 171,857.06 |
| | | 122 | 171,550.60 | 172,768.78 | 172,127.85 |
| | | 171 | 171,550.60 | 172,768.78 | 172,127.85 |
| 2434.4 | 472.5 | $x^*$ | 173,170.70 | 174,402.41 | 173,757.60 |
| | | 9 | 146,842.83 | 147,858.80 | 147,325.53 |
| | | 17 | 150,622.83 | 151,638.80 | 151,105.53 |
| | | 41 | 161,673.22 | 162,689.88 | 162,154.28 |
| | | 81 | 172,156.83 | 173,325.02 | 172,722.04 |
| | | 122 | 173,170.70 | 174,402.41 | 173,757.60 |
| | | 171 | 173,170.70 | 174,402.41 | 173,757.60 |
| 2434.4 | 756 | $x^*$ | 167,648.51 | 168,842.54 | 168,216.70 |
| | | 9 | 157,157.76 | 158,266.69 | 157,684.86 |
| | | 17 | 158,669.76 | 159,778.69 | 159,196.86 |
| | | 41 | 163,089.91 | 164,199.13 | 163,616.36 |
| | | 81 | 167,283.36 | 168,453.18 | 167,843.46 |
| | | 122 | 167,648.51 | 168,842.54 | 168,216.70 |
| | | 171 | 167,648.51 | 168,842.54 | 168,216.70 |

**Table A.2** Profit of This Study's Policy and the Airline Policy using Different Unconstraining Methods on Flight B.

| $r_1$ | $r_2$ | Booking Limit | Profit N1 | Profit N2 | Profit N3 |
|---|---|---|---|---|---|
| 1521.5 | 472.5 | $x^*$ | 186,880.34 | 186,549.75 | 186,305.45 |
| | | 9 | 158,922.42 | 158,642.75 | 158,442.29 |
| | | 17 | 162,702.42 | 162,422.75 | 162,222.29 |
| | | 41 | 174,024.57 | 173,743.63 | 173,544.74 |
| | | 81 | 186,105.67 | 185,753.09 | 185,552.89 |
| | | 122 | 186,880.34 | 186,549.75 | 186,305.45 |
| | | 171 | 186,880.34 | 186,549.75 | 186,305.45 |
| 1521.5 | 756 | $x^*$ | 181,072.14 | 180,750.93 | 180,515.26 |
| | | 9 | 169,945.85 | 169,642.77 | 169,424.50 |
| | | 17 | 171,457.85 | 171,154.77 | 170,936.50 |
| | | 41 | 175,986.72 | 175,683.12 | 175,465.47 |
| | | 81 | 180,819.16 | 180,486.90 | 180,268.73 |
| | | 122 | 181,072.14 | 180,750.93 | 180,515.26 |
| | | 171 | 181,072.14 | 180,750.93 | 180,515.26 |
| 2434.4 | 472.5 | $x^*$ | 182,818.99 | 182,491.63 | 182,255.12 |
| | | 9 | 154,804.18 | 154,529.99 | 154,337.47 |
| | | 17 | 158,584.18 | 158,309.99 | 158,117.47 |
| | | 41 | 169,906.34 | 169,630.88 | 169,439.92 |
| | | 81 | 181,987.44 | 181,640.34 | 181,448.07 |
| | | 122 | 182,818.99 | 182,491.63 | 182,255.12 |
| | | 171 | 182,818.99 | 182,491.63 | 182,255.12 |
| 2434.4 | 756 | $x^*$ | 177,010.79 | 176,692.81 | 176,464.94 |
| | | 9 | 165,827.62 | 165,530.01 | 165,319.68 |
| | | 17 | 167,339.62 | 167,042.01 | 166,831.68 |
| | | 41 | 171,868.48 | 171,570.36 | 171,360.65 |
| | | 81 | 176,700.92 | 176,374.15 | 176,163.91 |
| | | 122 | 177,010.79 | 176,692.81 | 176,464.94 |
| | | 171 | 177,010.79 | 176,692.81 | 176,464.94 |

**Table A.3** Loss Profit per Flight of Flight A

| $r_1$ | $r_2$ | Booking Limit | Loss Profit per Flight (Baht) | | |
|---|---|---|---|---|---|
| | | | N1 | N2 | N3 |
| 1521.5 | 472.5 | 9 | 26,226.90 | 26,439.61 | 26,329.62 |
| | | 17 | 22,446.90 | 22,659.61 | 22,549.62 |
| | | 41 | 11,396.50 | 11,608.52 | 11,500.87 |
| | | 81 | 912.89 | 973.38 | 933.11 |
| | | 122 | 0 | 0 | 0 |
| | | 171 | 0 | 0 | 0 |
| 1521.5 | 756 | 9 | 10,389.78 | 10,471.84 | 10,429.39 |
| | | 17 | 8,877.78 | 8,959.84 | 8,917.39 |
| | | 41 | 4,457.62 | 4,539.41 | 4,497.89 |
| | | 81 | 264.18 | 285.35 | 270.79 |
| | | 122 | 0 | 0 | 0 |
| | | 171 | 0 | 0 | 0 |
| 2434.4 | 472.5 | 9 | 26,327.87 | 26,543.61 | 26,432.07 |
| | | 17 | 22,547.87 | 22,763.61 | 22,652.07 |
| | | 41 | 11,497.48 | 11,712.53 | 11,603.32 |
| | | 81 | 1,013.87 | 1,077.39 | 1,035.56 |
| | | 122 | 0 | 0 | 0 |
| | | 171 | 0 | 0 | 0 |
| 2434.4 | 756 | 9 | 10,490.75 | 10,575.85 | 10,531.84 |
| | | 17 | 8,978.75 | 9,063.85 | 9,019.84 |
| | | 41 | 4,558.60 | 4,643.41 | 4,600.34 |
| | | 81 | 365.15 | 389.36 | 373.24 |
| | | 122 | 0 | 0 | 0 |
| | | 171 | 0 | 0 | 0 |

**Table A.4** Loss Profit per Flight of Flight B

| $r_1$ | $r_2$ | Booking Limit | Loss Profit per Flight (Baht) | | |
|---|---|---|---|---|---|
| | | | N1 | N2 | N3 |
| 1521.5 | 472.5 | 9 | 27,957.92 | 27,907.00 | 27,863.16 |
| | | 17 | 24,177.92 | 24,127.00 | 24,083.16 |
| | | 41 | 12,855.77 | 12,806.12 | 12,760.71 |
| | | 81 | 774.67 | 796.66 | 752.56 |
| | | 122 | 0 | 0 | 0 |
| | | 171 | 0 | 0 | 0 |
| 1521.5 | 756 | 9 | 11,126.29 | 11,108.16 | 11,090.76 |
| | | 17 | 9,614.29 | 9,596.16 | 9,578.76 |
| | | 41 | 5,085.42 | 5,067.81 | 5,049.79 |
| | | 81 | 252.98 | 264.03 | 246.53 |
| | | 122 | 0 | 0 | 0 |
| | | 171 | 0 | 0 | 0 |
| 2434.4 | 472.5 | 9 | 28,014.81 | 27,961.64 | 27,917.65 |
| | | 17 | 24,234.81 | 24,181.64 | 24,137.65 |
| | | 41 | 12,912.65 | 12,860.75 | 12,815.20 |
| | | 81 | 831.55 | 851.29 | 807.05 |
| | | 122 | 0 | 0 | 0 |
| | | 171 | 0 | 0 | 0 |
| 2434.4 | 756 | 9 | 11,183.17 | 11,162.80 | 11,145.26 |
| | | 17 | 9,671.17 | 9,650.80 | 9,633.26 |
| | | 41 | 5,142.31 | 5,122.45 | 5,104.29 |
| | | 81 | 309.87 | 318.66 | 301.03 |
| | | 122 | 0 | 0 | 0 |
| | | 171 | 0 | 0 | 0 |

**Table A.5** Loss Profit per Flight per Year of flight A

| $r_1$ | $r_2$ | Booking Limit | Loss Profit per Flight per Year (Baht) | | |
|---|---|---|---|---|---|
| | | | N1 | N2 | N3 |
| 1521.5 | 472.5 | 9 | 1,363,798.80 | 1,374,859.72 | 1,369,140.24 |
| | | 17 | 1,167,238.80 | 1,178,299.72 | 1,172,580.24 |
| | | 41 | 592,618.00 | 603,643.04 | 598,045.24 |
| | | 81 | 47,470.28 | 50,615.76 | 48,521.72 |
| | | 122 | 0 | 0 | 0 |
| | | 171 | 0 | 0 | 0 |
| 1521.5 | 756 | 9 | 540,268.56 | 544,535.68 | 542,328.28 |
| | | 17 | 461,644.56 | 465,911.68 | 463,704.28 |
| | | 41 | 231,796.24 | 236,049.32 | 233,890.28 |
| | | 81 | 13,737.36 | 14,838.20 | 14,081.08 |
| | | 122 | 0 | 0 | 0 |
| | | 171 | 0 | 0 | 0 |
| 2434.4 | 472.5 | 9 | 1,369,049.24 | 1,380,267.72 | 1,374,467.64 |
| | | 17 | 1,172,489.24 | 1,183,707.72 | 1,177,907.64 |
| | | 41 | 597,868.96 | 609,051.56 | 603,372.64 |
| | | 81 | 52,721.24 | 56,024.28 | 53,849.12 |
| | | 122 | 0 | 0 | 0 |
| | | 171 | 0 | 0 | 0 |
| 2434.4 | 756 | 9 | 545,519.00 | 549,944.20 | 547,655.68 |
| | | 17 | 466,895.00 | 471,320.20 | 469,031.68 |
| | | 41 | 237,047.20 | 241,457.32 | 239,217.68 |
| | | 81 | 18,987.80 | 20,246.72 | 19,408.48 |
| | | 122 | 0 | 0 | 0 |
| | | 171 | 0 | 0 | 0 |

