Implementing Innovative Ideas in Structural Engineering and Project Management
Edited by Saha, S., Zhang, Y., Yazdani, S., and Singh, A.

ISBN: 978-0-9960437-1-7

STRUCTURAL DAMAGE DETECTION BY MULTI-SCALE CROSS-SAMPLE ENTROPY

JUI-CHANG LIANG, MING-JING WANG, and TZU-KANG LIN

Dept of Civil Engineering, National Chiao Tung University, Hsinchu, Taiwan

This study proposes a structural health monitoring (SHM) system based on multi-scale entropy (MSE) and multi-scale cross-sample entropy (MSCE). By measuring the ambient vibration signal from a structure, the damage condition can be rapidly evaluated via a MSE analysis. The damage location can then be detected by analyzing the signals of different floors under the same damage condition via a MSCE analysis. Moreover, a damage index is proposed to efficiently quantify the SHM process. A numerical simulation of a four-story steel structure is used to verify that the damage location and condition can be detected by the proposed SHM algorithm, and the location can be efficiently quantified by the damage index. Based on the results, the damage condition can be correctly assessed, and accuracy rates of 60% and 86% for the damage location can be achieved using the MSCE and damage index methods, respectively.

Keywords: Structural health monitoring, Damage detection, Ambient vibration.

1 INTRODUCTION

Multi-scale entropy (MSE) was first proposed by Costa (2002) and Costa (2005). The method was successfully applied on a biological time series. Time series data of three groups, healthy subjects, subjects with congestive heart failure (CHF), and subjects with atrial fibrillation (AF), were analyzed by utilizing MSE with a satisfactory classification result.

Cross-approximate entropy (C-ApEn) was first proposed by Pincus and Singer (1966). Based on C-ApEn, cross-sample entropy (Cross-SampEn) was developed by Richman and Moorma (2000). The performance of Cross-SampEn has been shown to be much better than that of C-ApEn for evaluation of the synchronization degree between the heart rate and chest volume. Vocal disorder was analyzed using Cross-SampEn with two sound signals including the signals from a microphone (MIC) and electroglottograph (EGG) by Fabris (2013). Possible locations on a diseased person can be identified as the Cross-SampEn values are relatively small for healthy parts.

Based on the abovementioned research, a new SHM concept is proposed in this study. As the dynamic response of the structure is analyzed under a scale factor of 1 in the traditional method, where different damage conditions may not be reflected significantly, the MSE method is introduced for improved identification results. For identification of damage locations, the multi-scale cross-sample entropy (MSCE) method is applied by analyzing the signals of different floors under the same damage condition. In order to clearly illustrate the possible damage location, a damage index is proposed.

2 THE PROPOSED SHM METHOD

Suppose a time series $\{X_i\} = \{x_1, ..., x_i, ..., x_N\}$ with length N and a vector $u_m(i) = \{x_i, x_{i+1}, ..., x_{i+m-1}\}, 1 \le i \le N-m+1$ of length m can be defined as the pattern templates. The template space T, which is the $N-m+1$ combination of all samples with length m, is further defined as :

$$T = \begin{pmatrix} x_1 & x_2 & \cdots & x_m \\ x_2 & x_3 & \cdots & x_{m+1} \\ \vdots & \vdots & \ddots & \vdots \\ x_{N-m+1} & x_{N-m+2} & \cdots & x_N \end{pmatrix} \tag{1}$$

Let $n_i^m(r)$ be the number of similarities between templates $u_m(j)$ and $u_m(i)$ where a similarity is defined as $d[u_m(i), u_m(j)] \le r$; r is a predetermined threshold; and d_{ij} is the maximum distance between the two samples i and j:

$$d_{ij} = \max\{|x(i-k) - x(\mathrm{j+k})| : 0 \le k \le m-1\} \tag{2}$$

By substituting different pattern templates for similarity comparisons with template i, the total number of templates satisfying the criterion in Eq. (2) can be calculated, and the probability of sample similarity $U_i^m(\mathrm{r})$ can be defined as follows:

$$U_i^m(r) = \frac{n_i^m(r)}{(N-m-1)} \tag{3}$$

By calculating the average probability of templates with length m, the average similarity probability can be obtained as:

$$U_i^m(r) = \frac{1}{(N-m)} \sum_{i=1}^{N-m} U_i^m(r) \tag{4}$$

Here, $U^m(\mathrm{r})$ represents the template similarity of length m to the gross template space. Following the same procedure, the average similarity probability $U^{m+1}(r)$ of length $m+1$ can be estimated, and the SampEn of parameters m, r, and N can finally be calculated as:

$$S_E(\mathrm{m}, \mathrm{r}, N) = -\ln \frac{U^{m+1}(\mathrm{r})}{U^m(\mathrm{r})} \tag{5}$$

To evaluate the time series under different time scales, the MSE converts the original signal through a coarse-graining procedure, and the corresponding S_E values can be calculated individually to reflect any potential anomaly.

The estimation of Cross-SampEn can be summarized as follows. First, $\{X\} = \{x_1, ..., x_i, ..., x_N\}$ and $\{Y\} = \{y_1, ..., y_i, ..., y_N\}$ represent two individual time series with length N. The signals are detached into pattern templates of length m: $u_m(i) = \{x_i, x_{i+1}, ..., x_{i+m-1}\}, 1 \le i \le N-m+1$ and $v_m(j) = \{x_j, x_{j+1}, ..., x_{j+m-1}\}, 1 \le j \le N-m+1$.

The similarity number between $u_m(i)$ and $v_m(j)$, defined as $n^m(r)$, is calculated by Eq. (2) under the following criterion:

$$d\left[u_m(i), v_m(j)\right] \le r, \;\; 1 \le j \le N - m \tag{6}$$

The similarity probability of the template space can be defined as follows:

$$U_i^m(r)(v\|u) = \frac{n^m(r)}{(N-m)} \tag{7}$$

The average similarity probability of length *m* can be evaluated by the following equation:

$$U^m(r)(v\|u) = \frac{1}{(N-m)} \sum_{i=1}^{N-m} U_i^m(r)(v\|u) \tag{8}$$

Here, $U^m(r)(v\|u)$ quantifies the degree of asynchrony/dissimilarity between two template spaces, which can be treated as the *m*-point cross similarity of two time series. Similarly, the Cross-SampEn of length *m*+1 from the template spaces T_x and T_y can finally be derived by the average similarity probability $U^{m+1}(r)(v\|u)$
as follows:

$$CS_E(m, r, N) = -\ln\left\{\frac{U^{m+1}(r)(v\|u)}{U^m(r)(v\|u)}\right\} \tag{9}$$

Moreover, a floor damage index based on a multi-scale characteristic is proposed to improve the efficiency and accuracy of the SHM system. For a structure with *N* floors, the MSCE under the undamaged and damaged conditions can be expressed, respectively, as:

$$\mathrm{MSCE}_{\mathrm{undamaged}} = \{\mathrm{H}_1, \mathrm{H}_2, \cdots, \mathrm{H}_F\}^T \tag{10}$$

$$\mathrm{MSCE}_{\mathrm{damaged}} = \{\mathrm{D}_1, \mathrm{D}_2, \cdots, \mathrm{D}_F\}^T \tag{11}$$

where the symbols H and D represent the undamaged and damaged conditions, respectively, and the subscript *F* indicates the upper floor for the Cross-SampEn processing. The individual MSCE curve for a specific floor response and damage conditions can then be calculated under different scale factors accordingly. For example, the curve of the Cross-SampEn of signals between the foundation and the first floor, which represents the basic characteristics of the first floor of the undamaged structure, is expressed as:

$$\mathrm{H}_1 = \{CS_{E\,\mathrm{H}_1}^{\,1}, CS_{E\,\mathrm{H}_1}^{\,2} CS_{E\,\mathrm{H}_1}^{\,3}, \cdots\cdots, CS_{E\,\mathrm{H}_1}^{\,\tau}\} \tag{12}$$

where the superscript τ represents the scale factor.

