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EFFECTS OF FOUNDATION SETTLEMENT ON IN-PLANE BUCKLING OF SHALLOW ARCHES

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Foundation settlement of structures may take place and it may influence the structural behavior of structures. Shallow arches are used in engineering structures such as large span roofs and bridges. This paper is concerned with the effects of the foundation settlement on the in-plane buckling of shallow circular arches under a uniform radial load. Because the deformations of a shallow arch prior to buckling become nonlinear, their effects on the buckling also need to be considered. Differential equations of equilibrium for the nonlinear analysis are derived using the principle of virtual work. Analytical solutions for the nonlinear behavior and for the nonlinear buckling load are obtained. It is found that the foundation settlement plays an important role in the nonlinear structural response of the arch to the external load and that significantly influence the nonlinear buckling load of the arch.

Keywords: Analytical, Instability, Nonlinear, Structures.

1 INTRODUCTION

Foundation settlement of structures may take place from time to time, which influences the structural behavior of structures. Shallow arches are used in engineering structures such as large span roofs and bridges. It is known that shallow arches may buckle elastically in the plane of loading and fail in a limit point (snap-through) instability mode or in a bifurcation buckling mode (Pi *et al.* 2002). However, it is unknown if the foundation settlement influences the in-plane buckling behavior of shallow arches.

Because the deformations of a shallow arch prior to buckling become nonlinear, their effects on the buckling need to be considered in addition to the settlement. This paper investigates the influence of foundation settlements of shallow arches on their nonlinear equilibrium and buckling load. Differential equations of equilibrium for the nonlinear analysis are derived using the principle of virtual work. Analytical solutions for the nonlinear behavior and for the nonlinear buckling load are obtained and verified by comparisons with the finite element (FE) results.

2 NONLINEAR EQUILIBRIUM

The differential equations of equilibrium for non-liner analysis by using the principle of virtual work, which states that

$$\delta\Pi = \int_{-\Theta}^{\Theta} [-NR(\delta\tilde{w}' - \delta\tilde{v} + \tilde{v}'\delta\tilde{v}'\} - M\delta\tilde{v}'' - qR^2]\mathrm{d}\theta = 0 \tag{1}$$

where N and M are the axial compressive force and bending moment and they can be expressed as

$$N = AE(\tilde{w}' - \tilde{v} + \frac{1}{2}\tilde{v}'^2) \tag{2}$$

and

$$M = -EI\frac{\tilde{v}''}{R} \tag{3}$$

Integrating Eq. (1) by parts leads to the differential equations of equilibrium in the axial and radial directions as

$$N' = 0 \quad \text{and} \quad \frac{\tilde{v}^{iv}}{\mu^2} + \tilde{v}'' = Q \tag{4}$$

with

$$Q = \frac{qR - N}{N} \quad \text{and} \quad \mu^2 = \frac{NR^2}{EI} \tag{5}$$

and the boundary conditions can be written as

$$\tilde{v}'' = 0 \text{ at } \theta = \pm\Theta,\ \ \tilde{w} = 0, \tilde{v} = 0 \text{ at } \theta = -\Theta \tag{6}$$

and

$$\tilde{w} = \Delta\sin\Theta, \tilde{v} = \Delta\cos\Theta \quad \text{at} \quad \theta = \Theta \tag{7}$$

where Δ is the vertical settlement of the arch end at $\theta = \Theta$.

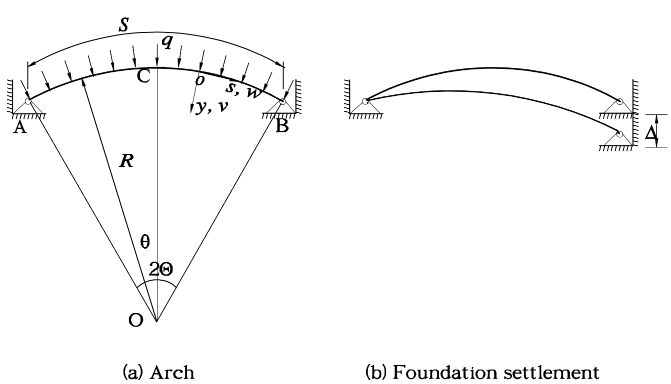

Figure 1. Arch and its foundation settlement.

The first equation of Eq. (4) indicates the axial compressive force N is a constant along the arch length. Solving the second equation of Eq. (4) under boundary conditions given by Eqs. (6) and (7) leads to the analytical solutions for the radial displacement as

$$\tilde{v} = \frac{Q}{\mu^2}\left(\frac{\cos\mu\theta}{\cos\beta} - 1 + \frac{1}{2}\mu^2\theta^2 - \frac{1}{2}\beta^2\right) + \frac{\Delta\cos\Theta}{2}\left(1 + \frac{\theta}{\Theta}\right) \tag{8}$$

where the axial force parameter $\beta = \mu\Theta$ (Pi *et al*.2014).

