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SHEAR STRAIN SAMPLING POINTS OF PLATE ELEMENT FROM DESIRABLE DISPLACEMENT FIELDS AND MIXED FINITE ELEMENTS

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A newly developed four-node bilinear plate element based on linked interpolation and Desirable Displacement Field (DDF) concept is formulated for the analysis of general plate problems. The proposed element is formulated on a mixed finite element for Reissner-mindlin plate theory and transverse displacement is linked to the rotation degree of freedom to ensure high-order interpolation capacity. By assumed strain method, the DDF is introduced for the investigation of strain sampling points. A high order polynomial for transverse displacement with linking shape function is used in finite element discretization to provide a better solution for the plate problems. A number of commonly selected problems will be tested using the present element to compare with other element models in the open literature to assess their relative convergence and accuracy.

Keywords: Optimum sampling points, Shear locking, Transverse shear strain, Shape functions, Linked interpolation, Quadrilateral element, Assumed strain method.

1 INTRODUCTION

Formulations of the element with the assumed strain methods have been successfully employed for development of a number of plate finite elements. These formulations can be very effective in eliminating the shear locking of plate and shell elements while maintaining the directional invariance and introducing no extra zero-energy modes or spurious modes. This advantage over the reduced or selective integration technique is significant in consideration of reliability of numerical results. The successful application of the assumed strain method to the development of degenerated plate elements has led to the establishment of a systematic procedure for the selection or locating optimal strain sampling points. This systematic procedure is proposed based on the concept of desirable displacement fields (DDF) (Ma and Kanok-Nukulchai 1989). With values at the optimal sampling points, each strain component can appropriately interpolated over the element domain in terms of nodal displacement variables. It is expected that when all the displacement fields in an element are in the form of corresponding desirable fields, the element performance will be defect-free.

The main objective of this study is to determine the possibility of applying the finite element method to the development of a new four-node plate element. The four-node Lagragian-type degenerated plate element is proposed to avoid the shear locking phenomena by the Desirable Displacement Field (DDF) concept. By assumed strain methods, this concept was introduced for the investigation of strain sampling points. A high order polynomial for transverse displacement with linking shape function based on mixed finite element formulation for Reissner-Mindlin plates (Zienckiewicz *et al.* 1993 and Xu *et al.* 1994) was used in finite element discretization to provide a better solution for the plate problems. The proposed element should be robust reliable, but simple and efficient, and can be incorporated with convenience in existing package for practical application.

2 FORMULATION OF THE ELEMENT

It is well know that strictly using the standard procedure to formulate the transverse shear stiffness will cause the transverse shear locking, In order to develop an element, which is applicable for both thick and thin plate, the assumed strain method will be used to eliminate the shear locking. The appropriate sampling points are required for transverse shear strain interpolation. Therefore, only transverse shear strain components are included in the following discussion.

2.1 Coordinate Systems

We aim to use three-dimensional Cartesian coordinate system for the convenience way to adapt or combine the proposed element in this study to the other elements in further study, although it may certainly unnecessary for the flat plate element. For example, a generalized shell element can be achieved by combining plate and membrane elements, but they have to formulate under the conforming coordinate system. Then, the three coordinate systems shown in Figure 1 to be used in the proposed element formulation are defined as follows:

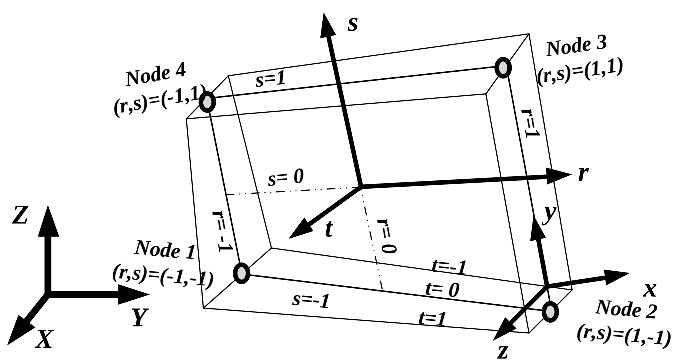

Figure 1. Coordinate system and geometric definition.

The geometry of the proposed element is defined by the natural coordinates (r, s, t). The global coordinates (X, Y, Z) of the mid-surface node and corresponding directional thickness are input to define the element geometry. The local coordinate (x, y, z) of any point i on mid-surface of the element can then be expressed in terms of nodal local coordinates (x_i, y_i) and nodal thickness h_i.