**Table A.6** Loss Profit per Flight per Year of flight B

| $r_1$ | $r_2$ | Booking Limit | Loss Profit per Flight per Year (Baht) | | |
|---|---|---|---|---|---|
| | | | N1 | N2 | N3 |
| 1521.5 | 472.5 | 9 | 1,453,811.84 | 1,451,164.00 | 1,448,884.32 |
| | | 17 | 1,257,251.84 | 1,254,604.00 | 1,252,324.32 |
| | | 41 | 668,500.04 | 665,918.24 | 663,556.92 |
| | | 81 | 40,282.84 | 41,426.32 | 39,133.12 |
| | | 122 | 0 | 0 | 0 |
| | | 171 | 0 | 0 | 0 |
| 1521.5 | 756 | 9 | 578,567.08 | 577,624.32 | 576,719.52 |
| | | 17 | 499,943.08 | 499,000.32 | 498,095.52 |
| | | 41 | 264,441.84 | 263,526.12 | 262,589.08 |
| | | 81 | 13,154.96 | 13,729.56 | 12,819.56 |
| | | 122 | 0 | 0 | 0 |
| | | 171 | 0 | 0 | 0 |
| 2434.4 | 472.5 | 9 | 1,456,770.12 | 1,454,005.28 | 1,451,717.80 |
| | | 17 | 1,260,210.12 | 1,257,445.28 | 1,255,157.80 |
| | | 41 | 671,457.80 | 668,759.00 | 666,390.40 |
| | | 81 | 43,240.60 | 44,267.08 | 41,966.60 |
| | | 122 | 0 | 0 | 0 |
| | | 171 | 0 | 0 | 0 |
| 2434.4 | 756 | 9 | 581,524.84 | 580,465.60 | 579,553.52 |
| | | 17 | 502,900.84 | 501,841.60 | 500,929.52 |
| | | 41 | 267,400.12 | 266,367.40 | 265,423.08 |
| | | 81 | 16,113.24 | 16,570.32 | 15,653.56 |
| | | 122 | 0 | 0 | 0 |
| | | 171 | 0 | 0 | 0 |

**Table A.7** Profit of This Study's Policy and Airline Policy when Varying *h* of Flight A.

| Unconstraining Method | Booking Limit | Profit | | |
|---|---|---|---|---|
| | | $h = 1,500$ | $h = 2,000$ | $h = 2,500$ |
| N1 | $x^*$ | 173,170.70 | 173,170.70 | 173,170.70 |
| | 9 | 146,842.83 | 146,842.83 | 146,842.83 |
| | 17 | 150,622.83 | 150,622.83 | 150,622.83 |
| | 41 | 161,673.22 | 161,673.22 | 161,673.22 |
| | 81 | 172,156.83 | 172,156.83 | 172,156.83 |
| | 122 | 173,170.70 | 173,170.70 | 173,170.70 |
| | 171 | 173,170.70 | 173,170.70 | 173,170.70 |
| N2 | $x^*$ | 174,402.41 | 174,402.41 | 174,402.41 |
| | 9 | 147,858.80 | 147,858.80 | 147,858.80 |
| | 17 | 151,638.80 | 151,638.80 | 151,638.80 |
| | 41 | 162,689.88 | 162,689.88 | 162,689.88 |
| | 81 | 173,325.02 | 173,325.02 | 173,325.02 |
| | 122 | 174,402.41 | 174,402.41 | 174,402.41 |
| | 171 | 174,402.41 | 174,402.41 | 174,402.41 |
| N3 | $x^*$ | 173,757.60 | 173,757.60 | 173,757.60 |
| | 9 | 147,325.53 | 147,325.53 | 147,325.53 |
| | 17 | 151,105.53 | 151,105.53 | 151,105.53 |
| | 41 | 162,154.28 | 162,154.28 | 162,154.28 |
| | 81 | 172,722.04 | 172,722.04 | 172,722.04 |
| | 122 | 173,757.60 | 173,757.60 | 173,757.60 |
| | 171 | 173,757.60 | 173,757.60 | 173,757.60 |

**Table A.8** Profit of This Study's Policy and Airline Policy when Varying *h* of Flight B.

| Unconstraining Method | Booking Limit | Profit | | |
|---|---|---|---|---|
| | | $h = 1,500$ | $h = 2,000$ | $h = 2,500$ |
| **N1** | $x^*$ | 182,818.99 | 182,818.99 | 182,818.99 |
| | 9 | 154,804.18 | 154,804.18 | 154,804.18 |
| | 17 | 158,584.18 | 158,584.18 | 158,584.18 |
| | 41 | 169,906.34 | 169,906.34 | 169,906.34 |
| | 81 | 181,987.44 | 181,987.44 | 181,987.44 |
| | 122 | 182,818.99 | 182,818.99 | 182,818.99 |
| | 171 | 182,818.99 | 182,818.99 | 182,818.99 |
| **N2** | $x^*$ | 154,529.99 | 154,529.99 | 154,529.99 |
| | 9 | 158,309.99 | 158,309.99 | 158,309.99 |
| | 17 | 169,630.88 | 169,630.88 | 169,630.88 |
| | 41 | 181,640.34 | 181,640.34 | 181,640.34 |
| | 81 | 182,491.63 | 182,491.63 | 182,491.63 |
| | 122 | 182,491.63 | 182,491.63 | 182,491.63 |
| | 171 | 154,529.99 | 154,529.99 | 154,529.99 |
| **N3** | $x^*$ | 182,255.12 | 182,255.12 | 182,255.12 |
| | 9 | 154,337.47 | 154,337.47 | 154,337.47 |
| | 17 | 158,117.47 | 158,117.47 | 158,117.47 |
| | 41 | 169,439.92 | 169,439.92 | 169,439.92 |
| | 81 | 181,448.07 | 181,448.07 | 181,448.07 |
| | 122 | 182,255.12 | 182,255.12 | 182,255.12 |
| | 171 | 182,255.12 | 182,255.12 | 182,255.12 |

**Table A.9** Profit of This Study's Policy when Varying $\theta_1$ of Flight A. Other Parameters are $p_1 = 3043, p_2 = 945, g_1 = g_2 = 0$ and $\theta_2 = 0.7$.

| $r_1$ | $r_2$ | $(\theta_1, \theta_2)$ | Profit | | |
|---|---|---|---|---|---|
| | | | **N1** | **N2** | **N3** |
| 1521.5 | 472.5 | (0.7, 0.7) | 164,017.50 | 165,180.55 | 177,668.75 |
| | | (0.8, 0.7) | 170,539.70 | 171,746.76 | 172,127.85 |
| | | (0.9, 0.7) | 177,072.79 | 178,328.65 | 173,757.60 |
| | | (0.95, 0.7) | 180,479.78 | 181,763.38 | 168,216.70 |
| 1521.5 | 756 | (0.7, 0.7) | 158,495.31 | 159,620.68 | 177,668.75 |
| | | (0.8, 0.7) | 165,017.51 | 166,186.89 | 172,127.85 |
| | | (0.9, 0.7) | 171,550.60 | 172,768.78 | 173,757.60 |
| | | (0.95, 0.7) | 174,957.59 | 176,203.51 | 168,216.70 |
| 2434.4 | 472.5 | (0.7, 0.7) | 152,282.24 | 153,365.45 | 177,668.75 |
| | | (0.8, 0.7) | 162,717.76 | 163,871.38 | 172,127.85 |
| | | (0.9, 0.7) | 173,170.70 | 174,402.41 | 173,757.60 |
| | | (0.95, 0.7) | 178,621.88 | 179,897.97 | 168,216.70 |
| 2434.4 | 756 | (0.7, 0.7) | 146,760.05 | 147,805.58 | 177,668.75 |
| | | (0.8, 0.7) | 157,195.57 | 158,311.51 | 172,127.85 |
| | | (0.9, 0.7) | 167,648.51 | 168,842.54 | 173,757.60 |
| | | (0.95, 0.7) | 173,099.70 | 174,338.10 | 168,216.70 |

**Table A.10** Profit of This Study's Policy when Varying $\theta_1$ of Flight B. Other Parameters are $p_1 = 3043, p_2 = 945, g_1 = g_2 = 0$ and $\theta_2 = 0.7$.