It is noted the radial displacement is a function of the dimensionless external load Q and the axial force parameter β. Hence, an equation of equilibrium between Q and β is required. For this, substituting the solution for the radial displacement given by Eq. (8) into the axial force expression given by Eq. (2) and integrating it over the entire arch and considering the boundary condition for the axial displacement given by Eq. (7) leads to the equation of equilibrium between Q and β as

$$A_1 Q^2 + A_2 Q + A_3 = 0 \tag{9}$$

where

$$A_1 = \frac{1}{4\beta^2}\left(5 - 5\frac{\tan\beta}{\beta} + \tan^2\beta\right) + \frac{1}{6} \tag{10}$$

$$A_2 = \frac{1}{\beta^2}\left(1 - \frac{\tan\beta}{\beta}\right) + \frac{1}{3} \tag{11}$$

$$A_3 = \frac{\beta^2}{\lambda^2} + \frac{\Delta^2\cos^2\Theta}{8\Theta^4} - \frac{\Delta(2\sin\Theta + \cos\Theta)}{2\Theta^2} \tag{12}$$

where the modified slenderness λ is defined as (Bradford *et al*. 2002)

$$\lambda = \frac{S}{r_x}\frac{\Theta}{2} \tag{13}$$

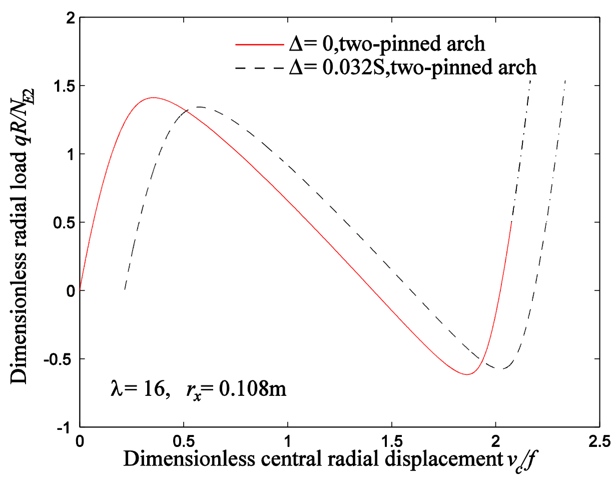

Figure 2. Variation of qR/N_{E2} with v_c/f.

The effects of the foundation settlement on the nonlinear equilibrium of shallow arches are shown in Figure 2 as variations of the dimensionless load qR/N_{E2} with the dimensionless central radial displacement v_c/f for an arch with $\lambda = 16$, where N_{E2} is the second mode flexural buckling load of a pin-ended column under uniform axial compression (Timoshenko and Gere 1961) and f is the rise of the arch..

It can be seen that the foundation settlement influences the nonlinear behavior of shallow arches. The influence is also seen in Figure 3, which shows variations of the dimensionless load qR/N_{E2} with the dimensionless axial force N/N_{E2} for the same arch.

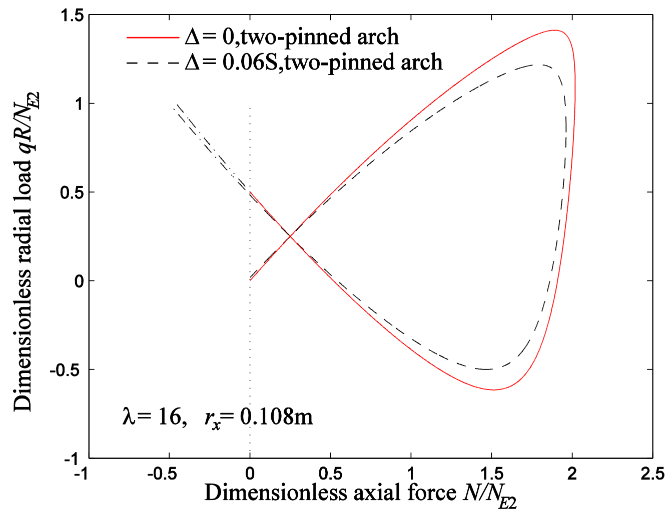

Figure 3. Variation of qR/N_{E2} with N/N_{E2}.

