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3-D LUMPED MASS METHOD OF DYNAMIC ANALYSIS FOR HIGH-RISE BUILDING

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This study is related to decreasing size of stiffness matrix of high-rise building by using floor diaphragm constraints. The combined actions of horizontal structural members such as slab and beams produce structural response that is much stiffer than vertical elements such as columns. In structural analysis of high-rise building model, absolute rigid slab and vertical structure are working together for lumped mass method. Each lumped mass node has 6 degree of freedoms which is very suitable for high-rise building calculation. The lumped mass is consisted of slab and beams, on the other hand the system's stiffness matrix is only assembly of columns and braces. High-rise building lumped mass model's stiffness matrix size is n=6 x m (n; total size of stiffness matrix, m; total floor number). In the step by step calculation, total unknown number is decrease enough which means we can easily calculate high-rise building structures for dynamic analysis also nonlinear step by step calculation using simple personal computer. If building model has large span and short height, in this case rigid slab theory will not satisfied. This idea is more suitable for high-rise building calculation.

Keywords: Structural analysis, Tall building, Earthquake analysis, Diaphragm constraints, Rigid slab, Programming.

1 INTRODUCTION

In the step by step calculation, high-rise building's 3-D model has a huge number of unknown freedoms which is impossible to calculate usual personal computer. On the problems of decreasing size of stiffness matrix, generally using constraints in the stiff elements such as "rigid floor diaphragm" Edward L.Wilson (2002) in multistory building model. This paper introduces one case of constraints which is absolute rigid slab (floor diaphragm) transformation is suitable for high-rise buildings with nondeterministic loadings. It is based on developed transforming between 3-D finite element methods and lumped mass method thus leading to dynamic step by step calculation with programming. Finally, tall building analytical model is calculated during the El-Centro Earthquake and checked results by using commercial software.

2 PRELIMINARY STRUCTURAL ANALYSIS ON HIGH-RISE BUILDING

High-rise building model has some special features such as short span, usually ratio of length and width of plan are similar or symmetrical in plan, and height is many times taller than sides. Vertical and horizontal structures stiffness's are perceptible different from each other for this reason deformations has large gap between vertically and horizontally. Slab and beams are connected each other more stiff than column. Therefore each floor slab is working same as the rigid slab, main deformation of structure is only vertical structures such as column and brace. This idea comes from the "elastic behavior of pile group foundation" written by Tominaga and Yamamoto (1986). Horizontal stiffness is much higher than vertical stiffness; also individual columns deformation is relatively similar. Based on these issues, rigid slab idea (floor diaphragm constraints) is developed in this research.

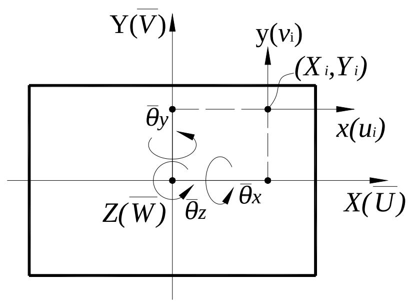

Figure 1. Slab center of mass and elements position.

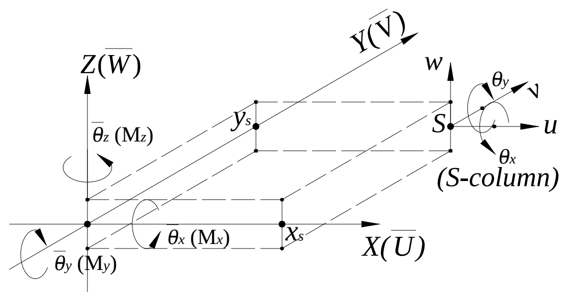

Figure 2. Freedom of element and slab.

