Implementing Innovative Ideas in Structural Engineering and Project Management
Edited by Saha, S., Zhang, Y., Yazdani, S., and Singh, A.

ISBN: 978-0-9960437-1-7

EXPLICIT PROBABILISTIC DEMAND AND CONSEQUENCE MODELS FOR SEISMIC RISK PREDICTION

MEHDI KIA and MEHDI BANAZADEH

Dept of Civil Engineering, Amirkabir University of Technology (Tehran Polytechnic), Tehran, Iran

This paper presents new probabilistic models intended for use with reliability methods for computing seismic risk on the component scale. In the present context, risk is defined as the probability of attaining or exceeding a specific level of monetary loss. The models are classified as explicit story-specific demand model and consequence models. The demand model predict maximum drift at the level of each story of low-rise to mid-rise steel moment resisting frames (SMRFs) and, consequence models estimate repair cost of some fragility groups in a building. A linear function of spectral acceleration at fundamental period in logarithmic space is considered for demand model. The demand model is coupled with a set of relations to explicitly estimate unknown statistical characteristics of the probabilistic demand model parameters. The consequence models are also formulated as a polynomial function respect to Engineering Demand Parameter in logarithmic space. Bayesian regression technique is employed to determine probability distribution of the consequence models parameters. Finally, as an application of the proposed models, seismic loss curve of an example building is developed for earthquake intensities.

Keywords: Earthquake, Uncertainty, Probabilistic, Reliability, Loss, Probabilistic models.

1 INTRODUCTION

Performance-Based Earthquake Engineering (PBEE), which is known as cornerstone of the modern earthquake engineering, tries to estimate the performance of structures over the full range of the probable structural response, from elastic to global instability against a likely range of ground-motion excitations. Performance has been introduced in various forms in literatures, but one of the most sophisticated forms is to be defined as an environmental, economical and societal consequence measure. This definition provides a framework which makes it possible to answer the question "how safe is safe?" Indeed, it implies making decisions that balance construction cost and safety, instead of only calibrating performance against what is deemed acceptable in the past. Generally, there are two different formulations for above definition. The first formulation referred to as Pacific Earthquake Engineering Research (PEER) center methodology employs total probability theorem as the basis to quantify building performance in terms of a decision variable (Cornell *et al.* 2000). The second method adopted reliability analysis as a foundation on which performance assessment can be based (Haukaas 2008). Since the essence of both methods is the probability concept, the assessment outcome, the probability a specific loss threshold occurs, could be the basis for decision maker involved in the investment and construction sector. However, authors believe that reliability framework has some interesting characteristics making it appealing for practical purposes. The main characteristics are: (1) The ability to explicitly yet effortlessly address uncertainties arise from randomness and lack of knowledge, (2) Reliability methods, especially gradient based methods, are tailor to compute probability on the tail of loss curve, where the probability is low but loss is high. Therefore, in this study, attention is directed towards reliability based approach. In a classical structural reliability analysis, the failure probability of a single component is evaluated by the integral (Haldar *et al.* 2000):

$$P_f = \int_{g(X)<0} \cdots \int f_X(x)dX \tag{1}$$

Where P_f is the failure probability, $X = (X_1, X_2, \ldots. X_n)$ is the vector of random variables, $f_X(x)$ is the joint probability density function (PDF) of X and $g()$ is a limit-state function that defines the performance event for which the probability is being assessed. Although the probability integration, i.e. Eq. (1), cannot be solved analytically, several reliability methods are introduced to numerically estimate the probability integration, including First/Second Order Reliability Method (FORM and SORM) and sampling technique. The foundation of these methods is based on simulating out-come space by computing the value of limit-state function for each realization of random variables. It must be considered that the required number of realizations depends on the applied reliability method. However in comparison to simulation techniques, the gradient-based methods (FORM/SORM) require remarkably less realizations. According to the above brief description, if reliability methods are intended to be implemented in a seismic risk assessment study, either with simulation or gradient-based techniques, limit-state function addressing the desired failure event should be defined first. In this paper, the following general folded limit-state function as the mean to address the problem of seismic risk prediction on the component scale is discussed and proposed for steel moment frames:

$$g(\ ) = dv - \sum DV(\theta_{dv}, EDP(\theta_d, IM)) \tag{2}$$

Where DV is a decision variable in the form of repair cost, dv is a threshold introduced by analyst for a target performance, EDP is an engineering demand parameter, θ is a vector of a model parameters, and IM is a random variable which reflects uncertainties embedded in seismic excitations and known as the earthquake intensity measure in literatures. To complete limit-state function in the form of Eq. (2), it needs developing modularized demand and consequence models. A modularized model can takes input from upstream model and/or return desired output to downstream models. According to above discussion, the main objective of the present study would be summarized to serve new probabilistic demand and consequence models to be employed in the codified and practical seismic risk evaluations. All these models are developed using Bayesian inference to explicitly consider statistical uncertainty, arising due to use of finite-size sample population, as well as aleatory uncertainty.

2 PROBABILISTIC STORY-DRIFT MODELS

Probabilistic drift demand models are specifically developed for a typical structure based on data obtained from numerous nonlinear response history analyses and/or experimental tests, when seismic vulnerability assessment is performed. Although, this methodology can be acceptable for research papers, it is not appealing for practical purpose because of its computational cost. Thus, in this paper, generic story-specific demand model with linear formulation in logarithmic space is proposed to predict maximum drift demand for each story of the low-rise to mid-rise multi-story steel moment resisting frames. Developing demand model in the logarithmic space approximately satisfies the normality (i.e., model error has normal distribution) and homoscedasticity assumptions (i.e., Standard deviation of model error is constant). Eq. (3) exhibits general form of story-drift demand model developed in the present study

$$Ln[D(Sa(T_1),\theta)] = a + bLn(Sa(T_1)) + \sigma\varepsilon \quad (3)$$

Where $Ln[D(Sa(T_1),\theta)]$ is a response that the model predicts and equal to natural logarithm of the maximum story drift, $\theta = (a, b, \sigma)$ is a vector of unknown normal parameters and $(Sa(T_1))$ is spectral acceleration at fundamental period reflecting uncertainties in seismic excitation and defined in terms of gravity coefficient g. In addition, Eq. (3) is linked with a set of relations in terms of building characteristics to compute unknown statistical characteristics of the model parameters. In the following, because of limitation put on the number of pages, only those relations applied to compute mean and standard deviation of the model parameter a is presented. Although, a full set of relations would be available upon request through authors email address. In developing these relations, a comprehensive structural data based is established by performing Incremental Dynamic Analysis (IDA) on 81 generic moment resisting frames under 82 suitably multiply-scaled ground motion records. The concept of generic moment frame is not new, and has been utilized by various researchers to evaluate seismic performance of moment resisting frames (Ruiz-Garcia *et al.* 2010, Medina *et al.* 2004, Chintanapakdee *et al.* 2003, Esteva *et al.* 1989, Zareian *et al.* 2006). Thus, in the present study, a family of three-bay generic moment frames, the details of such a family of generic moment frames are presented in Zareian *et al.* (2006), is implemented to simulate story-drift of steel moment resisting frames (SMRFs).

Mean and Standard deviation of a :

$$\mu_a = \alpha_1 \frac{T \times CY^2 \times ST\_Num^2}{N} + \alpha_2 \frac{1}{T \times CY} + \alpha_3 \frac{T \times ST\_Num^2}{SSD} + \alpha_4 .CY + \alpha_5 .ST\_Num + \sigma\varepsilon \quad (4)$$

$$\sigma_a = \alpha_1 .N + \alpha_2 \frac{1}{N \times T^2 \times SSD \times CY^2} + \alpha_3 \frac{T^2}{N^2 \times CY^2 \times ST\_Num} + \alpha_4 \frac{1}{N \times T \times ST\_Num} + \alpha_5 N^2 + \sigma\varepsilon \quad (5)$$

Where T is structural fundamental period determined in accordance with ASCE-7-10, Section 12.8.2 (i.e. $C_u T_a$), N denotes number of stories, and ST_Num represents story number. *CY* indicates yield base shear coefficient and equals V_y/W, where W represents effective seismic weight and V_y is yield base shear strength. V_y may be obtained from pushover analysis or estimated by multiplying design value of the seismic base shear by over-strength factor. Moreover, *SSD* is a numerical index for beam stiffness and strength distributions over the height of structure and varies from 1 to 3. In this paper, based on interpolation technique and averaging over the height, following equation is suggested to calculate *SSD* value:

$$SSD = \frac{\sum_{i=2}^{N}\left(1+2\frac{\left(\frac{I_i}{I_1}-\frac{V_i}{V_1}\right)}{\left(1-\frac{V_i}{V_1}\right)}\right)}{N-1} \tag{6}$$