Similarly, the Cross-SampEn of signals between the floor and the floor of the damaged structure can be expressed as:

$$D_F = \{CS_{E\,\mathrm{D}_F}^{\,1}, CS_{E\,\mathrm{D}_F}^{\,2}, CS_{E\,\mathrm{D}_F}^{\,3}, \cdots\cdots, CS_{E\,\mathrm{D}_F}^{\,\tau}\} \tag{13}$$

Based on this definition, the damage index of a specific floor F can be expressed as follows:

$$DI_F = \sum_{q=1}^{\tau} (CS_{E\,\mathrm{D}_F}^{\,q} - CS_{E\,\mathrm{H}_F}^{\,q}) \tag{14}$$

where F is the floor number for damage evaluation.

3 NUMERICAL EVALUATION

A series of numerical analyses are conducted on a four-story steel structure model. The ambient response of the structure is simulated by exciting a numerical model established using the finite element method. The height of each story is 160 cm and the length and width of the floor are both 200 cm. Four 120-kg mass blocks are affixed on each floor. The velocity response of the mass center of each floor is simulated to reflect the characteristics of the structure.

In total, eleven damage cases in five damage categories are simulated as a database for verification. Details of the damage groups and floors are listed in Table 1.

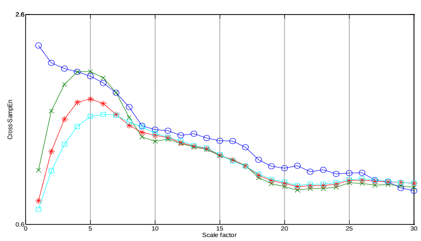
Table 1. List of damage groups and locations (numerical simulation).

Case Number	Damage Group	Damage Floors
1	Undamaged	None
2	Single floor	1F
3		2F
4		3F
5		4F
6	Two floors	1 & 2F
7		2 & 3F
8		3 & 4F
9	Three floors	1 & 2 & 3F
10		2 & 3 & 4F
11	All floors	1 & 2 & 3 & 4F

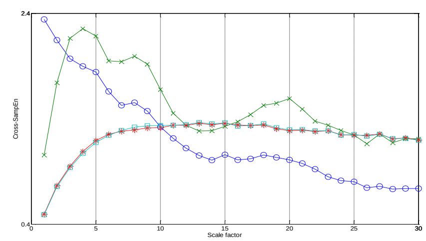
The MSE analysis is first conducted for five different damage cases. The trend analyzed from the signal of the third floor. The result demonstrates that the damage condition can be reflected correctly by calculating the SampEn. Following the increase of the damage condition, the entropy distance to the healthy condition is augmented.

Following the MSE analysis, signals are processed by Cross-SampEn to identify the damage location. As shown in Figure 1 (a), the undamaged case is first evaluated using signals of the two adjacent floors as a reference of healthy conditions. The established MSCE curve is used as a reference for damage detection under different conditions. The blue curve H_1 is the MSCE between the ground and first floors. The green, red, and cyan MSCE curves are represented by H_2, H_3, and H_4, respectively.

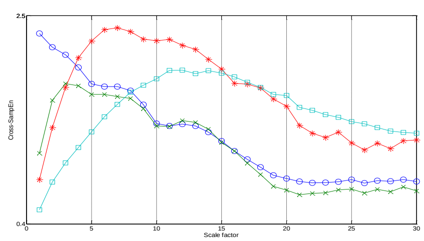
The MSCE curves for the damage locations are denoted as D_1, D_2, D_3, and D_4, respectively. For the case of damage on the second floor shown in Figure 1 (b). The result indicates that considerable change occurs between the first and second floors, and the damage location can be identified as the second floor. Figure 1 (c) shows the MSCE curves of the damage location on the third and fourth floors, the damages are classified to be on the third and fourth floors. The MSCE of the damage location from the first to third floor is shown in Figure 1 (d). Damage is misclassified on all floors owing to the compatibility between the MSCE curves. Finally, the MSCE of damages on all floors is shown in Figure 1 (e), all floors are identified as damaged.