2.2 Finite Element Discretization

In the development of a mixed finite element for Reissner – Mindlin plate theory, the transverse displacement $w$on the mid-plane and the rotations θ are interpolated by the following form (Zienkiewicz *et al.* 1993; Taylor and Auricchio 1993).

$$w = N_w\bar{w} + N_{w\theta}\bar{\theta} = \sum_{i=1}^{4} N_w^i \bar{w}^i + \sum_{k=1}^{4} N_{w\theta}^k (\theta_k^2 - \theta_k^1) h_k \quad (1)$$

Where i is the node number, k is the side number, h_k is the length of side $k$and θ_k^1, θ_k^2 are the rotation in the tangential direction of side k at its two ends. N_w^i is the simple bilinear interpolation, and $N_{w\theta}^k$ are linking shape function, which can be express as (Jirousek *et al.* 1995):

$$N_w^i = \frac{1}{4}(1 + r_i r)(1 + s_i s) \qquad \text{for } i = 1, 2, 3, 4 \quad (2)$$

$$N_{w\theta}^{12} = \frac{1}{2}(1 - r^2)(1 - s) \quad N_{w\theta}^{23} = \frac{1}{2}(1 - s^2)(1 + r) \quad (3a)$$

$$N_{w\theta}^{34} = \frac{1}{2}(1 - r^2)(1 + s) \quad N_{w\theta}^{41} = \frac{1}{2}(1 - s^2)(1 - r) \quad (3b)$$

2.3 Elimination of Transverse Shear Locking

The shear strain components in the local coordinate system can be written as:

$$\bar{\gamma} = \begin{bmatrix} \gamma_{13} \\ \gamma_{23} \end{bmatrix} = \begin{bmatrix} w,_x + \theta_y \\ w,_y - \theta_x \end{bmatrix} \quad (4)$$

To deal with the problem more effectively, the element-based natural strain components (Kanok-Nukulchai and Wong 1987 and 1988) are introduced, i.e.,

$$\begin{bmatrix} \tilde{\gamma}_{13} \\ \tilde{\gamma}_{23} \end{bmatrix} = \begin{bmatrix} x,_r & y,_r \\ x,_s & y,_s \end{bmatrix} \begin{bmatrix} \gamma_{13} \\ \gamma_{23} \end{bmatrix} = \begin{bmatrix} w,_r \\ w,_s \end{bmatrix} + \begin{bmatrix} x,_r & y,_r \\ x,_s & y,_s \end{bmatrix} \begin{bmatrix} \theta_y \\ -\theta_x \end{bmatrix} = \begin{bmatrix} w,_r \\ w,_s \end{bmatrix} + \begin{bmatrix} \tilde{\theta}_r \\ \tilde{\theta}_s \end{bmatrix} \quad (5)$$

The transverse shear strain $\tilde{\gamma}$ should vanish when the length/thickness ratio is very large or equivalently. In numerical procedure, this requirement will enforce the coefficient of higher order terms in $\tilde{\theta}_r$ and$\tilde{\theta}_s$ to be zerowhen they should not be. Employing the concept of desirable displacement field (Ma and Kanok-Nukulchai, 1989), w is approximated that all terms presented in the expression of $\tilde{\theta}_r$ and$\tilde{\theta}_s$have their counterparts in the expression of $w,_r$ and $w,_s$. Thus, the order of the desirable displacement field $\tilde{w}$ should be selected that $w,_r$ (or $w,_s$) includes all polynomial terms in pair with $\tilde{\theta}_r$ (or$\tilde{\theta}_s$) in Eq. (5). Then sampling points can be obtained corresponding to the locations, where:

$$\tilde{w},_r = w,_r \, ; \tilde{w},_s = w,_s \quad (6)$$

To start with, the expression for $\tilde{\gamma}_{13}$ is investigated. Terms included in expression for related fields are separated into two parts, one for standard bilinear shape function N_w and the other for additional shape function $N_{w\theta}$.

If we put the arbitrary constants (assumed as c_{ij}) into the desirable deflection terms in $\tilde{w}$ which include all polynomial expression, the derivative terms are also obtained as follows:

$$\tilde{w} = c_{00} + c_{10}r + c_{01}s + c_{20}r^2 + c_{11}rs + c_{02}s^2 + c_{21}r^2s + c_{12}rs^2 + c_{22}r^2s^2 \quad (7)$$

$$\tilde{w}_{,r} = \quad c_{10} + 2c_{20}r + c_{11}s + 2c_{21}rs + c_{12}s^2 + 2c_{22}rs^2 \quad (8)$$

