A strut and tie model for FRCM-strengthened reinforced concrete deep beams

Tadesse Wakjira[1] and Usama Ebead[2]

[1,2]*Dept of Civil and*
Architectural Engineering, College of Engineering, Doha, Qatar

Exiting literature revealed that fabric reinforced cementitious matrix (FRCM) is a promising material for the strengthening of shear deficient reinforced concrete (RC) beams. However, most of the available experimental studies are devoted to the use of FRCM system for the strengthening of slender beams and limited literature is available on the strengthening of deep beams using FRCM system. Moreover, there is no available literature on the analytical modelling of FRCM-strengthened deep beams. In this paper, a simple strut and tie model (STM) has been used to predict the ultimate load carrying capacity of RC deep beams strengthened in shear using FRCM system. The model accounts for the internal transverse reinforcement ratio and axial rigidity of the FRCM system. The proposed model is validated against an experimental results of RC deep beams strengthened with different types of FRCM system available in the literature.

Keywords: Reinforced concrete deep beam, fabric reinforced cementitious matrix (FRCM), Strengthening, strut and tie mode, Shear capacity.

1 INTRODUCTION

One of the major problems faced in construction industries worldwide is an early deterioration of reinforced concrete (RC) structures encountering their life cycle (Younis, Ebead, and Judd 2018). Fiber reinforced polymer (FRP) system has been widely used for the strengthening of deteriorated structures owing to their favorable properties over the traditional strengthening system (Aly, Benmokrane, and Ebead 2006; Ebead 2015; Ebead and Saeed 2013, 2014). However, FRP possess some drawbacks such as incompatibility with the concrete substrate and failure at high temperature due to its reliance on epoxy adhesives. To overcome these problems, fabric reinforced cementitious matrix (FRCM) has been introduced by replacing the epoxy adhesives with cementitious mortar and fibers with the textile fabrics. Exiting literature revealed that FRCM can be used to significantly enhance the flexural and shear capacity of RC beams (Ebead et al. 2017; Ebead and Wakjira 2018a, 2018b, Elghazy et al. 2017, 2018; Pino et al. 2017; Tetta, Koutas, and Bournas 2016; Wakjira and Ebead 2018c, 2018a; Younis, Ebead, and Shrestha 2017) possessing a good bond with the concrete (Ebead and Younis 2019; Younis and Ebead 2018). However, most of the available experimental studies are devoted to the use of FRCM system for the strengthening of slender beams and the literature available on the FRCM-strengthened deep beams are scarce (Azam et al. 2018; Tetta, Koutas, and Bournas 2018; Wakjira and Ebead 2018b, 2019). Moreover, there is no available literature on the analytical modelling of FRCM-strengthened deep beams. ACI-ASCE Committee

445 (Joint ACI-ASCE Committee 445 1998) defined deep beams as beams in which the ratio of the critical shear span to effective depth *(a/d)* is less than 2.50. In this type of beams, unlike slender beams in which the combined shear and bending action transfers the load to the support, the load is transferred to the support through the compressive stresses by arch action (He, Liu, and Ma 2012). A strut-and-tie model (STM) has shown to be an effective design method for this type of beam.

In this paper, a strut and tie model is used to predict the ultimate load carrying capacity of FRCM-strengthened RC deep beams. The model is then validated against an experimental data of FRCM-strengthened deep beams available in the literature.

