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LINEAR AND NONLINEAR THERMOELASTIC ANALYSES OF BEAMS UNDER TEMPERATURE GRADIENT

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In a structure, beams are often connected with other members such as columns, which provide considerable restraints against the rotation and extension of the beam ends. When a beam is subjected to an in-plane temperature gradient field, the temperature gradient tends to change the curvature of the beam in the transverse direction and expand the beam in the axial direction. The restrained actions will produce bending moments and compressive forces in the beam, which increase with an increase of the temperature differential and average temperature of the temperature gradient field. When these actions reach critical values, the elastically restrained beam may bifurcate from its primary in-plane equilibrium state to a lateral-torsional buckled equilibrium configuration. This paper carries out linear and nonlinear thermoelastic analyses of an elastically restrained beam of doubly symmetric open thin-walled cross-section that is subjected to a linear temperature gradient field over its cross-section. It is found that geometric nonlinearity influences the thermoelastic responses of the beam to the temperature changes significantly. The influence decreases with an increase of the stiffness of the elastic restraints.

Keywords: Thermal effects, Elastic, In-plane, Restrained, Effective centroid.

1 INTRODUCTION

In structures, the rotations of beam ends are often restrained by adjacent structural members. When a uniform straight beam with ends being restrained rotationally is subjected to an in-plane temperature gradient field that varies linearly from the bottom surface to the top surface of the cross-section of the beam, the expansion produced by the in-plane temperature gradient field is distributed linearly over the cross-section. The elastic end restraints with stiffness k_{ri} at both ends restrain the end thermal expansions and rotations and the restrained axial expansion will produce axial compressive action and the restrained rotation will produce bending action in the beam. When the combine bending and axial compressive actions produced reach certain values, the beam may suddenly deflect laterally and twist out of the plane of the temperature gradient and fail in a lateral-torsional buckling mode.

To perform corrects flexural-torsional buckling analysis, it is important to determine the distributions of internal actions accurately. The prebuckling analysis for homogeneous uniform beams under temperature gradient is quite different from that for the beams under transverse loads. Firstly, because the elastic modulus of a material is a function of the temperature (Pi and Bradford 2008, AS4100 2012) and the temperature at a material point of the beam in a linear temperature gradient field is a function of the coordinates of the point, the elastic modulus at the point is also a function of its coordinates. Hence, the effective centroid of the cross-section is different from its geometric one. The may cause an additional bending moment in the beams. Secondly, in classical theory, the prebuckling analysis for beams under transverse loads is linearized. However, it is not known if the nonlinear analysis is required for beams under the temperature gradient field.

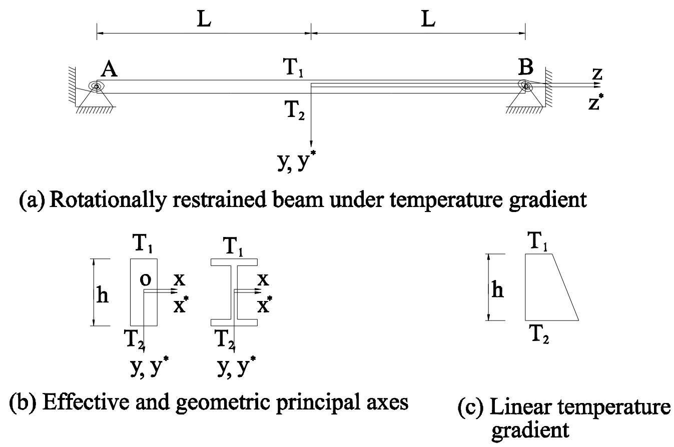

Figure 1. Elastically restrained beam.

The purpose of this paper is to carry out linear and nonlinear in-plane analyses for homogenous uniform beams with a doubly symmetric and in-plane having elastically restrained ends under a linear in-plane temperature gradient field.

