Chapter 1

Executive Summary

Let S be a non-empty set and $*$ be a binary operation on the set S. The algebraic structure $(S,*)$ is called grupoid and called semigroup if the following condition hold:

$$a*(b*c)=(a*b)*c,$$

for all $a,b,c \in S$. The binary relation $\leq$ on the set S is called patial order relation if the following conditions hold: Let $a,b,c \in S$

(1) $a \leq a$ for all $a \in S$.

(2) If $a \leq b$ and $b \leq a$, then $a = b$.

(3) If $a \leq b$ and $b \leq c$, then $a \leq c$.

The algebraic structure $(S,\leq)$ is called partial order set (poset).

The algebraic structure $(S,*,\leq)$ is called an ordered semigroup if the following conditions hold:

(1) $(S,*)$ is semigroup.

(2) $(S,\leq)$ is a partial order set.

(3) For $a,b,c \in S$, if $a \leq b$, then $a*c \leq b*c$ and $c*a \leq c*b$.

Throughout this report S denotes an ordered semigruop unless or otherwise mentioned. Let A,B be any two non-empty sets. We define

$$\text{AB}:=\{ab \quad | \quad a \in A, b \in B\}.$$

Let A be a non-empty subset of an ordered semigroup S. By a subsemigroup of S we mean a non-empty subset A of S such that $AA \subseteq A$. A non-empty subset R of S is called a right ideal of S if

(1) $RS \subseteq R$ and

(2) For $a \in R$ and $b \in S$, if $b \leq a$, then $b \in R$.

A non-empty subset L of S is called a left ideal of S if

(1) $LS \subseteq L$ and

(2) For $a \in L$ and $b \in S$, if $b \leq a$, then $b \in L$.

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A non-empty subset A of S is called an ideal of S if it is both a right ideal and a left ideal of S. Let B be a non-empty subset of S. Then B is called a generalized bi-ideal of S if

(1) $BSB \subseteq B$

(2) For $a \in B$ and $b \in S$, if $b \leq a$, then $b \in B$.

And it called bi-ideal if it is both a subsemigroup and a generalized bi-ideal of S.

Let A be a non-empty set. Then we define

$$(A] := \{x \in S \mid x \leq a, \exists a \in A\}.$$

A non-empty subset Q of S is called a quasi-ideal of S if

(1) $(SQ] \cap (QS] \subseteq Q$

(2) For $a \in Q$ and $b \in S$, if $b \leq a$, then $b \in Q$.

Let S be an ordered semigroup and Q be a non-empty set. A function f from a non-empty set $S \times Q$ to the unit interval $[0,1]$ is called a Q-fuzzy subset of a set S, that is, $f : S \times Q \rightarrow [0,1]$.

Let A be a non-empty subset of S. We denote by $f_{A \times Q}$ the characteristic mapping of $A \times Q$, that is the mapping of $S \times Q$ into $[0,1]$ defined by

$$f_{A \times Q}(x,q) := \begin{cases} 1, & (x,q) \in A \times Q, \\ 0, & (x,q) \notin A \times Q, \end{cases}$$

for all $(x,q) \in A \times Q$. Then $f_{A \times Q}$ is a Q-fuzzy subset f of S. Let S be an ordered semigroup. A Q-fuzzy subset f of S is called a Q-fuzzy subsemigroup of S if $f(xy,q) \geq \min\{f(x,q), f(y,q)\}$ for all $(x,q),(y,q) \in S \times Q$. A Q-fuzzy subset f of S is called a Q-fuzzy left (resp. right) ideal of S if

(1) If $x \leq y$, then $f(x,q) \geq f(y,q)$, and

(2) $f(xy,q) \geq f(y,q)$ (resp. $f(xy,q) \geq f(x,q)$) for all $x, y \in S$ and $q \in Q$.

A Q-fuzzy subset f of S is called a Q-fuzzy ideal of S if it is both a Q-fuzzy left and Q-fuzzy right ideal of S. A Q-fuzzy subset f of S is called a Q-fuzzy generalized bi-ideal of S if

(1) If $x \leq y$, then $f(x,q) \geq f(y,q)$, and

(2) $f(xyz,q) \geq \min\{f(x,q), f(z,q)\}$ for all $x, y, z \in S$ and $q \in Q$.

A Q-fuzzy subset f of S is called a Q-fuzzy bi-ideal of S if it is both Q-fuzzy subsemigroup and a Q-fuzzy generalized bi-ideal of S.

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Let x be an element in S. Then we define

$$A_x := \{(y,z) \in S \times S \mid x \leq yz\}.$$

Let f, g be any two Q-fuzzy subset of S. Then the product $f \circ g$ of f and g as defined by

$$(f \circ g)(x,q) := \begin{cases} \sup_{x \leq yz} \{\min\{f(y,q), f(z,q)\}\}, & A_x \neq \phi, \\ 0 & A_x = \phi, \end{cases}$$

$x, y \in S$ or and $q \in Q$.

A Q-fuzzy subset f of S is called a Q-fuzzy quasi-ideal of S if

(1) If $x \leq y$, then $f(x,q) \geq f(y,q)$ for all $x, y \in S, q \in Q$, and

(2) $(f \circ S) \cap (S \circ f) \subseteq f$, where $S(x,q) = 1$ for all $(x,q) \in S \times Q$.

The element a of an ordered semigroup S is called a regular element of S if there exists $x \in S$ such that $a \leq axa$ and an ordered semigroup S is called regular if for all element of S is a regular element.

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