2 STM FOR FRCM-STRENGTHENED RC DEEP BEAMS

The failure in deep beams is approximated to follow the shortest possible path (Tuchscherer, Birrcher, and Bayrak 2016). Figures 1a and 1b show the proposed STM for FRCM-strengthened beams in which the load is assumed to be transferred directly to the support in a form of a compressive strut. The diagonal compressive strut may fail because of concrete splitting caused by the diagonal tension stress or the crushing of concrete caused by formation of diagonal compressive stress. The latter is resisted by concrete compressive strength of the diagonal strut, while the diagonal tension stress is mainly resisted by transverse reinforcement; i.e., internal transverse reinforcement (ITR) and external transverse reinforcement (FRCM) as shown in Figure 1b.
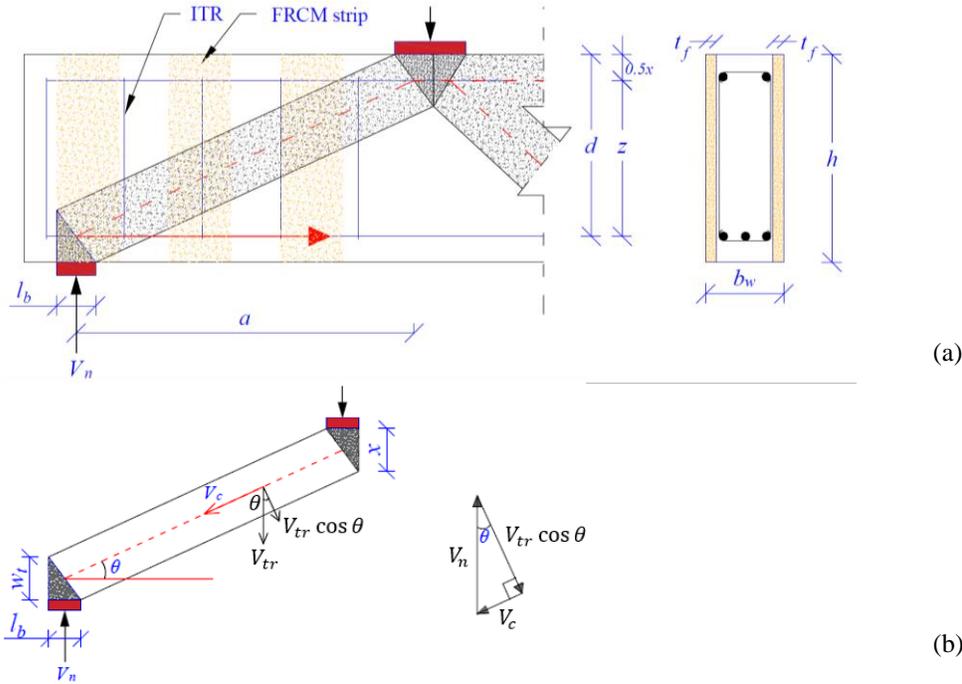

Figure 1. Detail of STM for FRCM-strengthened beams within the critical shear span: (a) layout of STM and (b) load transfer and equilibrium of forces.

From the equilibrium of forces on the strut, Figure 1b, the nominal shear capacity (V_n) of FRCM-strengthened beam can be given in Eq. (1).

$$V_n = V_c \sin\theta + V_{tr}\cos^2\theta \qquad (1)$$

where,

V_c and V_{tr} are the shear force in the concrete compressive strut and the shear strength provided by the transverse reinforcement, which are ITR and FRCM reinforcement.

The inclination (θ) of the compressive strut from the horizontal can be determined as follows.

$$\theta = \tan^{-1}(z/a) \tag{2}$$

where,

a is the critical shear span and

z is the lever arm determined from the cracked rectangular section of singly reinforced RC beam as given in Eq. (3).

$$z = d - \frac{kd}{3} \tag{3}$$

where,

d is the effective depth of the section and

kd is the position of the neutral axis from the extreme compression side of the beam.

The factor k is determined from the moment-curvature relationship of singly reinforced RC beams as given in Eq. (4) (Park and Kuchma 2007).

$$k = \sqrt{(n\rho)^2 + 2n\rho} - n\rho \tag{4}$$

where,

ρ is the flexural reinforcement ratio ($A_s/b_w d$) and

n is the steel-concrete modular ratio, which is the modulus of elasticity of steel ($E_s = 200,000$ MPa) to concrete (E_c).

The modulus of elasticity of concrete is given by Eq. (5) according to (ACI Committee 318 2014).

$$E_c = 4700\sqrt{f_c'} \tag{5}$$

The force in the concrete compressive strut (V_c) is given by Eq. (6).

$$V_c = f_{ce}A_{st} \tag{6}$$

where,

f_{ce} is the effective concrete strength and

A_{st} is the smaller of cross-sectional area of the strut at the two ends.