2 EFFECTIVE PRINCIPAL AXES

The basic assumptions used in this investigation for thermoelastic analysis are:

1. Deformations are assumed to satisfy the Euler-Bernoulli hypothesis.
2. The temperature at an arbitrary point P can be expressed as (Figure 1)

$$T(y^*) = T_{ave} + \frac{y^* \Delta T}{h} \quad \text{with} \quad T_{ave} = \frac{T_1 + T_2}{2} \quad \text{and} \quad \Delta T = T_2 - T_1 \tag{1}$$

Because the modulus of elasticity is a function of temperature and the linear temperature gradient field $T(y^*)$ is a function of the coordinate y^*, the modulus of elasticity $E(y^*)$ of the beam is also a function of the coordinate y^*. In this case, when an axial force is applied in the direction of the geometric centroidal axis o^*z^*, the beam is subjected to combined axial compression and bending because the modulus of elasticity $E(y^*)$ varies over the entire cross-section. Hence, using the axis system $o^*x^*y^*z^*$ (Fig.1) consisting of the geometric principal axes o^*x^* and o^*y^* of the cross-section and the geometric centroidal axis o^*z^* of the beam is not convenient for the thermoelastic analysis. In order to simplify the analysis, an effective centroidal axis oz can be defined such that when an axial compressive force N is applied in the direction of the effective centroidal axis, the beam is subjected to uniform compression only, i.e. the following requirements are satisfied

$$\int_A \sigma_{zz}\,\mathrm{d}A = N, \quad \int_A \sigma_{zz} x\,\mathrm{d}A = 0, \quad \int_A \sigma_{zz} y\,\mathrm{d}A = 0 \tag{2}$$

The axes *ox* and *oy* of the axes *oxyz* are parallel to the geometric principal axes o^*x^* and o^*y^* of the cross-section and is denoted as effective principal axes, and the axis *oz* is the locus of the effective centroids defined by Eq. (2) (Figure 1).

The longitudinal normal strain ε_{zz} and stress σ_{zz} due to pure uniform compression can be expressed as

$$\varepsilon_{zz} = w' \text{ and } \sigma_{zz} = E(y)w' \tag{3}$$

where *w* is the axial displacement of the effective centroid *o*, which is the origin of the axes *oxy*, ( )'=d( )/dz.

Substituting Eq (3) into the conditions given by Eq. (2) leads to

$$w'\int_A E(y)dA = w'\overline{EA} = -N, \quad w'\int_A E(y)xdA = 0, \quad w'\int_A E(y)ydA = 0 \tag{4}$$

From Eq. (4) and by replacing the coordinates *x*, *y* with $x = x^* + x_c$ and $y = y^* + y_c$, the coordinates x_c, y_c of the effective centroid *o* in the geometric principal axes $o^*x^*y^*$ of the cross-section can be obtained as (Figure 2)

$$x_c = \frac{\int_A E(y^*)x^*\mathrm{d}A}{\int_A E(y^*)\mathrm{d}A} \quad \text{and} \quad y_c = \frac{\int_A E(y^*)y^*\mathrm{d}A}{\int_A E(y^*)\mathrm{d}A} \tag{5}$$

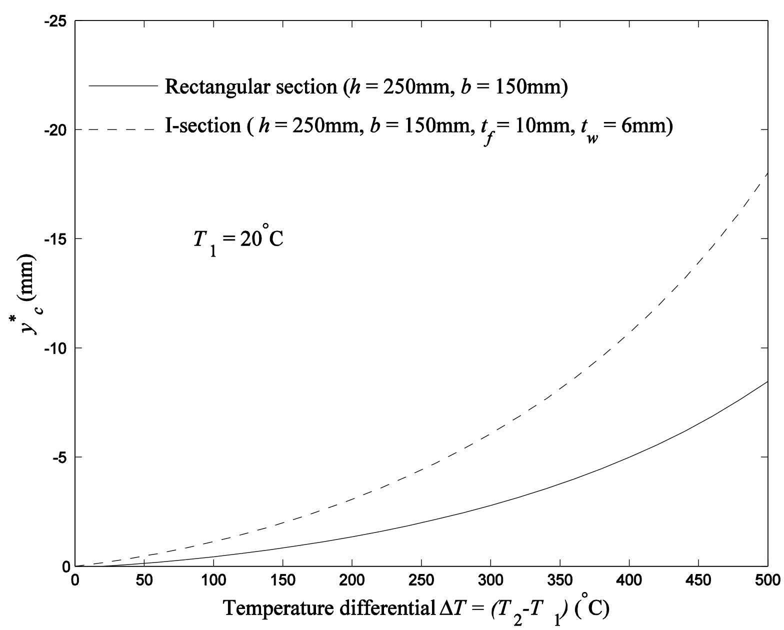

Figure 2. Effective centroids.

