Properties of polynomials over ordered fields with non-negative coefficients

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Abstract

This research project deals with polynomials with nonnegative coefficients over an ordered field. Certain relations between the researches work of Beli [3] and Brunotte [4, 5] are discovered.

The 2007 work of Beli [3] investigates polynomials with nonnegative coefficients over an ordered field. When applied to the field of real numbers, we have

Theorem. If $P(x)=\sum_{i=0}^{n} a_i x^i \in \mathbb{R}[\mathrm{X}]$ and $|P|(x)=\sum_{i=0}^{n} |a_i| x^i \in \mathbb{R}[\mathrm{X}]$ where $a_i \neq 0$ $(i=1, 2, 3, \ldots, n)$, then there is a polynomial $Q\in \mathbb{R}[\mathrm{X}]\setminus\{0\}$ such that $PQ \in \mathbb{R}_+[\mathrm{X}]$ if and only if there is a positive rational number ε with $\frac{|P(x)|}{|P|(x)} > \varepsilon$ for each positive real number x.

The 2009 work of Brunotte [4] is the discovery a real polynomial having proper complex roots or negative real roots can be turned into a polynomial with real positive coefficients by multiplying with a suitable polynomial, i.e.,

Theorem. If $f \in \mathbb{R}[X]$ has proper complex roots or nonnegative real roots, then there is a natural number m such that $(1+x)^m f(x)$ is a polynomial with positive coefficients.

In 2010, Brunotte [5] extended his 2009 work by proving:

Theorem. If $b, c, r \in \mathbb{R}$ are such that $r > 0$, $b^2 < 4c$, then there is a nonnegative integer n such that the polynomial $(x+r)^n(x^2+bx+c)$ has only positive coefficients. Moreover, such integer can be chosen to satisfy $n \le \max\left\{\left\lceil \frac{1-\beta}{\alpha} \right\rceil, \lceil 1-\gamma \rceil, 0\right\}$, where $\alpha = 4\delta c - (2c-br)^2$, $\beta = 12\delta c - 2\sigma(2c-br)$, $\gamma = 8\delta c - \sigma^2$, $\delta = r^2 - br + c$, $\sigma = 3c - 2br + r^2$

The main research finding is an alternative proof of Beli's theorem which is simpler and is based on the use of Brunotte's results.

Keywords: polynomials, coefficients of a polynomial, roots of a polynomial, ordered fields