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EFFECT OF STRAIN-STRESS RELATIONSHIP OF STEEL TUBE ON THE INITIAL STIFFNESS OF SQUARE CFT COLUMNS

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The restoring force characteristic of a concrete filled steel tubular (CFT) column is modeled with three straight lines which show the relation of *M-R*. The initial stiffness is calculated by assuming that the steel and concrete portions are fully effective and materials are elastic. The first break point is the yield bending moment of the CFT column. The yield bending moment is defined by the bending moment when a part of the cross-section yield or the allowable strength for short-term loading obtained by the superposed strength method. It is supposed that the theoretical initial stiffness obtained by assuming that the whole cross section is effective is different from the real one. This study aims to clarify effects of strain-stress relation on the yield strength and allowable strength for short-term loading and the initial stiffness. Three types of stress-strain relations for steel are used, elastic-perfectly plastic model, Menegotto-Pinto's model and Morino's model. According to the analytical results, the difference between allowable strength and yield strength becomes larger as the axial load ratio, indexes R_h and k which are used in Menegotto-Pinto's model and Morino's model increase.

Keywords: Steel concrete composite structure, Beam-column, Allowable strength, Yield strength, Restoring force characteristics, Axial load ratio.

1 INTRODUCTION

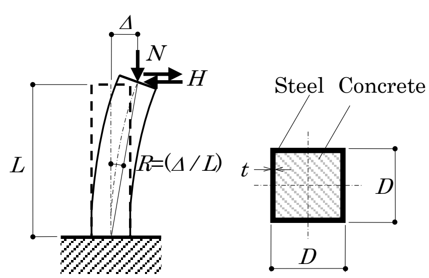
The restoring force characteristic of a concrete filled steel tubular (afterwards, referred to as CFT) column is shown in "Architectural Institute of Japan Recommendations for Design and Construction of Concrete Filled Steel Tubular Structures (afterwards, referred to as CFT Recommendations)" (AIJ 2008). The skeleton curve is modeled with three straight lines and represented by the relationship between rotation angle of the member and bending moment. The initial stiffness is calculated by assuming that the steel and concrete portions are fully effective and these are elastic. The first break point is the yield bending moment of the CFT column. In CFT Recommendations, it is mentioned that the yield strength of CFT short columns M_y is obtained as follows: The relationship between bending moments and curvatures is calculated by using the stress-strain relations and Navier's hypothesis, and then the yield strength of CFT column is obtained as the bending moment when the concrete portion or the steel tube has reached the yield stress of materials. However, this calculation method is relatively complex and the yield strength may be calculated by superposing the yield bending moment of a concrete portion and steel tube. This superposed strength is which is defined as the allowable strength of

CFT columns. Against this background, the past studies showed that the yield strength of the CFT column is different from the allowable strength of CFT columns for short-term loading (Liu *et al.* 2011). Therefore, the theoretical initial stiffness of a CFT column obtained by assuming that the whole cross section is effective is different from the actual one. These two initial stiffnesses were compared in past studies (Yoshida *et al.* 2016). However, the stress-strain relation of steel is limited to the elastic-perfectly plastic model. Objective of this study is to clarify the effects of strain-stress relation on the yield strength and the allowable strength for short-term loading and the initial stiffness.

2 ANALYTICAL WORK

2.1 Analytical Model and Loading Condition

Loading condition is shown in Figure 1. The analytical model is a square CFT cantilever column which has a length L, size of cross section D and thickness of steel tube t. The effective length of the column is 4 times longer than the diameter of the steel tube section.



(a) Loading condition. (b) Cross section.

Figure 1. Analytical model.

2.2 Analytical Method

The relationship between lateral load H and lateral deformation Δ is calculated by the column deflection curve method. The relationship between bending moment and curvature is calculated by using the Navier's assumption and the infinitesimal deformation theory.