If we put the arbitrary constants (assumed as a_{ij}) into the first part of transverse displacement field ($w = \sum_{i=1}^{4} N_w^i \overline{w}^i$), the bilinear approximation of deflection can be written as:

$$w = a_{00} + a_{10}r + a_{01}s + a_{11}rs \quad (9)$$

If we put the arbitrary constants (assumed as b_{ij}) into the additional part of transverse displacement field which includes linking shape function $N_{w\theta}$, terms of displacement may be expressed as

$$w = b_{00} + b_{10}r + b_{01}s + b_{20}r^2 + b_{02}s^2 + b_{21}r^2s + b_{12}rs^2 \quad (10)$$

Using the desirable displacement field concept from Eq. (6), the following four conditions at each node can be used to determine the locations of the optimal sampling points.

$$\tilde{w}_1 = w_1 \quad : \quad \tilde{w}_2 = w_2 \quad : \quad \tilde{w}_3 = w_3 \quad : \quad \tilde{w}_4 = w_4 \quad (11)$$

For assumed arbitrary value and some steps of mathematical manipulation, the solutions of above conditions are r = 0, s = 1 and r = 0, s = -1. Thus, two points, i.e., (0,1) and (0,-1) in the natural coordinate system, have been located where the derivatives of the assumed deflection field with respect to r is always equal to that of the desirable deflection field, provided that the nodal value of the two are equal. Therefore, this is the most suitable location for expression of the shear strain $\tilde{\gamma}_{13}$ in an element and therefore, these two points are considered to be optimal for shear strain sampling.

In the same way, the desirable deflection field for expression of $\tilde{\gamma}_{23}$ can be constructed, and two optimal points, (1,0) and (-1,0), can also be located for sampling this shear strain component. For shear strain component, no matter whether the element is a parallelogram or and arbitrary quadrilateral then, the same sampling points as those just located.

3 NUMERICAL RESULTS

A clamped rectangular plate problem, shown in Figure 2, is tested using the present element to compare with exact benchmark solution from literature (MacNeal and Harder 1985) to assess the relative accuracy and convergence. Two aspect ratios of one and five, i.e., $b/a = 1$ and 5, are considered.

Figure 3 shows that the proposed element is very good for rectangular plate of aspect ratio of 1. Also the good results can be obtained for the case of high aspect ratio of 5 when the mesh reaches to some state of refinement.

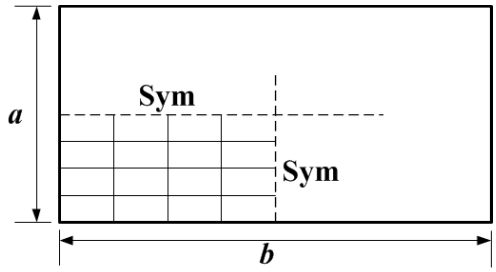

Figure 2. Clamped plate subjected to a center force. $a = 2; b = 2\ or\ 10$; $E = 1.7472 \times 10^7$; $\nu = 0.3$. The center concentrated load = 4.0× 10^{-4}.

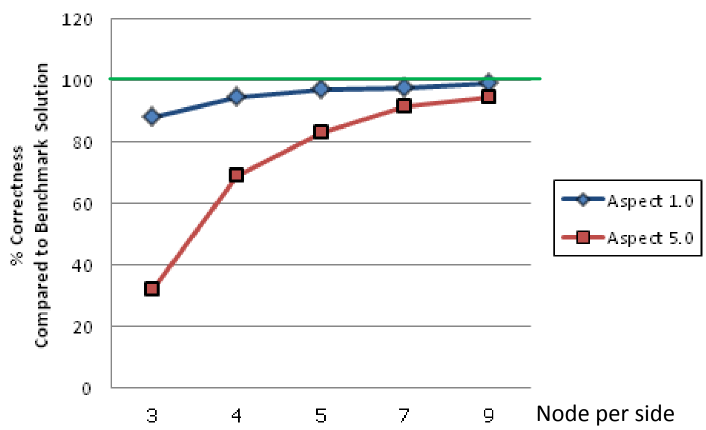

Figure 3. Normalized solution for plate bending problem.

4 CONCLUSIONS

This study has considered a plate element based on assumed strain method and mixed formulation using linked interpolation. The optimal sampling points can be located by the desirable displacement field concept. The process leads to reasonably good four-node quadrilateral plate element. This element can be further developed to get a more degree of freedom at each vertices by combining or assembling of finite element stiffness matrix with the other element. However, the performance of the developed element has to be tested further in future studies under the different circumstances and including mesh distortions as the distorted mesh may have the locking phenomena.

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