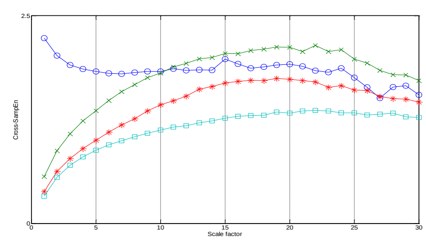
(a) Healthy condition.



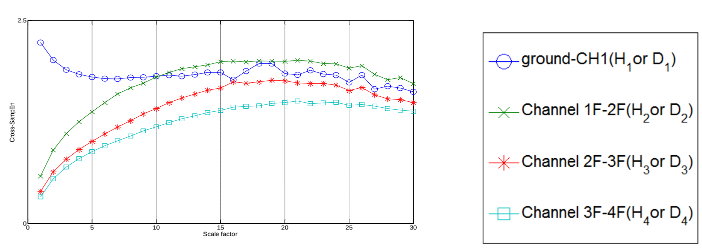
(b) Damage on the second floor.



(c) Damage on the third and fourth floors.



(d) Damage from the first to the third floor.

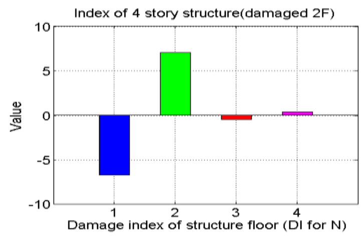


(e) Damage on all floors.

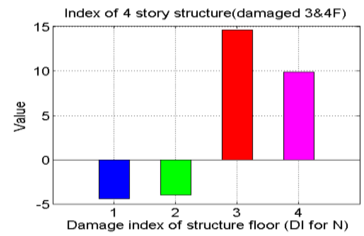
Figure 1. The numerical MSCE diagram.

Following the preliminary diagnosis of the damage location made from the MSCE curve, the damage index of each floor is calculated based on the healthy state of the structure. A partial list of the floor damage index is shown in Figure 2.

The accuracy of all damage cases is summarized. The total accuracy rate is estimated based on whether the damage location can be correctly detected. The accuracy based on the MSCE curve is approximately 60%. By introducing the damage index proposed in this study, the accuracy can be improved to 80%. The feasibility of the proposed SHM system has thus been successfully demonstrated.



(a) Damage on the second floor.



(b) Damage on the third and fourth floor.

Figure 2. The damage indices of different damage conditions.

4 SUMMARY AND CONCLUSION

A structural health monitoring (SHM) algorithm based on multi-scale entropy (MSE) and multi-scale cross entropy (MSCE) was proposed in this study. By measuring the ambient vibration signal from a structure, the damage condition can be rapidly evaluated via MSE analysis, and the damage location can be detected via MSCE analysis. Moreover, a damage index was also proposed to efficiently quantify the SHM process. A four-story steel structure with different damage conditions and damage locations was simulated and analyzed numerically. As shown by the results, the damage condition can be significantly identified via MSE analysis, and the damage location can also be determined through comparison of cross-sample entropy between undamaged and damaged cases.

References

Chiara Fabrisa, Wladimiro De Colleb, Giovanni Sparacinoa, Voice disorders assessed by (cross-) Sample Entropy of electroglottogram and microphone signals, *Biomedical Signal Processing and Control*, 8 920– 926, 2013.

M. Costa, A. L. Goldberger and C. K. Peng, Multi-scale Entropy Analysis of Complex Physiologic Time Series, *Physical Review Letters*, Vol. 89, No.6, pp. 068102-1 - 068102-4, 2002.

M. Costa, A. L. Goldberger and C. K. Peng, Multi-scale Entropy Analysis of Biological Signals, *Physical Review E*, Vol.71, pp. 021906-1 - 021906-18, 2005.

Richman, Joshua S., and J. Randall Moorman, Physiological time-series analysis using approximate entropy and sample entropy, *Am J Physiol Heart CircPhysiol*, 278: H2039–H2049, 2000.

S.M. Pincus, B.H. Singer, Randomness and degrees of irregularity, *Proc. Natl.Acad.* Sci. U.S.A. 93 2083–2088, 1996.