Static analysis of high-rise building model is based on Finite Element Method (FEM), and 3-D frame system consisted of several types of elements, such as vertical columns, braces, horizontal slabs and beams. Each elements are joined each other by the nodes, each node has 6 degree of freedoms also members local stiffness matrix size is 12x12. Eq. (1) shows transformation matrix, which is related to the rigid slab idea. After creating element's stiffness matrix, the global stiffness matrix is transformed by using transformation matrix ($[T]$). Creating stiffness matrix of each element is same as FEM analysis and only difference is assembling process of system's stiffness matrix. Figure 1 show individual elements position defined by center of mass node on slab and Figure 2, shown individual element displacement and rotation is determined by main axis of slab. Also there are 2 simplifications, which are:

1. Floor slab is absolute rigid.
2. Individual column's torsional deformation does not affect to the system.

In other words, individual vertical element's torsional displacement (θ_z) is neglected by Eq. (1) simplification for 3-D lumped mass system. Structures are deformed by center axis of system and easy to create lumped mass system.

$$\{u\} = [T]\{\overline{U}\}$$

$\{u\}^T = \lfloor u_i \; v_i \; w_i \; \theta_{xi} \; \theta_{yi} \rfloor$ - each elements displacement;

$$\{\overline{U}\}^T = \lfloor \overline{U}\,\overline{V}\,\overline{W}\,\overline{\theta_x}\,\overline{\theta_y}\,\overline{\theta_z} \rfloor \text{ - center of mass point's displacement vector;} \qquad (1)$$

$$[T] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -Y_i \\ 0 & 1 & 0 & 0 & 0 & X_i \\ 0 & 0 & 1 & Y_i & -X_i & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \text{ - transformation matrix;}$$

3-D lumped mass model is shown in Figure 3; each floor's lumped mass node has 6 freedoms and main axis of Multi Degree of Freedom system (MDOF) is located in center of mass. For the considered case of 3-D finite element frame model is changed to rigid slab and vertical structure model which is stiffness matrix assembly easy for lumped mass method. Rigid slab is nodal mass of MDOF system rotate the main axis and every column deformed by rigid slab's motion. After using the transformation matrix of rigid slab, assembly of stiffness matrix becomes simple band matrix. Lumped mass method is usually used for 2-D dynamic system, but in this paper the idea is used for 3-D calculation.

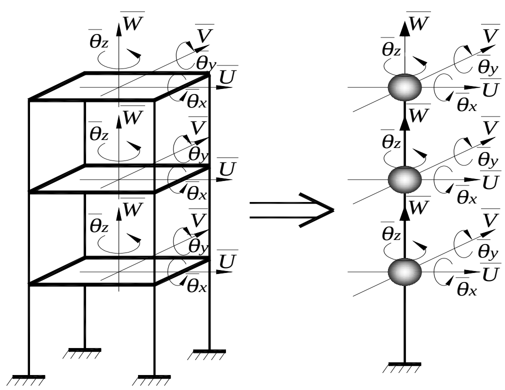

Figure 3. Lumped mass model.

In dynamic analysis of structure, damping coefficient that is proportional to stiffness given by Eq. (2) is frequently used. Damping coefficient of Rayleigh type, which was used this paper, is combination of mass and stiffness proportional damping, also a_0 and a_1 are constants, that are defined by Eq. (2).

w_1, w_2 - first and second order circular frequency of structure.

h_1, h_2 – damping ratio (in this case $h_1=h_2=0.05$)

$$[c] = a_0[m] + a_1[k] \qquad (2)$$

where,

$$a_0 = 2\omega_1\omega_2\left(h_1\omega_2 - h_2\omega_1\right)/\left(\omega_2^2 - \omega_1^2\right)$$
$$a_1 = 2\left(h_2\omega_2 - h_1\omega_1\right)/\left(\omega_2^2 - \omega_1^2\right)$$

Dynamic analysis of programming is used with average acceleration method of Newmark's beta method in the step by step calculation. In here coefficients are equal to $\gamma=1/2; \beta=1/4$. Programming code is developed successfully on FORTRAN language, by using subroutines which are inverse matrix solver and eigenvalue and eigenvector calculator.

3 ANALYTICAL MODEL

3.1 Static Loading

Figure 4 shows the analytical model of high-rise building with the rigid slab attached 400kN static horizontal load along the *X* axis at the roof and checked the results by comparing with result of commercial software "Lira 9.6" (2012).