| $r_1$ | $r_2$ | $(\theta_1, \theta_2)$ | Profit | | |
|---|---|---|---|---|---|
| | | | **N1** | **N2** | **N3** |
| 1521.5 | 472.5 | (0.7, 0.7) | 172,801.43 | 186,549.75 | 186,305.45 |
| | | (0.8, 0.7) | 179,899.58 | 180,750.93 | 180,515.26 |
| | | (0.9, 0.7) | 186,880.34 | 182,491.63 | 182,255.12 |
| | | (0.95, 0.7) | 190,227.05 | 176,692.81 | 176,464.94 |
| 1521.5 | 756 | (0.7, 0.7) | 166,993.24 | 186,549.75 | 186,305.45 |
| | | (0.8, 0.7) | 174,091.38 | 180,750.93 | 180,515.26 |
| | | (0.9, 0.7) | 181,072.14 | 182,491.63 | 182,255.12 |
| | | (0.95, 0.7) | 184,418.86 | 176,692.81 | 176,464.94 |
| 2434.4 | 472.5 | (0.7, 0.7) | 160,292.74 | 186,549.75 | 186,305.45 |
| | | (0.8, 0.7) | 171,649.77 | 180,750.93 | 180,515.26 |
| | | (0.9, 0.7) | 182,818.99 | 182,491.63 | 182,255.12 |
| | | (0.95, 0.7) | 188,173.73 | 176,692.81 | 176,464.94 |
| 2434.4 | 756 | (0.7, 0.7) | 154,484.54 | 186,549.75 | 186,305.45 |
| | | (0.8, 0.7) | 165,841.58 | 180,750.93 | 180,515.26 |
| | | (0.9, 0.7) | 177,010.79 | 182,491.63 | 182,255.12 |
| | | (0.95, 0.7) | 182,365.54 | 176,692.81 | 176,464.94 |

**Table A.11** Profit of This Study's Policy when Varying $\theta_2$ of Flight A. Other Parameters are $p_1 = 3043, p_2 = 945, g_1 = g_2 = 0$ and $\theta_1 = 0.7$.

| $r_1$ | $r_2$ | $(\theta_1, \theta_2)$ | Profit | | |
|---|---|---|---|---|---|
| | | | **N1** | **N2** | **N3** |
| 1521.5 | 472.5 | (0.7, 0.7) | 164,017.50 | 165,180.55 | 164,567.70 |
| | | (0.7, 0.8) | 167,094.45 | 168,278.04 | 167,656.61 |
| | | (0.7, 0.9) | 170,171.66 | 171,375.32 | 170,742.22 |
| | | (0.7, 0.95) | 171,655.46 | 172,871.03 | 172,232.66 |
| 1521.5 | 756 | (0.7, 0.7) | 158,495.31 | 159,620.68 | 159,026.80 |
| | | (0.7, 0.8) | 163,418.44 | 164,576.67 | 163,969.06 |
| | | (0.7, 0.9) | 168,341.98 | 169,532.31 | 168,906.03 |
| | | (0.7, 0.95) | 170,716.05 | 171,925.45 | 171,290.75 |
| 2434.4 | 472.5 | (0.7, 0.7) | 152,282.24 | 153,365.45 | 152,795.92 |
| | | (0.7, 0.8) | 155,359.19 | 156,462.94 | 155,884.83 |
| | | (0.7, 0.9) | 158,436.41 | 159,560.21 | 158,970.44 |
| | | (0.7, 0.95) | 159,920.20 | 161,055.93 | 160,460.89 |
| 2434.4 | 756 | (0.7, 0.7) | 146,760.05 | 147,805.58 | 147,255.02 |
| | | (0.7, 0.8) | 151,683.18 | 152,761.56 | 152,197.28 |
| | | (0.7, 0.9) | 156,606.72 | 157,717.20 | 157,134.26 |
| | | (0.7, 0.95) | 158,980.79 | 160,110.35 | 159,518.97 |

**Table A.12** Profit of This Study's Policy when Varying $\theta_2$ of Flight B. Other Parameters are $p_1 = 3043, p_2 = 945, g_1 = g_2 = 0$ and $\theta_1 = 0.7$.

| $r_1$ | $r_2$ | $(\theta_1, \theta_2)$ | Profit | | |
|---|---|---|---|---|---|
| | | | **N1** | **N2** | **N3** |
| 1521.5 | 472.5 | (0.7, 0.7) | 172,801.43 | 172,505.96 | 172,265.16 |
| | | (0.7, 0.8) | 176,005.42 | 175,706.31 | 175,459.73 |
| | | (0.7, 0.9) | 179,239.53 | 178,933.23 | 178,684.11 |
| | | (0.7, 0.95) | 180,878.02 | 180,570.51 | 180,316.23 |
| 1521.5 | 756 | (0.7, 0.7) | 166,993.24 | 166,707.14 | 166,474.98 |
| | | (0.7, 0.8) | 172,119.61 | 171,827.70 | 171,586.29 |
| | | (0.7, 0.9) | 177,294.20 | 176,990.77 | 176,745.30 |
| | | (0.7, 0.95) | 179,915.78 | 179,610.43 | 179,356.69 |
| 2434.4 | 472.5 | (0.7, 0.7) | 160,292.74 | 160,021.56 | 159,790.66 |
| | | (0.7, 0.8) | 163,496.72 | 163,221.91 | 162,985.23 |
| | | (0.7, 0.9) | 166,730.84 | 166,448.83 | 166,209.61 |
| | | (0.7, 0.95) | 168,369.32 | 168,086.11 | 167,841.74 |
| 2434.4 | 756 | (0.7, 0.7) | 154,484.54 | 154,222.74 | 154,000.48 |
| | | (0.7, 0.8) | 159,610.92 | 159,343.30 | 159,111.80 |
| | | (0.7, 0.9) | 164,785.50 | 164,506.37 | 164,270.80 |
| | | (0.7, 0.95) | 167,407.08 | 167,126.03 | 166,882.20 |

**Table A.13** Profit of This Study's Policy when Varying $r_1$ and $r_2$ of Flight A. Other Parameters are $p_1 = 3043, p_2 = 945, g_1 = g_2 = 0, \theta_1 = 0.9$ and $\theta_2 = 0.7$.

| $r_1$ | $r_2$ | Profit | | |
|---|---|---|---|---|
| | | **N1** | **N2** | **N3** |
| 1521.5 | 472.5 | 177,072.79 | 178,328.65 | 177,668.75 |
| 1521.5 | 756 | 171,550.60 | 172,768.78 | 172,127.85 |
| 2434.4 | 472.5 | 173,170.70 | 174,402.41 | 173,757.60 |
| 2434.4 | 756 | 167,648.51 | 168,842.54 | 168,216.70 |

**Table A.14** Profit of This Study's Policy when Varying $r_1$ and $r_2$ of Flight B. Other Parameters are $p_1 = 3043, p_2 = 945, g_1 = g_2 = 0, \theta_1 = 0.9$ and $\theta_2 = 0.7$.

| $r_1$ | $r_2$ | Profit | | |
|---|---|---|---|---|
| | | **N1** | **N2** | **N3** |
| 1521.5 | 472.5 | 186,880.34 | 186,549.75 | 186,305.45 |
| 1521.5 | 756 | 181,072.14 | 180,750.93 | 180,515.26 |
| 2434.4 | 472.5 | 182,818.99 | 182,491.63 | 182,255.12 |
| 2434.4 | 756 | 177,010.79 | 176,692.81 | 176,464.94 |

**Table A.15** Profit in Different Cases of Number of Update Booking Limit Points of Flight A.

| Block $(\theta_1, \theta_2)$ | Profit | | | | | |
|---|---|---|---|---|---|---|
| | **FCFS** | **No update** | **7 points** | **13 points** | **14 points** | **18 points** |
| (0.7,0.7) | 219,029.91 | 219,008.28 | 219,376.58 | 220,374.37 | 219,948.08 | 219,971.88 |
| (0.7,0.8) | 223,478.50 | 223,487.11 | 223,810.73 | 224,742.01 | 224,306.35 | 224,323.33 |
| (0.7,0.9) | 227,915.28 | 227,897.58 | 228,176.42 | 229,072.42 | 228,620.70 | 228,629.92 |
| (0.7,0.95) | 230,137.92 | 230,119.75 | 230,380.55 | 231,246.45 | 230,786.09 | 230,797.14 |
| (0.8,0.7) | 234,169.45 | 234,213.70 | 234,508.94 | 235,853.49 | 235,402.01 | 235,447.86 |
| (0.8,0.8) | 238,618.04 | 238,896.86 | 239,195.05 | 240,199.09 | 239,735.06 | 239,765.63 |
| (0.8,0.9) | 243,054.81 | 243,375.22 | 243,571.42 | 244,509.19 | 244,032.35 | 244,061.10 |
| (0.8,0.95) | 245,277.45 | 245,342.71 | 245,670.00 | 246,693.66 | 246,206.29 | 246,240.58 |
| (0.9,0.7) | 249,270.03 | 249,345.76 | 249,771.11 | 251,282.44 | 250,834.97 | 250,898.80 |
| (0.9,0.8) | 253,718.62 | 253,795.09 | 254,213.30 | 255,653.68 | 255,185.48 | 255,243.70 |
| (0.9,0.9) | 258,155.39 | 258,273.45 | 258,601.40 | 259,972.36 | 259,479.58 | 259,528.83 |
| (0.9,0.95) | 260,378.03 | 260,240.94 | 260,656.78 | 262,148.38 | 261,650.62 | 261,694.99 |
| (0.95,0.7) | 256,892.14 | 257,240.52 | 257,670.33 | 259,033.22 | 258,578.05 | 258,641.42 |
| (0.95,0.8) | 261,340.73 | 261,689.85 | 262,106.17 | 263,384.39 | 262,906.33 | 262,973.00 |
| (0.95,0.9) | 265,777.50 | 266,168.21 | 266,497.58 | 267,719.10 | 267,221.05 | 267,278.00 |
| (0.95,0.95) | 268,000.14 | 268,135.70 | 268,548.70 | 269,899.72 | 269,383.25 | 269,443.30 |