According to (ACI Committee 318 2014) the effective concrete strength is the smaller of the effective compressive strength of the concrete in strut ($f_{ce,st}$) and nodal zone ($f_{ce,n}$) as given in Eq. (7) and Eq. (8), respectively.

$$f_{ce,st} = 0.85\beta_s f_c' \tag{7}$$

where,

β_s is the factor that depends on the shape of the strut, which is unity for prismatic strut considered in this study.

$$f_{ce,n} = 0.85\beta_n f_c' \tag{8}$$

where β_n is given by Eq. (9) below as per (ACI Committee 318 2014).

$$\beta_n = \begin{cases} 1.0, & nodal\ zone\ bounded\ by\ strut\ only \\ 0.80, & nodal\ zone\ anchoring\ one\ tie \\ 0.60, & nodal\ zone\ anchoring\ more\ than\ one\ tie \end{cases} \tag{9}$$

Thus, β_n is 0.80 for the beams considered in this study.

Area of the strut is given by Eq. (10).

$$A_{st} = b_w a_{st} \tag{10}$$

where a_{st} is the minimum of the width of the strut at the two ends given by Eq. (11).

$$a_{st} = \min(l_b \sin\theta + w_t \cos\theta, \quad l_p \sin\theta + x \cos\theta) \tag{11}$$

where,

l_b is the with of the support plate; l_p is the width of the loading plate;

w_t is the depth of the tie, which is taken as twice the distance between the centroid of flexural reinforcement and extreme tension side of the beam; and

x is the depth of the top horizontal strut, which is taken as twice the distance between the centroid of the compression zone in the moment-curvature and extreme compression side of the beam (2kd/3).

The shear strength provided by the transverse reinforcements (V_{tr}) is the sum of the shear provided by the ITR (V_{itr}) and FRCM reinforcement (V_f) as given in Eq. (12).

$$V_{tr} = f_{itr}A_{itr} + f_f A_f \tag{12}$$

where,

f_{itr} and A_{itr} are the tensile stress and area of ITR;

f_f and A_f are the effective tensile stress and area of FRCM reinforcement.

The effective tensile stress of the FRCM system is determined as the product of its elastic modulus (E_f) and effective strain (ε_f) as given in Eq. (13).

$$f_f = E_f \varepsilon_f \tag{13}$$

The effective strain in FRCM system is limited to the minimum of 0.004 and 75% of its ultimate strain as per (ACI Committee 549 2013).

Therefore, the shear capacity of FRCM-strengthened beam is given by Eq. (14).

$$V_n = f_{ce}A_{st}\sin\theta + f_{itr}A_{itr}cos^2\theta + E_f\varepsilon_f A_f cos^2\theta \tag{14}$$

Finally, the ultimate load carrying capacity of the beam can be determined based on the loading condition.

3   VERIFICATION OF THE MODEL

The proposed model is validated against an experimental database of RC deep beams strengthened with FRCM system available in the literature (Azam et al. 2018; Tetta et al. 2018; Wakjira and Ebead 2018b). (Azam et al. 2018) tested rectangular RC beams of dimensions $1700 \times 250 \times 400$ mm (length $\times$ width $\times$ height) with critical shear span to effective depth (*a/d*) ratio of 1.62. The beams were strengthened with externally bonded cement-based composites and internally reinforced with or without internal ITR. The beams tested by (Wakjira and Ebead 2018b) were of 2100 mm long rectangular beams with cross-sectional dimensions 150×330 mm (width $\times$ height) tested under three-point bending with *a/d* ratio of 1.96. The average 28-day compressive strength of concrete used to cast the beams was 30 MPa. The beams were strengthened with both continuous and discontinuous configurations of three different types of FRCM composites (carbon, glass, PBO) applied using a near surface embedded technique. The beams tested by (Tetta et al. 2018) had rectangular cross section of dimensions $1677 \times 102 \times 203$ mm) (length $\times$ width $\times$ height) tested under three-point loading with *a/d* ratio of 1.6. The beams were strengthened with an externally bonded carbon FRCM. However, the slender beams with *a/d* ratio greater than 2.50 tested by (Tetta et al. 2018) are not included in this study.