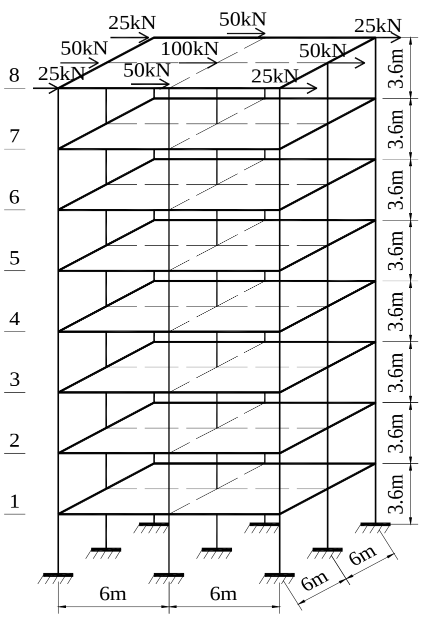

Figure 4. Analytical model of 8th floors building.

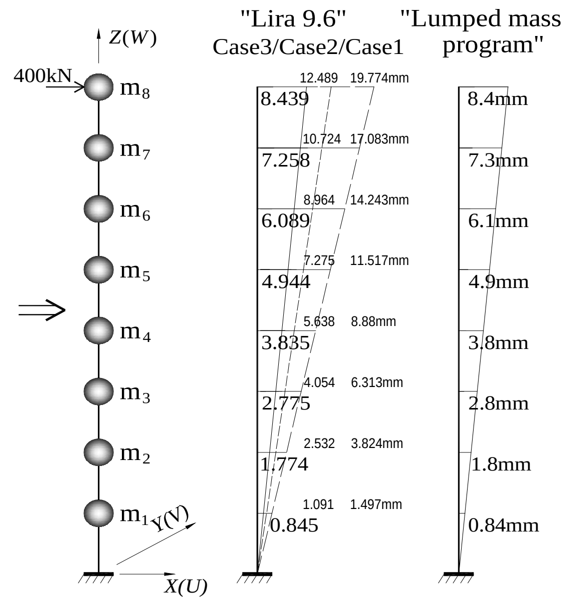

Figure 5. Comparison of displacement under static load.

3-D lumped mass program calculation is based on rigid slab idea consequently deformation result is different from simple system with member information shown as Table 1 in Lira 9.6. There are three different calculations with Young's module of member. Firstly stiffness is given as Table 1 (Case-1), displacement of top floor is 19.77mm in Figure 5. In second case only beam and slab stiffness is 3 times higher than first case $E=6x10^6$ kN/m^2, displacement is 12.4mm and third case stiffness is $E=2x10^{10}$ kN/m^2 same as absolute rigid horizontal structure, displacement is 8.439mm almost same as Lumped mass programming results. We can observe if horizontal structure stiffness is increased as much as higher, deformation will be close to our program result.

Table 1. Structural member information of analytical model.

№	Element type	Material	BxH (mm)	Young's module (kN/m^2)
1	Column	Reinforced	600x600	2x10^7
2	Beam	concrete	500x700	2x10^7
3	Slab		200	2x10^7

3.2 Dynamic Loading

After the Step-by-step successful calculation, program gave those results, shown in Figure 6. In this calculation, sequence of time step was Δt=0.02s and total duration of El-Centro 1940 earthquake is 50 seconds.

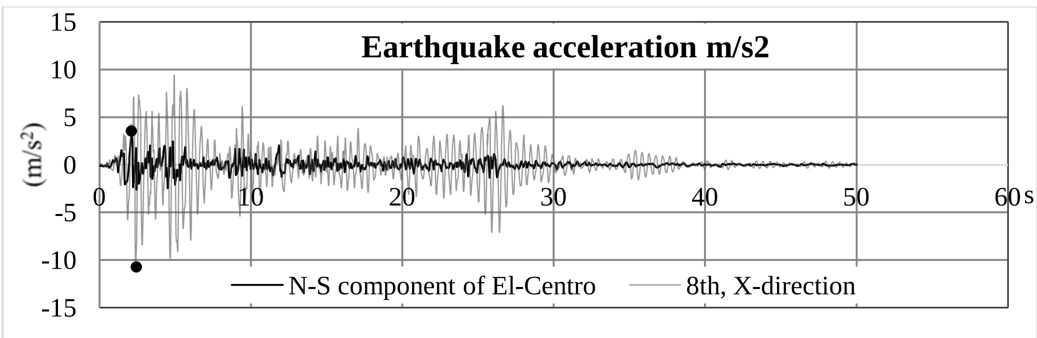

Figure 6. 8th floor and ground acceleration responses during earthquake (N-S component).