**Table A.16** Profit in Different Cases of Number of Update Booking Limit Points of Flight B.

| **Block** $(\theta_1,\theta_2)$ | **Profit** | | | | | |
|---|---|---|---|---|---|---|
| | **FCFS** | **Not-update** | **7 points** | **13 points** | **14 points** | **18 points** |
| (0.7,0.7) | 218,745.10 | 218,669.24 | 219,031.33 | 220,004.35 | 219,571.92 | 219,590.66 |
| (0.7,0.8) | 223,184.24 | 223,142.87 | 223,460.28 | 224,374.91 | 223,934.86 | 223,945.36 |
| (0.7,0.9) | 227,618.65 | 227,588.59 | 227,840.07 | 228,705.66 | 228,248.60 | 228,250.39 |
| (0.7,0.95) | 229,838.93 | 229,810.76 | 230,042.31 | 230,882.59 | 230,417.37 | 230,420.52 |
| (0.8,0.7) | 233,857.85 | 233,834.28 | 234,125.33 | 235,441.08 | 234,985.21 | 235,025.30 |
| (0.8,0.8) | 238,296.99 | 238,513.67 | 238,805.31 | 239,778.90 | 239,313.53 | 239,344.64 |
| (0.8,0.9) | 242,731.40 | 242,984.94 | 243,172.03 | 244,094.69 | 243,614.61 | 243,636.88 |
| (0.8,0.95) | 244,951.68 | 244,955.73 | 245,272.03 | 246,284.77 | 245,793.49 | 245,820.35 |
| (0.9,0.7) | 248,956.00 | 249,175.87 | 249,443.67 | 250,833.90 | 250,376.09 | 250,428.23 |
| (0.9,0.8) | 253,395.14 | 253,379.23 | 253,788.30 | 255,201.37 | 254,726.81 | 254,773.81 |
| (0.9,0.9) | 257,829.55 | 257,850.50 | 258,169.32 | 259,517.28 | 259,017.35 | 259,058.41 |
| (0.9,0.95) | 260,049.83 | 260,039.28 | 260,360.50 | 261,696.88 | 261,189.59 | 261,224.57 |
| (0.95,0.7) | 256,563.50 | 256,793.87 | 257,221.31 | 258,586.78 | 258,121.26 | 258,174.35 |
| (0.95,0.8) | 261,002.64 | 261,261.82 | 261,669.00 | 262,921.73 | 262,435.90 | 262,491.35 |
| (0.95,0.9) | 265,437.05 | 265,733.08 | 266,053.33 | 267,255.02 | 266,749.21 | 266,795.61 |
| (0.95,0.95) | 267,657.33 | 267,703.88 | 268,105.87 | 269,435.85 | 268,912.08 | 268,962.53 |

**Table A.17** Multiple Comparison of Different Number of Update Booking Limit Points of Flight A.

| | **diff** | **lwr** | **upr** | ***p*-adj** |
|---|---|---|---|---|
| 2-1 | 126.05 | -43.81 | 295.90 | 0.26 |
| 3-1 | 471.32 | 301.47 | 641.18 | 0.00 |
| 4-1 | 1,660.63 | 1,490.77 | 1,830.48 | 0.00 |
| 5-1 | 1,191.39 | 1,021.54 | 1,361.25 | 0.00 |
| 6-1 | 1,232.85 | 1,062.99 | 1,402.70 | 0.00 |
| 3-2 | 345.27 | 175.42 | 515.13 | 0.00 |
| 4-2 | 1,534.58 | 1,364.72 | 1,704.43 | 0.00 |
| 5-2 | 1,065.35 | 895.49 | 1,235.20 | 0.00 |
| 6-2 | 1,106.80 | 936.94 | 1,276.65 | 0.00 |
| 4-3 | 1,189.31 | 1,019.45 | 1,359.16 | 0.00 |
| 5-3 | 720.07 | 550.22 | 889.93 | 0.00 |
| 6-3 | 761.53 | 591.67 | 931.38 | 0.00 |
| 5-4 | -469.23 | -639.09 | -299.38 | 0.00 |
| 6-4 | -427.78 | -597.64 | -257.93 | 0.00 |
| 6-5 | 41.45 | -128.40 | 211.31 | 0.98 |

**Note:** *p*-adj is the p-value after adjustment for the multiple comparisons.

**Table A.18** Multiple Comparison of Different Number of Update Booking Limit Points of Flight B.

| | **diff** | **lwr** | **upr** | ***p*-adj** |
|---|---|---|---|---|
| 2-1 | 82.61 | -72.32 | 237.53 | 0.63 |
| 3-1 | 402.76 | 247.83 | 557.68 | 0.00 |
| 4-1 | 1,556.24 | 1,401.32 | 1,711.17 | 0.00 |
| 5-1 | 1,080.75 | 925.82 | 1,235.68 | 0.00 |
| 6-1 | 1,114.19 | 959.27 | 1,269.12 | 0.00 |
| 3-2 | 320.15 | 165.22 | 475.08 | 0.00 |
| 4-2 | 1,473.64 | 1,318.71 | 1,628.56 | 0.00 |
| 5-2 | 998.14 | 843.22 | 1,153.07 | 0.00 |
| 6-2 | 1,031.59 | 876.66 | 1,186.51 | 0.00 |
| 4-3 | 1,153.48 | 998.56 | 1,308.41 | 0.00 |
| 5-3 | 677.99 | 523.07 | 832.92 | 0.00 |
| 6-3 | 711.43 | 556.51 | 866.36 | 0.00 |
| 5-4 | -475.49 | -630.42 | -320.56 | 0.00 |
| 6-4 | -442.05 | -596.98 | -287.12 | 0.00 |
| 6-5 | 33.44 | -121.49 | 188.37 | 0.99 |

**Note:** *p*-adj is the p-value after adjustment for the multiple comparisons.

**Table A.19** Profit in Different Cases of Incorrect Initial Mean Demand of 7 Points Case.

| **Block** | **Profit** | | | | |
|---|---|---|---|---|---|
| **($\theta_1$,$\theta_2$)** | **Int 0.50** | **Int 0.75** | **Int 1.00** | **Int 1.25** | **Int 1.50** |
| (0.7,0.7) | 220,558.90 | 220,406.37 | 219,376.58 | 211,338.56 | 198,654.78 |
| (0.7,0.8) | 225,088.49 | 224,962.48 | 223,810.73 | 215,886.12 | 202,456.99 |
| (0.7,0.9) | 229,604.44 | 229,460.95 | 228,176.42 | 219,687.86 | 205,553.75 |
| (0.7,0.95) | 231,877.63 | 231,725.17 | 230,380.55 | 222,433.44 | 207,191.44 |
| (0.8,0.7) | 235,695.52 | 235,542.99 | 234,508.94 | 226,884.44 | 213,387.98 |
| (0.8,0.8) | 240,227.00 | 240,062.19 | 239,195.05 | 230,578.44 | 217,175.06 |
| (0.8,0.9) | 244,744.44 | 244,561.68 | 243,571.42 | 235,232.99 | 220,277.97 |
| (0.8,0.95) | 247,019.74 | 246,867.27 | 245,670.00 | 237,119.21 | 221,803.20 |
| (0.9,0.7) | 250,801.98 | 250,612.70 | 249,771.11 | 241,709.05 | 228,155.52 |
| (0.9,0.8) | 255,329.95 | 255,127.91 | 254,213.30 | 245,341.15 | 231,936.93 |
| (0.9,0.9) | 259,852.46 | 259,669.69 | 258,601.40 | 249,916.15 | 234,959.99 |
| (0.9,0.95) | 262,126.60 | 261,934.86 | 260,656.78 | 251,800.95 | 237,354.15 |
| (0.95,0.7) | 258,428.96 | 258,232.37 | 257,670.33 | 249,469.91 | 235,913.95 |
| (0.95,0.8) | 262,956.93 | 262,747.58 | 262,106.17 | 253,102.02 | 238,863.30 |
| (0.95,0.9) | 267,475.92 | 267,288.29 | 266,497.58 | 256,778.54 | 242,718.42 |
| (0.95,0.95) | 269,749.59 | 269,552.98 | 268,548.70 | 259,552.08 | 244,168.53 |