The ratio of the theoretically predicted to experimental load carrying capacity (P_{th}/P_{ex}) is depicted in Figure 2. This ratio ranged between 0.81 and 1.11 with an average of 0.94 and standard deviation of 0.09 as shown in Figure 2. The correlation of determination (R^2) between the experimental and theoretically predicted ultimate load carrying capacity was 0.96. This observation shows that the proposed STM can predict the load carrying capacity of the FRCM-strengthened deep beams with an acceptable accuracy.
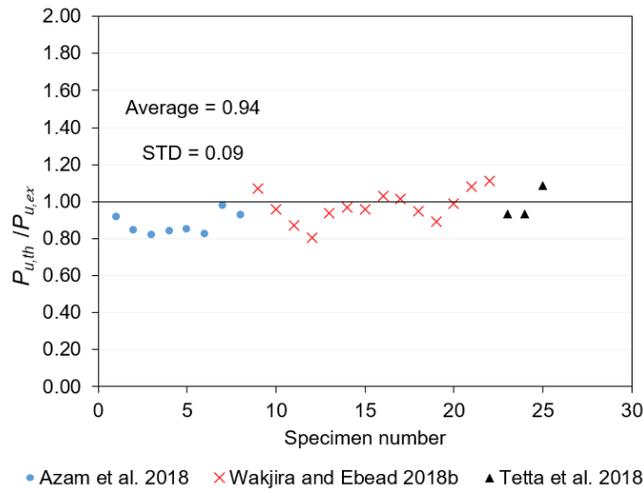
Figure 2. Verification of the model against experimental data.

4 CONCLUSION

A simple strut and tie model was used to predict the ultimate load carrying capacity of FRCM-strengthened RC deep beams. The model accounts for internal transverse reinforcement ratio and axial rigidity of the FRCM system. An experimental database of FRCM-strengthened deep beams available in the literature was used to validate the model and a reasonable agreement between the experimental and theoretically predicted ultimate load of the beams was obtained. The correlation of determination (R^2) between the experimental (P_{ex}) and theoretically predicted (P_{th}) ultimate load carrying capacity was 96%, while the average of P_{th}/P_{ex} ratio was 0.94 with a standard deviation of 0.09.

References

ACI Committee 318. 2014. *Building Code Requirements for Structural Concrete (ACI 318-14)*. American Concrete Institute, Farmington Hills, MI, USA.
ACI Committee 549. 2013. *Guide to Design and Construction of Externally Bonded Fabric-Reinforced Cementitious Matrix (FRCM) Systems for Repair and Strengthening Concrete and Masonry Structures (ACI 549.4R-13), American Concrete Institute, Farmington Hills, MI, USA, 2013.*
Aly, Ragi, Brahim Benmokrane, and Usama Ebead. 2006. "Tensile Lap Splicing of Fiber-Reinforced Polymer Reinforcing Bars in Concrete." *ACI Structural Journal* 103(6):857–64.
Azam, Rizwan, Khaled Soudki, Jeffrey West, and Martin Noël. 2018. "Shear Strengthening of RC Deep Beams with Cement-Based Composites." *Engineering Structures* 172(June):929–37.
Ebead, Usama. 2015. "Inexpensive Strengthening Technique for Partially Loaded Reinforced Concrete Beams : Experimental Study." *Journal of Materials in Civil Engineering* (10).
Ebead, Usama and Huda Saeed. 2013. "Hybrid Shear Strengthening System for Reinforced Concrete Beams: An Experimental Study." *Engineering Structures* 49(April 2013):421–33.
Ebead, Usama and Huda Saeed. 2014. "Flexural and Interfacial Behavior of Externally Bonded/ Mechanically Fastened Fiber-Reinforced Polymer Strengthened Reinforced Concrete Beams." *ACI*

Structural Journal 111(4).

Ebead, Usama, Kshitij C. Shrestha, Muhammad S. Afzal, Ahmed El Refai, and Antonio Nanni. 2017. "Effectiveness of Fabric-Reinforced Cementitious Matrix in Strengthening Reinforced Concrete Beams." *Journal of Composites for Construction* 21(2):04016084.

Ebead, Usama and Tadesse Wakjira. 2018a. "Behaviour of RC Beams Strengthened in Shear Using near Surface Embedded FRCM." *IOP Conference Series: Materials Science and Engineering* 431:072001.