Input data of El-Centro earthquake has N-S, E-W and U-D components accordingly analytical model has given along with *X*, *Y*, *Z* direction and θ_x, θ_y, θ_z rotation displacement, velocity, acceleration meanings. N-S component of earthquake's maximum acceleration was 3.417m/s^2 higher than other components therefore 3-D lumped mass model *X* direction is same as N-S direction and maximum acceleration is 11m/s^2 at roof. As shown in Figure 6, maximum acceleration amplitude time is first 6 seconds, and therefore Figure 7 shows XY plane displacement curves of 2nd, 5th, 8th floors during that time.

As a result, displacement curves shape is similar each floors and meaning of amplitude is reasonable. Lastly Figure 8 shows the acceleration response of three floors along *Y* direction during E-W component. We can see easily almost linear relationship between those responses of maximum acceleration.

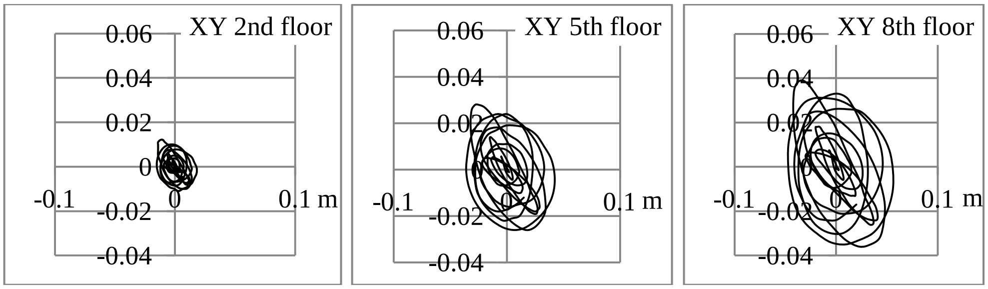

Figure 7. 2, 5, 8th floors *XY* displacement curves during El-Centro (N-S horizontal, E-W).

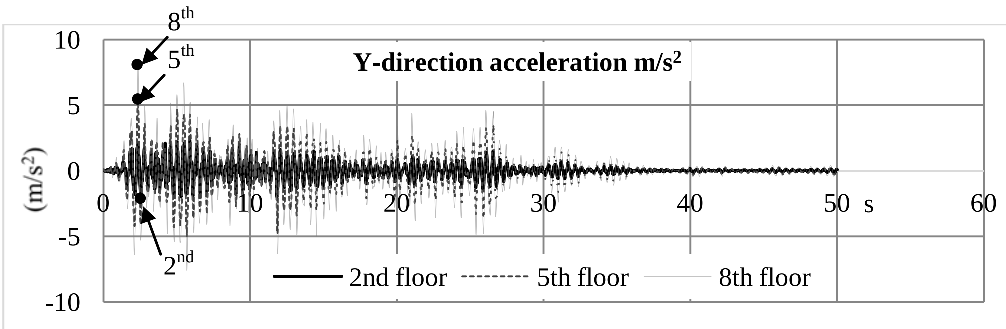

Figure 8. 2, 5, 8th floors acceleration responses during El-Centro (E-W component).

4 CONCLUSIONS

In this paper, dynamic analysis on high-rise building were performed by using rigid slab idea with lumped mass system and step by step calculation with Newmark's beta method. Commercial software and 3-D lumped mass program's static calculation results difference were only 0.5 percent on the other hand, rigid slab idea calculation's accuracy is possible to utilize for structural analyzing software. In dynamic analysis to check the 3-D lumped mass program using the model structure, smooth and reasonable results could be obtained without numerical trouble in case El-Centro earthquake simultaneous input data of N-S, E-W and U-D.

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