**Table A.20** Profit in Different Cases of Incorrect Initial Mean Demand of 13 Points Case.

| **Block** | **Profit** | | | | |
|---|---|---|---|---|---|
| **($\theta_1,\theta_2$)** | **Int 0.50** | **Int 0.75** | **Int 1.00** | **Int 1.25** | **Int 1.50** |
| (0.7,0.7) | 220,374.37 | 220,374.37 | 220,374.37 | 220,374.37 | 220,389.07 |
| (0.7,0.8) | 224,742.01 | 224,742.01 | 224,742.01 | 224,742.01 | 224,754.28 |
| (0.7,0.9) | 229,072.42 | 229,072.42 | 229,072.42 | 229,072.42 | 229,083.75 |
| (0.7,0.95) | 231,246.45 | 231,246.45 | 231,246.45 | 231,246.45 | 231,256.83 |
| (0.8,0.7) | 235,853.49 | 235,853.49 | 235,853.49 | 235,853.49 | 235,875.44 |
| (0.8,0.8) | 240,199.09 | 240,199.09 | 240,199.09 | 240,199.09 | 240,221.09 |
| (0.8,0.9) | 244,509.19 | 244,509.19 | 244,509.19 | 244,509.19 | 244,530.25 |
| (0.8,0.95) | 246,693.66 | 246,693.66 | 246,693.66 | 246,693.66 | 246,716.08 |
| (0.9,0.7) | 251,282.44 | 251,282.44 | 251,282.44 | 251,282.44 | 251,300.55 |
| (0.9,0.8) | 255,653.68 | 255,653.68 | 255,653.68 | 255,653.68 | 255,670.15 |
| (0.9,0.9) | 259,972.36 | 259,972.36 | 259,972.36 | 259,972.36 | 259,990.46 |
| (0.9,0.95) | 262,148.38 | 262,148.38 | 262,148.38 | 262,148.38 | 262,166.06 |
| (0.95,0.7) | 259,033.22 | 259,033.22 | 259,033.22 | 259,033.22 | 259,052.61 |
| (0.95,0.8) | 263,384.39 | 263,384.39 | 263,384.39 | 263,384.39 | 263,403.99 |
| (0.95,0.9) | 267,719.10 | 267,719.10 | 267,719.10 | 267,719.10 | 267,742.06 |
| (0.95,0.95) | 269,899.72 | 269,899.72 | 269,899.72 | 269,899.72 | 269,922.21 |

**Table A.21** Profit in Different Cases of Incorrect Initial Mean Demand of 14 Points Case.

| **Block** | **Profit** | | | | |
|---|---|---|---|---|---|
| **($\theta_1,\theta_2$)** | **Int 0.50** | **Int 0.75** | **Int 1.00** | **Int 1.25** | **Int 1.50** |
| (0.7,0.7) | 219,948.08 | 219,948.08 | 219,948.08 | 219,948.08 | 219,948.08 |
| (0.7,0.8) | 224,306.35 | 224,306.35 | 224,306.35 | 224,306.35 | 224,306.35 |
| (0.7,0.9) | 228,620.70 | 228,620.70 | 228,620.70 | 228,620.70 | 228,620.70 |
| (0.7,0.95) | 230,786.09 | 230,786.09 | 230,786.09 | 230,786.09 | 230,786.09 |
| (0.8,0.7) | 235,402.01 | 235,402.01 | 235,402.01 | 235,402.01 | 235,402.01 |
| (0.8,0.8) | 239,735.06 | 239,735.06 | 239,735.06 | 239,735.06 | 239,735.06 |
| (0.8,0.9) | 244,032.35 | 244,032.35 | 244,032.35 | 244,032.35 | 244,032.35 |
| (0.8,0.95) | 246,206.29 | 246,206.29 | 246,206.29 | 246,206.29 | 246,206.29 |
| (0.9,0.7) | 250,834.97 | 250,834.97 | 250,834.97 | 250,834.97 | 250,834.97 |
| (0.9,0.8) | 255,185.48 | 255,185.48 | 255,185.48 | 255,185.48 | 255,185.48 |
| (0.9,0.9) | 259,479.58 | 259,479.58 | 259,479.58 | 259,479.58 | 259,479.58 |
| (0.9,0.95) | 261,650.62 | 261,650.62 | 261,650.62 | 261,650.62 | 261,650.62 |
| (0.95,0.7) | 258,578.05 | 258,578.05 | 258,578.05 | 258,578.05 | 258,578.05 |
| (0.95,0.8) | 262,906.33 | 262,906.33 | 262,906.33 | 262,906.33 | 262,906.33 |
| (0.95,0.9) | 267,221.05 | 267,221.05 | 267,221.05 | 267,221.05 | 267,221.05 |
| (0.95,0.95) | 269,383.25 | 269,383.25 | 269,383.25 | 269,383.25 | 269,383.25 |

**Table A.22** Profit in Different Cases of Incorrect Initial Mean Demand of 18 Points Case.

| **Block** | **Profit** | | | | |
|---|---|---|---|---|---|
| **($\theta_1,\theta_2$)** | **Int 0.50** | **Int 0.75** | **Int 1.00** | **Int 1.25** | **Int 1.50** |
| (0.7,0.7) | 219,971.88 | 219,971.88 | 219,971.88 | 219,971.88 | 219,971.88 |
| (0.7,0.8) | 224,323.33 | 224,323.33 | 224,323.33 | 224,323.33 | 224,323.33 |
| (0.7,0.9) | 228,629.92 | 228,629.92 | 228,629.92 | 228,629.92 | 228,629.92 |
| (0.7,0.95) | 230,797.14 | 230,797.14 | 230,797.14 | 230,797.14 | 230,797.14 |
| (0.8,0.7) | 235,447.86 | 235,447.86 | 235,447.86 | 235,447.86 | 235,447.86 |
| (0.8,0.8) | 239,765.63 | 239,765.63 | 239,765.63 | 239,765.63 | 239,765.63 |
| (0.8,0.9) | 244,061.10 | 244,061.10 | 244,061.10 | 244,061.10 | 244,061.10 |
| (0.8,0.95) | 246,240.58 | 246,240.58 | 246,240.58 | 246,240.58 | 246,240.58 |
| (0.9,0.7) | 250,898.80 | 250,898.80 | 250,898.80 | 250,898.80 | 250,898.80 |
| (0.9,0.8) | 255,243.70 | 255,243.70 | 255,243.70 | 255,243.70 | 255,243.70 |
| (0.9,0.9) | 259,528.83 | 259,528.83 | 259,528.83 | 259,528.83 | 259,528.83 |
| (0.9,0.95) | 261,694.99 | 261,694.99 | 261,694.99 | 261,694.99 | 261,694.99 |
| (0.95,0.7) | 258,641.42 | 258,641.42 | 258,641.42 | 258,641.42 | 258,641.42 |
| (0.95,0.8) | 262,973.00 | 262,973.00 | 262,973.00 | 262,973.00 | 262,973.00 |
| (0.95,0.9) | 267,278.00 | 267,278.00 | 267,278.00 | 267,278.00 | 267,278.00 |
| (0.95,0.95) | 269,443.30 | 269,443.30 | 269,443.30 | 269,443.30 | 269,443.30 |

**Table A.23** Multiple Comparison of Different Initial Mean Demand of 7 Points Case.

| | **diff** | **lwr** | **upr** | ***p*-adj** |
|---|---|---|---|---|
| 2-1 | -173.94 | -698.14 | 350.26 | 0.88 |
| 3-1 | -1,173.97 | -1,698.17 | -649.77 | 0.00 |
| 4-1 | -9,669.23 | -10,193.43 | -9,145.03 | 0.00 |
| 5-1 | -23,810.41 | -24,334.61 | -23,286.21 | 0.00 |
| 3-2 | -1,000.03 | -1,524.23 | -475.83 | 0.00 |
| 4-2 | -9,495.29 | -10,019.49 | -8,971.09 | 0.00 |
| 5-2 | -23,636.47 | -24,160.67 | -23,112.27 | 0.00 |
| 4-3 | -8,495.26 | -9,019.46 | -7,971.06 | 0.00 |
| 5-3 | -22,636.44 | -23,160.64 | -22,112.24 | 0.00 |
| 5-4 | -14,141.19 | -14,665.39 | -13,616.99 | 0.00 |

**Note:** *p*-adj is the p-value after adjustment for the multiple comparisons.

**Table A.24** Profit in Different Cases of Smoothing Constant.

| **Block** | **Profit** | | | | |
|---|---|---|---|---|---|
| $(\theta_1,\theta_2)$ | $\rho = 0.1$ | $\rho = 0.2$ | $\rho = 0.3$ | $\rho = 0.4$ | $\rho = 0.5$ |
| (0.7,0.7) | 156,111.48 | 172,137.52 | 185,336.54 | 195,773.77 | 203,570.06 |
| (0.7,0.8) | 159,283.37 | 175,633.08 | 189,095.28 | 199,727.65 | 207,620.60 |
| (0.7,0.9) | 162,460.93 | 179,120.13 | 192,820.94 | 203,660.74 | 211,633.61 |
| (0.7,0.95) | 164,033.89 | 180,853.26 | 194,695.82 | 205,625.40 | 213,651.19 |
| (0.8,0.7) | 166,956.73 | 184,184.71 | 198,277.73 | 209,388.29 | 217,841.44 |
| (0.8,0.8) | 170,128.62 | 187,684.05 | 202,030.80 | 213,330.83 | 221,869.56 |
| (0.8,0.9) | 173,306.19 | 191,167.79 | 205,759.77 | 217,250.95 | 225,891.27 |
| (0.8,0.95) | 174,879.14 | 192,897.61 | 207,634.18 | 219,216.55 | 227,897.98 |
| (0.9,0.7) | 177,716.78 | 196,092.12 | 211,109.85 | 223,013.34 | 232,136.36 |
| (0.9,0.8) | 180,888.67 | 199,588.62 | 214,867.64 | 226,959.66 | 236,143.63 |
| (0.9,0.9) | 184,066.23 | 203,074.26 | 218,587.16 | 230,880.47 | 240,148.54 |
| (0.9,0.95) | 185,639.19 | 204,802.66 | 220,462.98 | 232,828.11 | 242,157.40 |
| (0.95,0.7) | 183,293.99 | 202,149.53 | 217,544.40 | 229,778.10 | 239,232.15 |
| (0.95,0.8) | 186,465.88 | 205,646.03 | 221,300.77 | 233,705.52 | 243,238.48 |
| (0.95,0.9) | 189,643.44 | 209,131.19 | 225,022.65 | 237,625.38 | 247,234.41 |
| (0.95,0.95) | 191,216.40 | 210,860.54 | 226,901.31 | 239,582.00 | 249,242.54 |