Where I indicates beam moment of inertia and V represents story shear when subjected to ASCE-7-10 lateral load pattern. i and 1 indicate specific story and the first story respectively. Table 1 shows posterior statistics of the unknown parameters α_1 of Eq. (4) to Eq. (5).

Table 1. Posterior statistics of the unknown model parameters.

		α_1	α_2	α_3	α_4	α_5	σ
μ_a	Mean	7.05E-01	-3.33E-01	2.23E-02	-5.62E+00	-4.39E-01	2.77E-01
	Standard deviation	7.12E-02	7.10E-03	1.23E-03	8.01E-02	1.15E-02	8.96E-03
σ_a	Mean	2.76E-03	4.12E-04	6.16E-03	2.77E-02	-1.45E-04	2.73E-03
	Standard deviation	1.60E-04	5.56E-05	3.00E-04	1.34E-03	2.07E-05	8.84E-05

Of course, for simplicity, someone can pass up epistemic uncertainty on the proposed relations. To this end, only mean value of the coefficients of the relations requires to be considered. Otherwise, one can suppose normal marginal distribution for α_1 to reflect epistemic uncertainty on the proposed relations. This assumption is supported with the fact that t-distribution asymptotically approaches a normal distribution when the number of data is large. With these equations, story-specific demand models can be directly developed for a designated SMRF. For this purpose, it is only required defining demand models in the form of Eq. (3). Then, unknown statistical characteristics of the model parameters are calculated using proposed relations.

3 CONSEQUENCE MODELS

In this part of the study the probabilistic consequence models are developed to describe relation between structural demand parameters, i.e., the maximum drift at the level of each story, and unit repair cost of 25 fragility groups of a building. To develop consequence model for each component, a comprehensive database should be initially developed. In generation of such a database, fragility models with cost functions presented in FEMA P-58 (FEMA 2012) are used. A simulation based- procedure is applied to develop database. This method is a three-stage process that starts by random generation of structural demand. Next, for a given structural demand and using fragility model introduced in FEMA (2012), the probability that the component is in a specific damage-state for a given value of demand parameter $P(DS_i | D)$ is computed as:

$$P(DS_i | D) = F(DS_i | D) - F(DS_{i+1} | D) \tag{7}$$

Where F $(DS_i | D)$ represents the probability that damage to the component exceeds i^th^ damage-state for a given demand parameter. Then, for a given damage-state, a repair cost value is randomly produced with consideration of uncertainty in the cost data proposed in FEMA (2012). The generated cost-value multiplies by the probability obtained in the second stage. This step is repeated for all damage-states considered for the component and the results are summed to compute total repair cost for a given structural demand. It is noted that the cost values presented in FEMA (2012) are valid for California in 2011. Thus, to keep the generality of the consequence models, a non-dimensional quantity λ is suggested to be used instead of repair cost in consequence models. This quantity is defined as the ratio of obtained total repair cost to the mean value of repair cost for the most severe damage state defined for the component. According to developed database, a fourth-order polynomial function respect to demand parameter is suggested to express consequence models. Similar to story-specific demand models, consequence models are also developed in logarithmic space to satisfy normality and homoscedasticity assumption.

$$Ln(\lambda) = \alpha_1 + \alpha_2\, Ln(D) + \alpha_3\, Ln^2(D) + \alpha_4\, Ln^3(D) + \alpha_5\, Ln^4(D) + \sigma.\varepsilon \tag{8}$$

Once the initial model has been developed, a stepwise deletion procedure proposed in Gardoni (2002) is employed to reduce number of terms in the models to compromise between model simplicity and model accuracy. For an example, in the following the Post-Northridge steel connection is presented.

$$\begin{aligned} Ln(\lambda) = \alpha_1 &+ (0.8753\alpha_1 + 4.895)\, Ln(D) + (0.24687\alpha_1 + 3.5286) Ln^2(D) \\ &+ (0.02245\alpha_1 + 0.6332) Ln^3(D) + \sigma.\varepsilon \end{aligned} \tag{9}$$