2.3 Stress-Strain Relationship

The stress-strain relationship of concrete is calculated by Eq. (1) (AIJ 1991).

$$\left.\begin{array}{ll}\sigma = \left\{1-\left(1-\dfrac{\varepsilon}{\varepsilon_m}\right)^a\right\}{}_c\sigma_B & (\varepsilon < \varepsilon_m) \\ \sigma = {}_c\sigma_B & (\varepsilon_m \le \varepsilon)\end{array}\right\} \tag{1}$$

In Eq. (1), ${}_c\sigma_B$ is the compressive strength of concrete. The index a, the initial Young's modulus E_c and the strain at the compressive strength ε_m are calculated by Eq. (2), (3) and (4), respectively.

$$a = \frac{E_c \times \varepsilon_m}{{}_c\sigma_B} \tag{2}$$

$$E_c = \left(3.32 \times \sqrt{{}_c\sigma_B} + 6.9\right) \times 10^3 \tag{3}$$

$$\varepsilon_m = 0.93 \times {}_c\sigma_B^{\frac{1}{4}} \times 10^{-3} \tag{4}$$

In this study, the tensile stress of concrete is neglected. Three stress-strain relationships of a steel tube are used as follows: (a) the elastic-perfectly plastic model, (b) Menegotto-Pinto's model, and (c) stress-strain relationship recommended by Morino and Atsumi (1990) (hereinafter referred to as Morino's model). The stress-strain relationship of the Menegotto-Pinto's model is calculated by Eq. (5).

$$\frac{\sigma}{\sigma_y} = \frac{(1-b)\left(\frac{\varepsilon}{\varepsilon_y}\right)}{\left\{1+\left(\frac{\varepsilon}{\varepsilon_y}\right)^{R_h}\right\}^{\frac{1}{R_h}}} + b\left(\frac{\varepsilon}{\varepsilon_y}\right) \tag{5}$$

In Eq. (5), ε_y is the yield strain of steel and σ_y is the yield stress of the steel, R_h is the index and b is the index related to strain hardening. The stress-strain relationships of the Morino's model are calculated by Eq. (6) to (9).

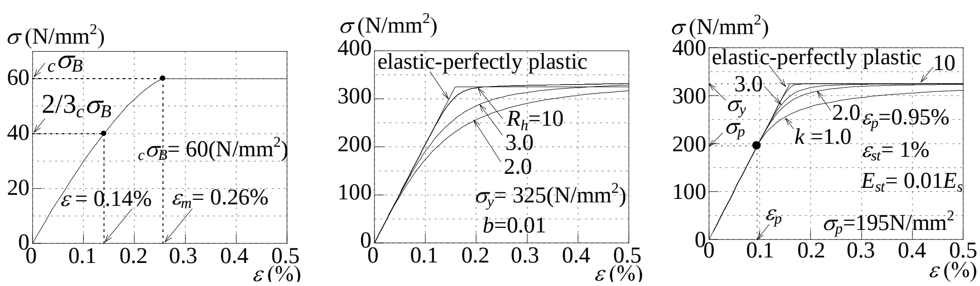
$$\sigma = E_s \varepsilon \qquad (0<\varepsilon<=\varepsilon_p) \tag{6}$$

$$\sigma = \sigma_1 + \sigma_2 \qquad (\varepsilon_p<\varepsilon<=\varepsilon_{st}) \tag{7}$$

$$\left.\begin{aligned} \sigma_1 &= \frac{(\varepsilon-\varepsilon_p)(\sigma_y-\sigma_p)/(\varepsilon_y-\varepsilon_p)}{\left[1+\left\{(\varepsilon-\varepsilon_p)/(\varepsilon_y-\varepsilon_p)^k\right\}\right]^{1/k}} + \sigma_p \\ \sigma_2 &= \frac{\sigma_y - \sigma_1(\varepsilon=\varepsilon_{st})}{(\varepsilon_{st}-\varepsilon_p)} \cdot (\varepsilon-\varepsilon_p) \end{aligned}\right\} \tag{8}$$

$$\sigma = E_{st}(\varepsilon-\varepsilon_{st}) + \sigma_y \qquad (\varepsilon_{st}<=\varepsilon) \tag{9}$$

In Eq. (6) to (9), E_s is Young's modulus of steel, k is the index, E_{st} is the index related to strain hardening, σ_p is a proportional limit stress and ε_p is a proportional limit strain. Figure 2 shows the stress-strain curves of the steel tube and concrete.



(a) Concrete. (b) Menegotto-Pinto's model. (c) Morino's model.

Figure 2. The stress-strain relationship.