**Table A.24** (Continued)

| **Block** | **Profit** | | | | |
|---|---|---|---|---|---|
| $(\theta_1,\theta_2)$ | $\rho=0.6$ | $\rho=0.7$ | $\rho_{\min SSE}$ | $\rho=0.8$ | $\rho=0.9$ |
| (0.7,0.7) | 209,415.51 | 213,769.45 | 220,374.37 | 216,789.91 | 218,443.73 |
| (0.7,0.8) | 213,501.82 | 217,886.28 | 224,742.01 | 220,944.85 | 222,613.55 |
| (0.7,0.9) | 217,534.40 | 221,982.31 | 229,072.42 | 225,060.22 | 226,737.38 |
| (0.7,0.95) | 219,589.51 | 224,037.94 | 231,246.45 | 227,143.47 | 228,829.13 |
| (0.8,0.7) | 224,202.59 | 228,893.90 | 235,853.49 | 232,166.75 | 233,896.18 |
| (0.8,0.8) | 228,277.75 | 232,995.22 | 240,199.09 | 236,312.20 | 238,076.86 |
| (0.8,0.9) | 232,324.50 | 237,103.80 | 244,509.19 | 240,420.36 | 242,196.44 |
| (0.8,0.95) | 234,371.85 | 239,170.37 | 246,693.66 | 242,509.67 | 244,285.36 |
| (0.9,0.7) | 238,994.84 | 244,067.54 | 251,282.44 | 247,545.33 | 249,378.53 |
| (0.9,0.8) | 243,038.40 | 248,164.63 | 255,653.68 | 251,706.28 | 253,550.55 |
| (0.9,0.9) | 247,070.98 | 252,263.69 | 259,972.36 | 255,817.56 | 257,680.94 |
| (0.9,0.95) | 249,109.47 | 254,325.15 | 262,148.38 | 257,895.62 | 259,776.47 |
| (0.95,0.7) | 246,435.22 | 251,676.19 | 259,033.22 | 255,263.61 | 257,197.30 |
| (0.95,0.8) | 250,484.81 | 255,780.35 | 263,384.39 | 259,409.79 | 261,372.69 |
| (0.95,0.9) | 254,517.10 | 259,872.76 | 267,719.10 | 263,538.95 | 265,498.62 |
| (0.95,0.95) | 256,549.66 | 261,927.61 | 269,899.72 | 265,614.91 | 267,590.64 |

**Table A.25** Multiple Comparison of Different Smoothing Constant.

| | **diff** | **lwr** | **upr** | ***p*-adj** |
|---|---|---|---|---|
| 2-1 | 18,058.26 | 16,185.41 | 19,931.11 | 0.00 |
| 3-1 | 32,834.81 | 30,961.96 | 34,707.66 | 0.00 |
| 4-1 | 44,515.99 | 42,643.14 | 46,388.84 | 0.00 |
| 5-1 | 53,338.64 | 51,465.79 | 55,211.49 | 0.00 |
| 6-1 | 59,957.97 | 58,085.12 | 61,830.82 | 0.00 |
| 7-1 | 64,864.14 | 62,991.29 | 66,736.99 | 0.00 |
| 8-1 | 72,230.81 | 70,357.96 | 74,103.66 | 0.00 |
| 9-1 | 68,253.04 | 66,380.18 | 70,125.89 | 0.00 |
| 10-1 | 70,064.59 | 68,191.74 | 71,937.44 | 0.00 |
| 3-2 | 14,776.55 | 12,903.70 | 16,649.40 | 0.00 |
| 4-2 | 26,457.73 | 24,584.88 | 28,330.58 | 0.00 |
| 5-2 | 35,280.38 | 33,407.53 | 37,153.23 | 0.00 |
| 6-2 | 41,899.71 | 40,026.86 | 43,772.56 | 0.00 |
| 7-2 | 46,805.88 | 44,933.03 | 48,678.73 | 0.00 |
| 8-2 | 54,172.55 | 52,299.70 | 56,045.40 | 0.00 |
| 9-2 | 50,194.78 | 48,321.92 | 52,067.63 | 0.00 |
| 10-2 | 52,006.33 | 50,133.48 | 53,879.18 | 0.00 |
| 4-3 | 11,681.18 | 9,808.33 | 13,554.03 | 0.00 |
| 5-3 | 20,503.84 | 18,630.98 | 22,376.69 | 0.00 |
| 6-3 | 27,123.16 | 25,250.31 | 28,996.01 | 0.00 |
| 7-3 | 32,029.34 | 30,156.48 | 33,902.19 | 0.00 |
| 8-3 | 39,396.01 | 37,523.16 | 41,268.86 | 0.00 |
| 9- | 35,418.23 | 33,545.38 | 37,291.08 | 0.00 |
| 10-3 | 37,229.78 | 35,356.93 | 39,102.63 | 0.00 |
| 5-4 | 8,822.65 | 6,949.80 | 10,695.50 | 0.00 |
| 6-4 | 15,441.98 | 13,569.13 | 17,314.83 | 0.00 |
| 7-4 | 20,348.15 | 18,475.30 | 22,221.00 | 0.00 |
| 8-4 | 27,714.82 | 25,841.97 | 29,587.67 | 0.00 |

**Table A.25** (Continued)

| | **diff** | **lwr** | **upr** | ***p*-adj** |
|---|---|---|---|---|
| 9-4 | 23,737.05 | 21,864.19 | 25,609.90 | 0.00 |
| 10-4 | 25,548.60 | 23,675.75 | 27,421.45 | 0.00 |
| 6-5 | 6,619.33 | 4,746.47 | 8,492.18 | 0.00 |
| 7-5 | 11,525.50 | 9,652.65 | 13,398.35 | 0.00 |
| 8-5 | 18,892.17 | 17,019.32 | 20,765.02 | 0.00 |
| 9-5 | 14,914.39 | 13,041.54 | 16,787.24 | 0.00 |
| 10-5 | 16,725.95 | 14,853.10 | 18,598.80 | 0.00 |
| 7-6 | 4,906.17 | 3,033.32 | 6,779.02 | 0.00 |
| 8-6 | 12,272.85 | 10,400.00 | 14,145.70 | 0.00 |
| 9-6 | 8,295.07 | 6,422.22 | 10,167.92 | 0.00 |
| 10-6 | 10,106.62 | 8,233.77 | 11,979.47 | 0.00 |
| 8-7 | 7,366.67 | 5,493.82 | 9,239.52 | 0.00 |
| 9-7 | 3,388.89 | 1,516.04 | 5,261.74 | 0.00 |
| 10-7 | 5,200.45 | 3,327.60 | 7,073.30 | 0.00 |
| 9-8 | -3,977.78 | -5,850.63 | -2,104.93 | 0.00 |
| 10-8 | -2,166.22 | -4,039.07 | -293.37 | 0.01 |
| 10-9 | 1,811.56 | -61.30 | 3,684.41 | 0.07 |