Ebead, Usama and Tadesse Wakjira. 2018b. "FRCM/Stirrups Interaction in RC Beams Strengthened in Shear Using NSE-FRCM." *IOP Conference Series: Materials Science and Engineering*.

Ebead, Usama and Adel Younis. 2019. "Pull-off Characterization of FRCM/Concrete Interface." *Composites Part B* 165(November 2018):545–53.

Elghazy, Mohammed, Ahmed Refai, Usama Ebead, and Antonio Nanni. 2017. "Effect of Corrosion Damage on the Flexural Performance of RC Beams Strengthened with FRCM Composites." *Composite Structures* 180:994–1006.

Elghazy, Mohammed, Ahmed El Refai, Usama Ebead, and Antonio Nanni. 2018. "Post-Repair Flexural Performance of Corrosion-Damaged Beams Rehabilitated with Fabric-Reinforced Cementitious Matrix (FRCM)." *Construction and Building Materials* 166(January):732–44.

He, Zhi Qi, Zhao Liu, and Zhongguo John Ma. 2012. "Investigation of Load-Transfer Mechanisms in Deep Beams and Corbels." *ACI Structural Journal* 109(4):467–76.

Joint ACI-ASCE Committee 445. 1998. "Recent Approaches to Shear Design of Structural Concrete." *Journal of Structural Engineering* 124(12):1374–1417.

Park, Jung-woong and Daniel Kuchma. 2007. "Strut-and-Tie Model Analysis for Strength Prediction of Deep Beams." *ACI Structural Journal* 104(6):657–61, 663–66.

Pino, Vanessa, Houman Hadad, Francisco Basalo, Antonio Nanni, Usama Ebead, and Ahmed Refai. 2017. "Performance of FRCM-Strengthened RC Beams Subject to Fatigue." *Journal of Bridge Engineering* 22(10):4017079.

Tetta, Zoi, Lampros Koutas, and Dionysios Bournas. 2016. "Shear Strengthening of Full-Scale RC T-Beams Using Textile-Reinforced Mortar and Textile-Based Anchors." *Composites Part B: Engineering* 95:225–39.

Tetta, Zoi, Lampros Koutas, and Dionysios Bournas. 2018. "Shear Strengthening of Concrete Members with TRM Jackets: Effect of Shear Span-to-Depth Ratio, Material and Amount of External Reinforcement." *Composites Part B: Engineering* 137(February 2017):184–201.

Tuchscherer, Robin G., David B. Birrcher, and Oguzhan Bayrak. 2016. "Reducing Discrepancy between Deep Beam and Sectional Shear-Strength Predictions." *ACI Structural Journal* 113(1):3–16.

Wakjira, Tadesse and Usama Ebead. 2018a. "A New Approach for Predicting the Shear Capacity of FRCM Strengthened RC Beams in Shear." *IOP Conference Series: Materials Science and Engineering*.

Wakjira, Tadesse and Usama Ebead. 2018b. "FRCM/Internal Transverse Shear Reinforcement Interaction in Shear Strengthened RC Beams." *Composite Structures* 201(June):326–39.

Wakjira, Tadesse and Usama Ebead. 2018c. "Hybrid NSE/EB Technique for Shear Strengthening of Reinforced Concrete Beams Using FRCM: Experimental Study." *Construction and Building Materials* 164:164–77.

Wakjira, Tadesse G. and Usama Ebead. 2019. "Internal Transverse Reinforcement Configuration Effect of EB/NSE-FRCM Shear Strengthening of RC Deep Beams." *Composites Part B: Engineering* 166:758–772.

Younis, Adel and Usama Ebead. 2018. "Bond Characteristics of Different FRCM Systems." *Construction and Building Materials* 175(June):610–20.

Younis, Adel, Usama Ebead, and Simon Judd. 2018. "Life Cycle Cost Analysis of Structural Concrete Using Seawater , Recycled Concrete Aggregate , and GFRP Reinforcement." *Construction and Building Materials* 175:152–60.

Younis, Adel, Usama Ebead, and Kshitij C. Shrestha. 2017. "Different FRCM Systems for Shear-Strengthening of Reinforced Concrete Beams." *Construction and Building Materials* 153:514–26.