The typical differences between the effective centroid (x_c, y_c) and geometric centroid (x_c, y_c^*) are shown in Figure 2. It can be seen that the distance between the effective and geometric centroids increases with an increase of the temperature differential ΔT between the bottom and top surfaces.

The temperature T_o at the effective axis ox can be expressed as

$$T_o = T_{ave} + \frac{y_c^* \Delta T}{h} \tag{6}$$

3 LINEAR ANALYSIS

For the linear in-plane thermoelastic analysis of a beam having elastic axial and rotational restraints at its both ends, the differential equations of equilibrium can be derived by using the principle of virtual work based on the effective axes $oxyz$, which can be stated as

$$\Pi = \int_{-L}^{L} [-N\delta w' - M\delta v'']\mathrm{d}z + \sum_{i=\pm L} k_{ri} v_i' \delta v_i' = 0 \tag{7}$$

where the axial force N and the bending moment M are given by

$$N = -\int_A \sigma \mathrm{d}A = -\overline{EA}(w' - \alpha T_o) \text{ and } M = \int_A \sigma y \mathrm{d}A = -\overline{EI}(v'' - \frac{\alpha \Delta T}{h}) \tag{8}$$

Integrating Eq. (7) by parts leads to the differential equations of equilibrium as

$$\overline{EA} w'' = 0 \quad \text{and} \quad \overline{EI} v^{iv} = 0 \tag{9}$$

in the axial and transverse directions respectively; and to the static boundary conditions

$$EI(v'' + \frac{\alpha \Delta T}{h}) \pm k_r v' = 0 \quad \text{at} \quad z = \pm L \tag{10}$$

The essential geometric or kinematic boundary conditions of end-supported beams are

$$w = 0 \quad \text{and} \quad v = 0 \quad \text{at} \quad z = \pm L \tag{11}$$

The solutions of Eq. (10) can then be obtained as ((Pi and Bradford 2015)

$$v = \frac{\alpha \Delta T (L^2 - z^2)}{2h(1+\eta_r)} \quad \text{and} \quad w = \alpha T_o z \tag{12}$$

where the dimensionless stiffness of rotational restraints $\eta_r = k_r L / \overline{EI}$.

By substituting Eq. (12) into Eq. (8), the axial force N and bending moment M can be obtained as

$$N = \alpha T_o \overline{EA} \quad \text{and} \quad M = -\frac{\overline{EI} \eta_r \alpha \Delta T}{h(1+\eta_r)} \tag{13}$$

which indicates that the linear thermal axial force and bending moment are uniform along the span of the beam.

4 NONLINEAR ANALYSIS

For nonlinear in-plane analysis, the strain can be expressed as

$$\varepsilon = w' + \frac{1}{2}v'^2 - yv'' \tag{14}$$

and the axial force becomes

$$N = -\int_A \sigma \mathrm{d}A = -\overline{EA}(w' + \frac{1}{2}v'^2 - \alpha T_o) \tag{15}$$

Accordingly, the differential equations of equilibrium can be expressed as (Pi and Bradford 2015)

$$N' = 0 \quad \text{and} \quad M'' - (Nv')' = 0 \tag{16}$$

in the axial and transverse directions.

Hence, the axial force N = constant, and the differential equation of equilibrium in the transverse direction can be written as

$$v^{iv} + \mu^2 v'' = 0 \quad \text{with} \quad \mu^2 = \frac{N}{\overline{EI}} \tag{17}$$

The solution of Eq. (20) that satisfies the boundary conditions is then obtained as

$$v = \frac{L^2 \alpha \Delta T \left[\cos\theta - \cos(\theta z / L)\right]}{h\theta(\eta_r \sin\theta - \theta\cos\theta)} \tag{18}$$

where $\theta = \mu L$.