**Note:** *p*-adj is the p-value after adjustment for the multiple comparisons.

# Appendix B

# R SYNTAX

## R Syntax for Calculate the Expected profit of Two-class Overbooking Model

```
expprofit = function(x, K, p1, p2, g1, g2, r1, r2, h, theta1, theta2, lambda1, lambda2){
#-------------------------EXPECTED B2--------------------------
expb2 <- function(x){
cdfb2_1 <- function(t){ppois(t,lambda2,FALSE)}
exp_B2<-0
if (x==0){
  exp_B2 <- 0
}else{
  t <- seq(0,x-1)
  exp_B2 <- sum(sapply(t, cdfb2_1))
}
exp_B2
}
#-------------PROBABILITY MASS FUNCTION OF B2----------------
dPrB2 <- function(x,j){
pmfB2 <- if (x==0){ 0
                }else if (j<x) {dpois(j,lambda2)
                }else if (j==x) {ppois(x-1,lambda2,FALSE)
                }else {0
                }
pmfB2
}
#---------CUMULATIVE DISTRIBUTION FUNCTION OF B2-------------
pPrB2 <- function(k){
cmfB2 <-0
j <- seq(0,k)
cmfB2_1 <- function(j){dPrB2(x,j)}
cmfB2_2 <- sapply(j, cmfB2_1)
cmfB2 <- sum(cmfB2_2)
cmfB2
}
#----------PROBABILITY MASS FUNCTION OF (K-B2)+-------------
drcap <- function(K,k){
pmfrcap <-          if (k==0){ 1-pPrB2(K-1)
                              }else if (k>0 & k<=K){ dPrB2(x,K-k)
                              }else 0
```

```
pmfrcap
}
#---------CUMULATIVE DISTRIBUTION FUNCTION OF (C-B2)+-------
prcap <- function(n){
cmfrcap <- 0
i <- seq(0,n)
cmfrcap_1 <- function(i){drcap(K,i)}
cmfrcap_2 <- sapply(i, cmfrcap_1)
cmfrcap <- sum(cmfrcap_2)
cmfrcap
}
#------------------------EXPECTED OF B1--------------------
expb1 <- function(x,K){
exp_B1 <- 0
if (x==0){
i <- seq(0,(K-1))
exp_B1.1 <- function(i){ppois(i,lambda1,FALSE)}
exp_B1 <- sum(sapply(i, exp_B1.1))
exp_B1
}else{
i <- seq(0,K)
exp_B1.2 <- function(i){(1-prcap(i))*(1-ppois(i,lambda1))}
exp_B1 <- sum(sapply(i, exp_B1.2))
exp_B1
}
}
#------------------------PROBABILITY MASS FUNCTION OF SHOW-UP--------
pshowup <- function(j){
p_showup <- 0
if (j==0){
m <- seq(1,x)
p_showup.1 <- function(m){dbinom(j,m,theta2)*dPrB2(x,m)}
p_showup.2 <- sapply(m, p_showup.1)
p_showup <- dPrB2(x,0) + sum(p_showup.2)
}else
m <- seq(j,x)
p_showup.1 <- function(m){dbinom(j,m,theta2)*dPrB2(x,m)}
p_showup.2 <- sapply(m, p_showup.1)
p_showup <- sum(p_showup.2)
p_showup
}
#------------------------EXPECTED OF DENIED BOARDING-----------#
expbump <- function(x,K){
exp_bump <- 0
if (x > K){
j <- seq(K+1,x)
exp_bump.1 <- function(j){(j-K)*pshowup(j)}
exp_bump.2 <- sapply(j, exp_bump.1)
```

```
exp_bump <- sum(exp_bump.2)
}else 0
exp_bump
}
profit = (p2+g2-r2+r2*theta2)*expb2(x)+(p1+g1-r1+r1*theta1)*expb1(x,K)-g2*lambda2-g1*lambda1-
h*expbump(x,K)
return(profit)
}
```

## R Syntax for Numerical Experiment using Simulation Data

```
p1 <- 100
g1 <- p1
for (k in 1:144){
h <- rep(seq(100,500,200),each = 48)
p2 <- rep(rep(seq(20,80,30), each = 16),3)
r1 <- rep(rep(c(0.5*p1,0.8*p1), each = 8),9)
r2 <- rep(c(rep(rep(20*c(0.5,0.8),each = 4),2),rep(rep(50*c(0.5,0.8),each =
4),2),rep(rep(80*c(0.5,0.8),each = 4),2)),3)
theta1 <- rep(rep(c(0.7,0.9),each = 2),36)
theta2 <- rep(c(0.7,0.9),72)
Z <- cbind(h,p2,r1,r2,theta1,theta2)
lambda1 <- rep(seq(40,140,20),each = 6)
lambda2 <- rep(seq(40,140,20),6)
lambda <- cbind(lambda1, lambda2)
profit = array(0,dim=c(10836,4))
for(i in 1:36){
for(j in 0:300){
profit[301*(i-1)+(j+1),] <-
cbind(j,expprofit(j,K,p1,Z[k,2],g1,Z[k,2],Z[k,3],Z[k,4],Z[k,1],Z[k,5],Z[k,6],lambda[i,1],lambda[i,2]),la
mbda[i,1],lambda[i,2])
}
}
colnames(profit)=c("x","Profit","lambda1","lambda2")
}
```