The integration of these two classes of proposed models in the form of Eq. (2) presents a closed-form generic limit state function which is continuously differentiable. Hence, the limit state function could be specifically intended for use with First Order Reliability Method (FORM) to compute seismic risk probability. This makes the run-time requires drawing a loss curve out dramatically reduce. In fact, the main vision behind this paper is to balance between accuracy and computational cost of a seismic risk analysis which could be appealing for practical purposes. As an application of the proposed model, seismic risk analysis for an example building was done at earthquake intensity equals 1.0g. The example building is a four story-building designed with respect to American Institute of Steel Construction (AISC) specifications and satisfies all seismic requirements of ASCE-7-10. For simplicity, only Post-Northridge steel connection is considered as a fragility group in the seismic loss. Fig (1) demonstrates loss curve of the example building.

Table 2. Posterior statistics of the consequence models parameters.

	α_1	σ
Mean	9.622E+00	2.539E-01
Standard Deviation	2.872E-02	2.000E-04

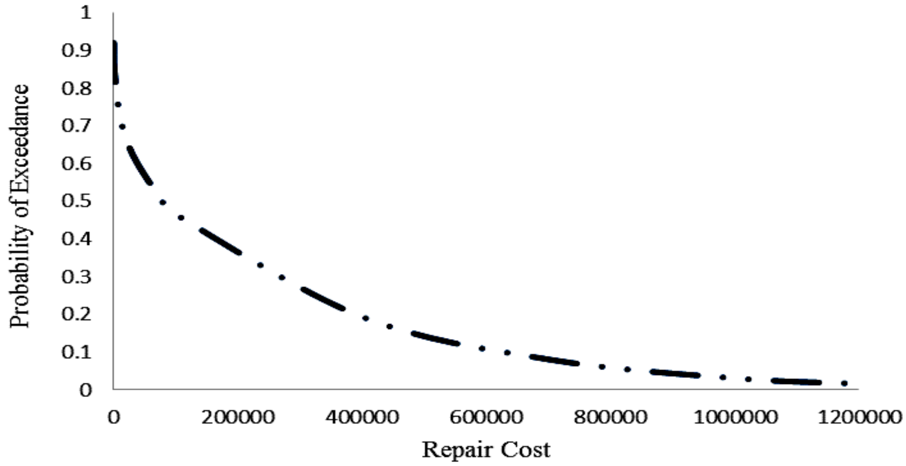

Figure 1. Loss curves of the example building at different hazard levels.

References

Chintanapakdee, C., and Chopra, A.K., Evaluation of modal pushover analysis using generic frames, *Earthquake engineering & structural dynamics*, 32(3), 417-442, 2003.

Cornell, C.A., and H. Krawinkler, Progress and challenges in seismic performance assessment. *PEER Center News*, 3(2), 1-3, 2000.

Esteva, L., and Ruiz, S.E., Seismic failure rates of multistory frames, *Journal of Structural Engineering*, 115(2), 268-284, 1989.

Federal Emergency Management Agency, FEMA P-58, Seismic performance assessment of buildings, volume 1-methodology,(2012).

Gardoni, P., Der Kiureghian, A., and Mosalam, K.M., Probabilistic capacity models and fragility estimates for reinforced concrete columns based on experimental observations, *Journal of Engineering Mechanics*, 128(10), 1024-1038, 2002.

Haldar, A., and Mahadevan, S., *Probability, reliability, and statistical methods in engineering design*, John Wiley & Sons, Incorporated, 2000.

Haukaas, T., Unified reliability and design optimization for earthquake engineering, *Probabilistic Engineering Mechanics*, 23(4), 471-481, 2008.

Medina, R.A., and Krawinkler, H., Seismic demands for non-deteriorating frame structures and their dependence on ground motions, 2004, Pacific Earthquake Engineering Research Center.

Ruiz-García, J., and Miranda, E., Probabilistic estimation of residual drift demands for seismic assessment of multi-story framed buildings, *Engineering Structures*, 32(1), 11-20, 2010.

Zareian, F., and Krawinkler, H., Simplified performance-based earthquake engineering, Stanford University, 2006.