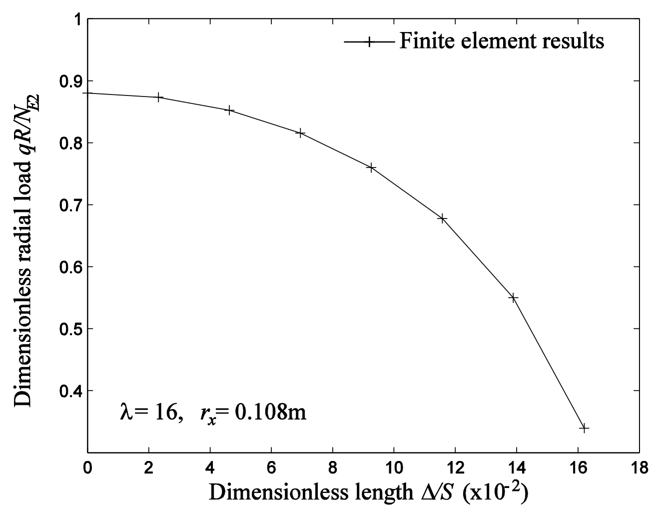

Figure 4. Effects of foundation settlements on the buckling load.

3 NONLINEAR BUCKLING

The non-linear buckling load is the local maximum and so differentiating the non-linear equilibrium equation given by Eq. (9) with respect to β leads to the equilibrium equation between Q and β at the limit point as being

$$B_1 Q^2 + B_2 Q + B_3 = 0 \tag{14}$$

where

$$B_1 = 2A_1 - \frac{\beta \partial A_1}{2\partial \beta}, \quad B_2 = 4A_1, \quad B_3 = A_2 - \frac{\beta_2}{\lambda^2} \tag{15}$$

Solving Eqs. (9) and (14) simultaneously leads to the non-linear limit point buckling load as shown in Figures 2 and 3. It can be seen from Figures 2 and 3 that the nonlinear buckling load of arches with foundation settlements is lower than that of the arch without foundation settlements.

4 INFLUENCE OF FOUNDATION SETTLEMENTS

The influence of foundation settlements on the in-plane buckling load is shown in Figure 4 as variations of the dimensionless buckling load with the settlement. It can be seen that the in-plane buckling load decreases as the foundation settlement increases.

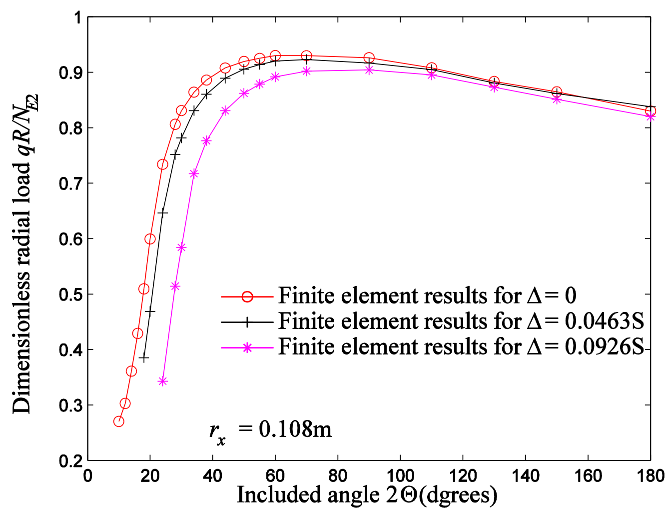

Figure 5. Effects of foundation settlements on the buckling load.

The influence of foundation settlements on the in-plane buckling load is also shown in Figure 5 as variations of the dimensionless buckling load with the included angle for different settlements. It can be seen that the influence of the foundation settlement on the in-plane buckling load is more significant for shallow arches but not significant for deep arches.

5 VERIFICATIONS

To verify the analytical solutions, the finite element results for an arch with a modified slenderness $\lambda = 16$ are compared with the corresponding analytical solutions in Figure 6 as the variations of the dimensionless load with the dimensionless central displacement. It can be seen that the agreements between them are extremely good.

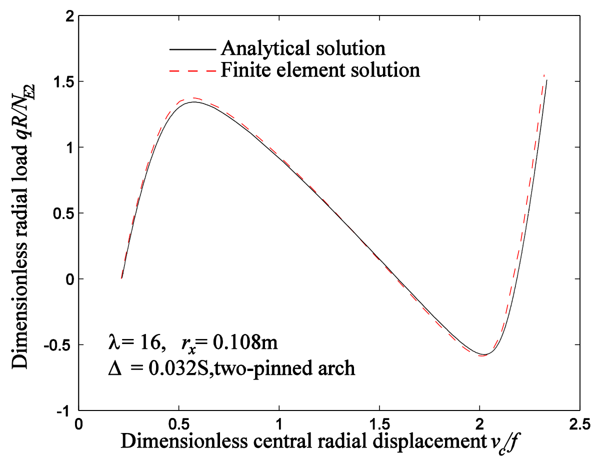

Figure 6. Comparison with FE results.

6 CONCLUSIONS

The influence of foundation settlement on the nonlinear in-plane equilibrium and buckling of shallow arches under a uniform radial load has been investigated. Analytical solutions for the nonlinear behavior and for the nonlinear buckling load were obtained. It was found that the foundation settlement plays an important role in the nonlinear structural response of the arch to the external load and that significantly influence the nonlinear buckling load of the arch.

Acknowledgments

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