**R Syntax for Compare Performance of Our policy and Airline's Policy**

```
#FlightA_FCFS
K <- 162
p1 <- 3043
p2 <- 945
g1 <- 0
g2 <- 0
h <- 2000
r1 <- 0.8*p1
r2 <- 0.5*p2
theta1 <- rep(c(rep(0.7,4),rep(0.8,4),rep(0.9,4),rep(0.95,4)))
theta2 <- rep(rep(c(0.7, 0.8, 0.9, 0.95)),4)
theta <- cbind(theta1, theta2)
avgprofit <- array(0,16)
obs <- c(133, 138, 154, 144, 164, 157, 145, 115, 129, 103, 118,  92,  90, 132, 162, 124, 127, 124, 113,
72, 82,   73,  92,  46,  63,  88,  68,  70,  61, 136, 136, 163, 120,  85,  94,  76,  53,  64,  94, 124, 89,  100,
104,  93, 128,  87,  92, 108,  90, 107, 164, 163)
#forecast total booking week 53
out.hw=HoltWinters(obs, gamma=FALSE, beta = FALSE)
totalbk.hw=as.numeric(forecast(out.hw,h=1)$mean)
totalbk.hw
lambda1_0 <- 0.4*totalbk.hw
lambda2_0 <- 0.6*totalbk.hw
for (i in 1:16){
  profit <- array(0,1000)
  alpha1 <- p1+g1-r1+r1*theta[i,1]
  alpha2 <- p2+g2-r2+r2*theta[i,2]
  tau1 <- alpha2/alpha1
  tau2 <- alpha2/(h*theta[i,2])
  set.seed(4520389)
  M <- 1000
    for(k in 1:M){
      d1 <- rpois(360, lambda1_0/360)  #Generate demand of class 1 360 day
      d2 <- rpois(360, lambda2_0/360)  #Generate demand of class 2 360 day
      b1 <- array(0,360)
      b2 <- array(0,360)
      recap <- array(0,360)
      recap_0 <- K
        for(m in 1:360){
          if(recap_0 == 0){
            b2[m] <- 0
            b1[m] <- 0
          }
          else{
            b2[m] <- pmin(d2[m],recap_0)
            recap_0 <- pmax(recap_0-b2[m],0)
            b1[m] <- pmin(d1[m],recap_0)
```

```
          recap_0 <- pmax(recap_0-b1[m],0)
         }
         recap[m] <- recap_0
       }
  w1 <- round(theta[i,1]*sum(b1),0)  #generate number of class1 show up
  w2 <- round(theta[i,2]*sum(b2),0)  #generate number of class2 show up
  profit[k] <- p1*sum(b1) + p2*sum(b2) -r1*(sum(b1) - w1) - r2*(sum(b2) - w2) - h*pmax(w2 - K,0)
  print(1000*(i-1)+k)
  }
  avgprofit[i] <- mean(profit)
}

#FlightA_not-update
K <- 162
p1 <- 3043
p2 <- 945
g1 <- 0
g2 <- 0
h <- 2000
r1 <- 0.8*p1
r2 <- 0.5*p2
theta1 <- rep(c(rep(0.7,4),rep(0.8,4),rep(0.9,4),rep(0.95,4)))
theta2 <- rep(rep(c(0.7, 0.8, 0.9, 0.95)),4)
theta <- cbind(theta1, theta2)
avgprofit <- array(0,16)

obs <- c(133, 138, 154, 144, 164, 157, 145, 115, 129, 103, 118,  92,  90, 132, 162, 124, 127, 124, 113,
72, 82,   73,  92,  46,  63,  88,  68,  70,  61, 136, 136, 163, 120,  85,  94,  76,  53,  64,  94, 124, 89,  100,
104,  93, 128,  87,  92, 108,  90, 107, 164, 163)
#forecast total booking next week
out.hw=HoltWinters(obs, gamma=FALSE, beta = FALSE)
totalbk.hw=as.numeric(forecast(out.hw,h=1)$mean)
totalbk.hw
lambda1_0 <- 0.4*totalbk.hw
lambda2_0 <- 0.6*totalbk.hw
for (i in 1:16){
  profit <- array(0,1000)
  alpha1 <- p1+g1-r1+r1*theta[i,1]
  alpha2 <- p2+g2-r2+r2*theta[i,2]
  tau1 <- alpha2/alpha1
  tau2 <- alpha2/(h*theta[i,2])
  set.seed(4520389)
  M=1000
    for(k in 1:M){
      d1 <- rpois(360, lambda1_0/360) #Generate demand of class 1 360 day
      d2 <- rpois(360, lambda2_0/360) #Generate demand of class 2 360 day
      recap <- array(0,360)
      recap_0 <- K
      b1 <- rep(0,360)
```

```
 b2 <- rep(0,360)
 x <- rep(0,360)
 #calculate opt_BL and opt_OBL for initial step
 if (0 < tau1 & tau1 < ppois(K-1,lambda1_0,FALSE)){
  BL_opt <- 0
 }else{
  if (ppois(K-1,lambda1_0,FALSE) < tau1 & tau1 < ppois(0,lambda1_0,FALSE)){
   BL_opt <- K-qpois(1-tau1, lambda1_0)
  }else{
   BL_opt<- K-2
  }
 }
OBL_opt <- K
while(pbinom(K-1,OBL_opt,theta[i,2],FALSE)<tau2) {OBL_opt <- OBL_opt+1}

profit_BL0 <- expprofit(BL_opt, K, p1, p2, g1, g2, r1, r2, h, theta[i,1], theta[i,2], lambda1_0, 
lambda2_0)
profit_OBL0 <- expprofit(OBL_opt, K, p1, p2, g1, g2, r1, r2, h, theta[i,1], theta[i,2], lambda1_0, 
lambda2_0)
profit_K <- expprofit(K-1, K, p1, p2, g1, g2, r1, r2, h, theta[i,1], theta[i,2], lambda1_0, lambda2_0)
x_0 <- K-1
if ((profit_BL0 > profit_K ) | (profit_OBL0 > profit_K) ){
 x_0 <- OBL_opt
if (profit_BL0 > profit_OBL0){
 x_0 <- BL_opt
 }
}
for(m in 1:360){
  if(x_0==0){
   b2[m] <- 0
  }else{
   b2[m] <- pmin(d2[m],x_0)
  }
  x[m] <- x_0-b2[m]
  x_0 <- x[m]
  recap[m] <- pmax(recap_0-b2[m],0)
  if(recap[m]==0){
    b1[m] <- 0
  }else{
    b1[m] <- pmin(d1[m],recap[m])
  }
  recap[m] <- recap[m]-b1[m]
  recap_0 <- recap[m]
 }
w1 <- round(theta[i,1]*sum(b1),0) #generate number of class1 show up
w2 <- round(theta[i,2]*sum(b2),0) #generate number of class2 show up
profit[k] <- p1*sum(b1) + p2*sum(b2) -r1*(sum(b1) - w1) - r2*(sum(b2)-w2) - h*pmax(w2-K,0)
profit
print(1000*(i-1)+k)
```

```
  }
  avgprofit[i] <- mean(profit)
}
#FlightA_18points
K <- 162
p1 <- 3043
p2 <- 945
g1 <- 0
g2 <- 0
h <- 2000
r1 <- 0.8*p1
r2 <- 0.5*p2
theta1 <- rep(c(rep(0.7,4),rep(0.8,4),rep(0.9,4),rep(0.95,4)))
theta2 <- rep(rep(c(0.7, 0.8, 0.9, 0.95)),4)
theta <- cbind(theta1, theta2)
re_cal <- c(seq(30,330,30), seq(354,360)) #set recalculate point
L <- length(re_cal)
n_re_cal <- re_cal-c(0,re_cal[-L])  #set start period point
sre_cal <- re_cal-n_re_cal+1        #set end period point
avgprofit <- array(0,16)
obs <- c(133, 138, 154, 144, 164, 157, 145, 115, 129, 103, 118,  92,  90, 132, 162, 124, 127, 124, 113,
72, 82,   73,  92,  46,  63,  88,  68,  70,  61, 136, 136, 163, 120,  85,  94,  76,  53,  64,  94, 124, 89,  100,
104,  93, 128,  87,  92, 108,  90, 107, 164, 163)
#forecast total booking next week
out.hw <- HoltWinters(obs, gamma=FALSE, beta = FALSE)
totalbk.hw <- as.numeric(forecast(out.hw,h=1)$mean)
lambda1_0 <- 0.4*totalbk.hw
lambda2_0 <- 0.6*totalbk.hw
for (i in 1:16){
  profit <- array(0,1000)
  alpha1 <- p1+g1-r1+r1*theta[i,1]
  alpha2 <- p2+g2-r2+r2*theta[i,2]
  tau1 <- alpha2/alpha1
  tau2 <- alpha2/(h*theta[i,2])
    set.seed(4520389)
  M=1000
  for(k in 1:M){
    d1 <- rpois(360, lambda1_0/360) #Generate demand of class 1 360 day
    d2 <- rpois(360, lambda2_0/360) #Generate demand of class 2 360 day
    recap <- array(0,360)
    recap_0 <- K
    b1 <- rep(0,360)
    b2 <- rep(0,360)
    x <- rep(0,360)
    #start calculate opt_BL and opt_OBL for initial step
    if (0 < tau1 & tau1 < ppois(K-1,lambda1_0,FALSE)){
      BL_opt <- 0
    }else{
      if (ppois(K-1,lambda1_0,FALSE) < tau1 & tau1 < ppois(0,lambda1_0,FALSE)){
```

```
      BL_opt <- K-qpois(1-tau1, lambda1_0)
    }else{
      BL_opt<- K-2
    }
  }
  BL_opt
  OBL_opt <- K
  while(pbinom(K-1,OBL_opt,theta[i,2],FALSE)<tau2) {OBL_opt <- OBL_opt+1}
  OBL_opt
  profit_BL0 <- expprofit(BL_opt, K, p1, p2, g1, g2, r1, r2, h, theta[i,1], theta[i,2], lambda1_0,
lambda2_0)
  profit_OBL0 <- expprofit(OBL_opt, K, p1, p2, g1, g2, r1, r2, h, theta[i,1], theta[i,2], lambda1_0,
lambda2_0)
  profit_K <- expprofit(K-1, K, p1, p2, g1, g2, r1, r2, h, theta[i,1], theta[i,2], lambda1_0, lambda2_0)
  x_0 <- K-1
  if ((profit_BL0 > profit_K ) | (profit_OBL0 > profit_K) ){
    x_0 <- OBL_opt
    if (profit_BL0 > profit_OBL0){
      x_0 <- BL_opt
    }
  }
  #end calculate opt_BL and opt_OBL for initial step
  for(j in 1:L){
    for(m in sre_cal[j]:re_cal[j]){
      if(x_0==0){
        b2[m] <- 0
      }else{
        b2[m] <- pmin(d2[m],x_0)
      }

      x[m] <- x_0-b2[m]
      x_0 <- x[m]
      recap[m] <- pmax(recap_0-b2[m],0)
      if(recap[m]==0){
        b1[m] <- 0
      }else{
        b1[m] <- pmin(d1[m],recap[m])
      }
      recap[m] <- recap[m]-b1[m]
      recap_0 <- recap[m]
    }
    #use current data to forecast remaining demand
    hw.1 <- HoltWinters(d1[1:re_cal[j]], gamma=FALSE, beta = FALSE)
    forecast.1 <- as.numeric(forecast(hw.1,h=1)$mean)
    hw.2 <- HoltWinters(d2[1:re_cal[j]], gamma=FALSE, beta = FALSE)
    forecast.2 <- as.numeric(forecast(hw.2,h=1)$mean)
    lambda1 <- (360-re_cal[j])*forecast.1
    lambda2 <- (360-re_cal[j])*forecast.2
    #start calculate opt_BL and opt_OBL for update booking
```

```
      if (0 < tau1 & tau1 < ppois(recap[re_cal[j]]-1,lambda1,FALSE)){
        BL_opt <- 0
      }else{
        if (ppois(recap[re_cal[j]]-1,lambda1,FALSE) < tau1 & tau1 < ppois(0,lambda1,FALSE)){
          BL_opt <- recap[re_cal[j]]-qpois(1-tau1, lambda1)
        }else{
          BL_opt <- recap[re_cal[j]]-2
        }
      }
      OBL_opt <- recap[re_cal[j]]
      while(pbinom(recap[re_cal[j]]-1,OBL_opt,theta[i,2],FALSE)<tau2) {OBL_opt <- OBL_opt+1}
      profit_BL <- expprofit(BL_opt, recap[re_cal[j]], p1, p2, g1, g2, r1, r2, h, theta[i,1], theta[i,2],
lambda1, lambda2)
      profit_OBL <- expprofit(OBL_opt, recap[re_cal[j]], p1, p2, g1, g2, r1, r2, h, theta[i,1], theta[i,2],
lambda1, lambda2)
      profit_K <- expprofit(recap[re_cal[j]]-1, recap[re_cal[j]], p1, p2, g1, g2, r1, r2, h, theta[i,1],
theta[i,2], lambda1, lambda2)
      x_0 <- recap[re_cal[j]]-1
      if ((profit_BL0 > profit_K ) | (profit_OBL0 > profit_K) ){
        x_0 <- OBL_opt
        if (profit_BL0 > profit_OBL0){
          x_0 <- BL_opt
        }
      }
      if (profit_BL > profit_OBL){
        x_0 <- BL_opt
      }else{
        x_0 <- OBL_opt
      }
      #end calculate opt_BL and opt_OBL for update booking
    }
    w1 <- round(theta[i,1]*sum(b1),0) #generate number of class1 show up
    w2 <- round(theta[i,2]*sum(b2),0) #generate number of class2 show up
    profit[k] <- p1*sum(b1) + p2*sum(b2) -r1*(sum(b1) - w1) - r2*(sum(b2)-w2) - h*pmax(w2-K,0)
    print(1000*(i-1)+k)
  }
  avgprofit[i] <- mean(profit)
}
```

# BIOGRAPHY

| | |
|---|---|
| **NAME** | Miss Murati Somboon |
| **ACADEMIC BACKGROUND** | Bachelor's Degree with a major in Mathematics from Prince of Songkla University, Songkla, Thailand in 2006 and Master's Degree in Applied Statistics and Information Technology, National Institute of Development Administration, Bangkok, Thailand in 2009 |