2.4 Yield Strength and Superposed Strength

In this study, the concrete portion is assumed to yield when the stress of the compressive edge reaches the two-third of the compressive strength of concrete. The steel tube is assumed to yield when the stress of the outer edge of the steel tube reaches the yield strength of the steel tube. In CFT Recommendations, the allowable strength for short-term loading of beam-columns is calculated by Eq. (10) and (11).

$$\left.\begin{array}{l} N = {}_cN \\ M_a = {}_sM_0 + {}_cM \end{array}\right\} \qquad (0 <= N <= {}_cN_c \text{ or } M >= {}_sM_0) \tag{10}$$

$$\left.\begin{array}{l} N = {}_cN_c + {}_sN \\ M_a = {}_sM \end{array}\right\} \qquad (0 > {}_cN_c \text{ or } M < {}_sM_0) \tag{11}$$

In Eq. (10) and (11), ${}_cN_c = {}_cA\ f_c$, ${}_sM_0 = {}_sZ\ {}_sf_t$, ${}_cA$ is the cross-sectional area of concrete portion, f_c is the allowable stress of concrete, ${}_sf_t$ is the allowable tensile stress of steel tube and ${}_sZ$ is the section modulus of the steel tube.

2.5 Initial Stiffness K_e and K_{cal}

2.5.1 *Initial stiffness based on the theory*

The theoretical lateral stiffness K_e is calculated by Eq. (12).

$$K_e = \frac{H}{R} = \frac{ZN}{\tan Z - Z} \tag{12}$$

In Eq. (12), the variable Z and the bending stiffness EI are defined by $Z = L\sqrt{N/EI}$ and $EI = E_cI_c + E_sI_s$, respectively. I_c and I_s are the moment of inertia of concrete portion and steel tube and calculated by assuming that the whole cross sections are effective.

2.5.2 *Initial stiffness by analysis (secant stiffness)*

The initial stiffness K_{cal} obtained by the numerical analysis is defined as the secant stiffness, which is the slope of the line obtained by connecting the original point and the allowable strength on the *H-R* curve (Refer to figure 3). In CFT Recommendations, the yield bending strength is determined as the strength when the outermost edge of the cross section reaches the yield stress.

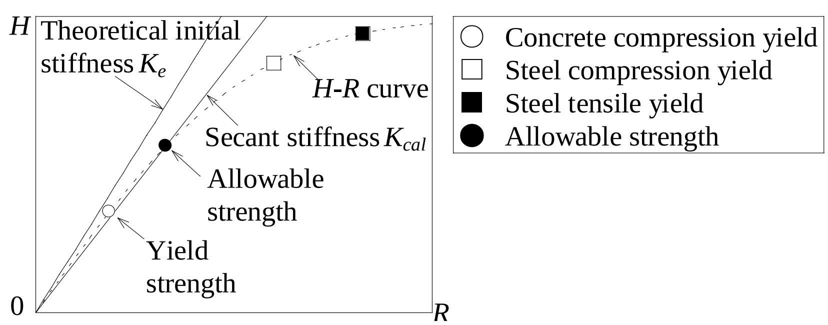


Figure 3. Definition of initial stiffness.

3 RESULTS OF ANALYSIS AND DISCUSSION

3.1 Analytical Parameters

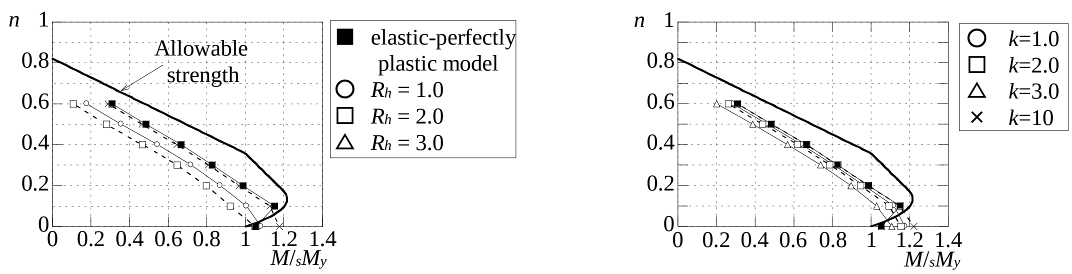
Analytical parameters are as follows:

- The axial force ratio n is 0, 0.1, 0.2, 0.3, 0.4, 0.5, and 0.6.
- The index R_h of the Menegotto-Pinto's model are 2.0, 3.0, and 10.0.
- The index k of the Morino's model are 1.0, 2.0, 3.0 and 10.0.