The nonlinear displacement is a function of the temperature gradient and the axial compressive force N (through θ). Hence, an equation of equilibrium of the temperature gradient and the axial force is needed. For this, substituting Eq. (18) into the axial force given by Eq. (15) and integrating over the entire beam leads to

$$\frac{\theta^2}{\lambda^2} = \alpha T_o + \frac{\alpha L^2 \Delta T^2 (\sin\theta\cos\theta - \theta)}{h^2\theta(\eta_r \sin\theta - \theta\cos\theta)} \tag{19}$$

where the slenderness λ of the beam is defined by

$$\lambda = \frac{L}{r} \quad \text{with} \quad r = \sqrt{\frac{\overline{EI}}{\overline{AE}}} \tag{20}$$

For a given beam under a given temperature gradient, the parameter θ can be obtained Eq. (19). The nonlinear transverse displacement v can then be calculated using Eq. (18).

Typical distributions of linear and nonlinear transverse displacement along the beam are compared in Figure 3. It can be seen that under the temperature gradient, the transverse displacements of pin-ended beam are larger than those of beams with end rotational restraints. As the dimensionless stiffness η_r of rotational restraints increases, the transverse displacements decrease. It can also be seen that the nonlinear transverse displacements are larger than the linear counterparts.

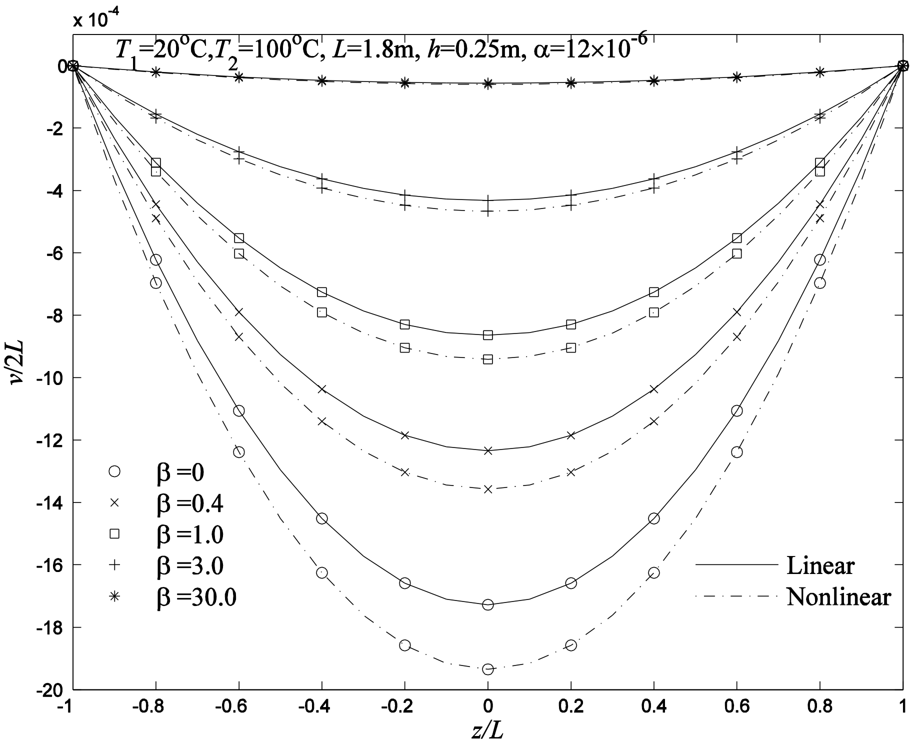

Figure 3. Distributions of transverse displacements.

5 CONCLUSIONS

This paper carried out linear and nonlinear thermoelastic analyses of an elastically restrained beam of doubly symmetric open thin-walled cross-section that is subjected to a linear temperature gradient field over its cross-section. It was found that geometric nonlinearity influences the thermoelastic responses of the beam to the temperature changes significantly. The influence decreases with an increase of the stiffness of the elastic restraints.

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