The yield stress of steel is 325 N/mm^2 and the compressive strength of concrete is 60 N/mm^2.

3.2 Comparison of Yield Strength and Allowable Strength

Figure 4 shows the yield strength and allowable strength on the relationship between the axial force ratio n and $M/{}_sM_y$ (${}_sM_y$: yield bending moment of steel tube subjected to bending alone). The thick lines are short-term allowable strength, and other white marks show the yield strength in the case of (a) Menegotto-Pinto's and (b) Morino's model, respectively. For reference, the yield strength obtained in the case of the elastic-perfectly plastic model is shown by the black squares. The yield strength was determined by the tensile yielding of the steel tube only when n equals 0 and the stress-strain relationship is the elastic-perfectly plastic model and Menegotto-Pinto's model with index R_h = 10. In the case of other axial force ratios and index, the yield strength was determined by the compression yielding of concrete. As the value of the index R_h, k decreases, the difference between the allowable strength and the yield strength increases.



(a)Menegotto-Pinto's Model (b)Morino's Model

Figure 4. n - $M/{}_sM_y$ interaction.

3.3 Comparison of Initial Stiffness

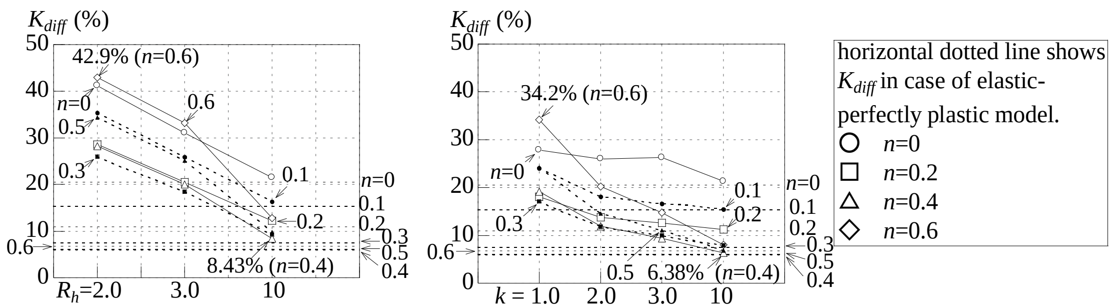
The difference between the theoretical initial stiffness K_e and the initial stiffness K_{cal} is calculated by Eq. (13).

$$K_{diff} = \frac{K_e - K_{cal}}{K_e} \times 100 \tag{13}$$

Figures 5(a) and (b) show the relationship between K_{diff} and index R_h and k, and the parameter is axial force ratio n. According to these figures, Menegotto-Pinto's K_{diff} is in the range of 8.43% to 42.9%. In the case of the Morino's model, K_{diff} is in the range of 6.38% to 34.2%.

In case of the elastic-perfectly plastic model, K_{diff} increases as the axial force ratio n decreases between n = 0.3 and n = 0.4 although n = 0.5 and 0.6 are the exception. In case of Menegotto-

Pinto's model, the relationship between K_{diff} and n is different from the case of elastic-perfectly plastic model. For example, when the value of R_h is 2.0 or 3.0, K_{diff} is the largest in case of n = 0.6 followed by n = 0, 0.1 and 0.5. The case of Morino's model shows a similar tendency. When the axial force ratio is 0.5 and 0.6, the value of K_{diff} is large because concrete yields in an early stage. When the axial force ratio is 0 and 0.1, the tensile area becomes large, and it is considered that the stiffness K_{cal} is small because the tensile stress of concrete is neglected.



(a) Menegotto-Pinto's Model (b) Morino's Model

Figure 5. Comparison of initial stiffness.

4 CONCLUSIONS

The comparison of the yield strength with the allowable strength and the comparison of theoretical initial stiffness K_e with the initial stiffness by numerical calculation K_{cal} are presented. The details are shown as follows:

(i) As the value of the index R_h and k decreases, the difference between the allowable strength and the yield strength increases.

(ii) The range of difference between K_e and K_{cal} is from 6.38% to 34.2% in case of Menegotto-Pinto's model and from 6.38% to 34.2% in the case of Morino's model.

Acknowledgements

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